On the Physical Relevance of the Study of $\gamma^\star\gamma \to \pi^0\pi^0$ at Small t and Large Q^2

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We discuss the relevance of a dedicated measurement of exclusive production of a pair of neutral pions in a hard $\gamma^\star\gamma$ scattering at small momentum transfer. In this case, the virtuality of one photon provides us with a hard scale in the process, enabling us to perform a QCD calculation of this reaction rate using the concept of Transition Distribution Amplitudes (TDA). Those are related by sum rules to the pion axial form factor F_A^π and, as a direct consequence, a cross-section measurement of this process at intense beam electron-positron colliders such as CLEO, KEK-B and PEP-II, or Super-B would provide us with a unique measurement of the neutral pion axial form factor $F_A^{\pi^0}$ at small scale.

1 Introduction

In a series of papers [1, 2, 3, 4, 5, 6, 7, 8], we have advocated that factorisation theorems [9] for exclusive processes may be extended to the case of other reactions such as (M_i stands for a meson and B_i for a baryon) $B_1\,\overline{B}_2 \to \gamma^\star\,\gamma$, $B_1\,\overline{B}_2 \to \gamma^\star\,M_1$ $\gamma^\star_T\,B_1 \to B_2\gamma$, $\gamma^\star_T\,B_1 \to B_2M_1$ or $\gamma^\star_L\,\gamma \to M_1M_2$, in the kinematical regime where the off-shell photon is highly virtual (Q^2 of the order of the energy squared of the reaction) but the momentum transfer t is small. This enlarges the successful description of deep-exclusive $\gamma\gamma$ reactions in terms of distribution amplitudes [10] and/or generalised distribution amplitudes [11] on the one side and perturbatively calculable coefficient functions describing hard scattering at the partonic level on the other side.

Intense beam electron colliders, such as B factories, are ideal places to study such reactions as

$$\gamma^\star_L\,\gamma \to \rho^\pm\pi^\mp,\ \gamma^\star_L\ \ \gamma \to \pi^\pm\pi^\mp,\ \gamma^\star_L\gamma \to \pi^0\pi^0,$$

in the near forward region and for large virtual photon invariant mass Q. Recently BaBar reported a new measurement of the reaction $\gamma^\star\gamma \to \pi^0$ up to photon virtualities squared of 40 GeV^2 [12]. In the latter study, the reaction $\gamma^\star\gamma \to \pi^0\pi^0$ was investigated in the $f_2(1270)$ and $f_0(980)$ resonance region as a potential background for the study of the π^0 transition form factor. This low-$W^2_{\pi\pi}$ kinematical region should be analysed in the framework of generalised two-meson distribution amplitudes [11] and in particular should solve the much discussed problem of its phase structure around the f_0 mass [13] which is of crucial importance for the ability to detect Pomeron-Odderon interference effects in high energy electro-production of meson pairs [14].

We want here to emphasise another kinematical region, namely the small-t large-$W^2_{\pi\pi}$ region at moderate Q^2 (2 GeV^2 and more) which may provide us with unique information on the π^0 axial form factor at small scale which so far has never been experimentally measured. It has been argued that a new duality [15] relates these two factorisation regimes.

In principle, another possibility to study this quantity would be the crossed channel, that is DVCS on a virtual neutral pion along the lines exposed in Ref. [16] for π^+.

2 Pion-pair production in the TDA regime

Let us recall the main ingredients of the analyses developed in [1, 4] focusing on the neutral pion case. With the kinematics described in Fig. 1, we define the axial $\gamma \to \pi$ transition distribution amplitude (TDA) $A(x, \xi, t)$ as the Fourier transform of matrix element $\langle \pi^0(p_\pi)| \mathcal{O}_A |\gamma(p_\gamma)\rangle$ where $\mathcal{O}_A = \bar\psi(\frac{-z}{2})[\frac{-z}{2}, \frac{z}{2}]\, \gamma^\mu \gamma^5 \psi(\frac{z}{2})$. The Wilson line $[\frac{-z}{2}, \frac{z}{2}]$ ensures the QCD-gauge invariance for non-local operators and equals unity in a light-like (axial) gauge. We do not write the electromagnetic Wilson line, since we choose an electromagnetic axial gauge for the photon. We then factorise the amplitude of the process $\gamma^\star_L \gamma \to \pi^0\pi^0$ as

$$\sum_{q=u,d} \int dx dz\, \Phi^q_\pi(z) M^q_h(z, x, \xi) \frac{A^{\pi^0}_q(x, \xi, t)}{f_\pi}\ , \tag{1}$$

with a hard amplitude $M^q_h(z, x, \xi)$, $\Phi^q_\pi(z)$ the distribution amplitude (DA) for q quark content of the π meson with momentum p'_π and $A^{\pi^0}_q(x, \xi, t)$ the axial $\gamma \to \pi$ TDA for the quark q.

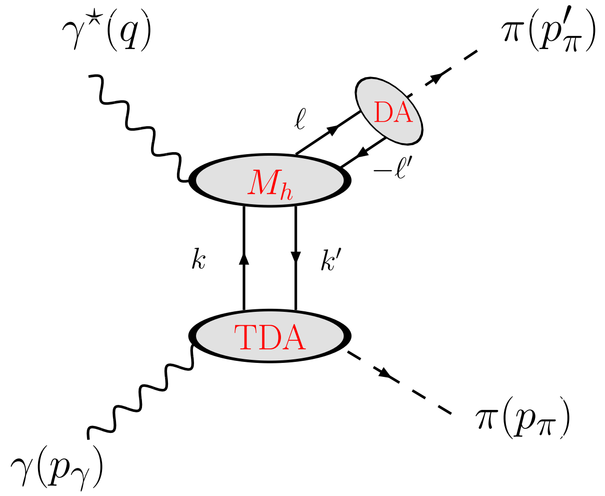

Figure 1: The factorised amplitude for $\gamma^\star\gamma \to \pi^0\pi^0$ at small transfer momentum.

The variable z is as usual the light-cone momentum fraction carried by the quark entering the pion with momentum p'_π, $x + \xi$ (resp. $x - \xi$) is the corresponding one for the quark leaving (resp. entering) the TDA. The skewness variable ξ describes the loss of light-cone momentum of the incident photon and is connected to the Bjorken variable x_B.

Contrarily to the case of generalised parton distributions (GPD) where the forward limit is related to the conventional parton distributions measured in the deep inelastic scattering (DIS), there is no such interesting constraints for the TDAs. The constraints we have here are sum rules obtained by taking the local limit of the corresponding operators and soft limits when the momentum of the meson in the TDA vanishes.

Let us consider in more detail the $\gamma \to \pi^0$ axial TDAs which is defined by ($P = \frac{p_\pi + p_\gamma}{2}$, $\Delta = p_\pi - p_\gamma$):

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi^0 | \bar{q}(\frac{-z}{2}) \Big[\frac{-z}{2}; \frac{z}{2}\Big] \gamma^\mu \gamma^5 q(\frac{z}{2}) | \gamma \rangle = \frac{1}{P^+} \frac{e}{f_\pi} (\varepsilon \cdot \Delta) P^\mu A_q^{\pi^0}(x, \xi, t) \tag{2}$$

A sum rule may be derived for this photon to meson TDA by integrating on x both side of Eq. (2) and we get

$$\frac{e}{f_\pi} (\varepsilon \cdot \Delta) P^\mu \int_{-1}^{1} dx \; A_q^{\pi^0}(x, \xi, t) = \langle \pi^0 | \bar{q}(0) \gamma^\mu \gamma^5 q(0) | \gamma \rangle, \tag{3}$$

The latter matrix element of a local quark–anti-quark operator is directly related to the quark q contribution $F_{A,q}^{\pi^0}$ to the axial form factor of the π^0 meson. Similarly, we have in the vector charged pion case [4]:

$$\int_{-1}^{1} dx \, V^{\pi^\pm}(x, \xi, t) = \frac{f_\pi}{m_\pi} F_V^{\pi^\pm}(t), \tag{4}$$

with $F_V^{\pi^\pm} = 0.017 \pm 0.008$ [17].

This sum rule constrains possible parametrisations of the TDAs. Note, in particular, the ξ-independence of the right hand side of the relation.

3 Models and cross section evaluation

3.1 Amplitude

Let us thus consider the $\pi^0\pi^0$ production case when the π^0 with momentum p'_π flies in the direction of the virtual photon and the other π^0 emerges from the TDA. For definiteness, we choose, in the CMS of the meson pair, $p = \frac{Q^2 + W_{\pi\pi}^2}{2(1+\xi)W_{\pi\pi}}(1, 0, 0, -1)$ and $n = \frac{(1+\xi)W_{\pi\pi}}{2(Q^2 + W_{\pi\pi}^2)}(1, 0, 0, 1)$ and we express the momenta through a Sudakov decomposition (with $\Delta_T^2 = \frac{1-\xi}{1+\xi} t$ and neglecting the pion mass):

$$p_\gamma = (1+\xi)p, \quad q = \frac{Q^2 + W_{\pi\pi}^2}{1+\xi} n - \frac{Q^2}{Q^2 + W_{\pi\pi}^2}(1+\xi)p, \quad p_\pi = (1-\xi)p - \frac{\Delta_T^2}{1-\xi} n + \Delta_T. \tag{5}$$

We can see that ξ is determined by the external kinematics through $\xi \simeq \frac{Q^2}{Q^2 + 2W_{\pi\pi}^2}$ – similarly to $x_B = \frac{Q^2}{Q^2 + W_{\pi\pi}^2}$ to which it is linked via the simple relation $\xi \simeq \frac{x_B}{2 - x_B}$.

The hard amplitude amplitude in Eq. (1) thus reads :

$$M_h^q(z, x, \xi) = \frac{4\,\pi^2\, \alpha_{em}\, \alpha_s\, C_F\, Q_q}{N_C\, Q} \frac{1}{z\,\bar{z}} \left(\frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right) \varepsilon \cdot \Delta\,, \tag{6}$$

where $Q_u = 2/3$, $Q_d = -1/3$ and with $\bar{z} = 1 - z$. Note that the factor f_π in the π DA $\Phi^q_\pi(z)$ cancels with the one from the TDA definition and does not appear in Eq. (7). Now, if we choose the asymptotic form for the neutral pion DA, $\Phi^u_{\pi^0}(z) = -\Phi^d_{\pi^0}(z) = \frac{6f_\pi}{\sqrt{2}} z(1-z)$, the z-integration is readily carried out and after separating the real and imaginary parts of the amplitude, the x-integration gives:

$$\mathcal{I}^A_x = \frac{1}{\sqrt{2}} \sum_{q=u,d} |Q_q| \int_{-1}^{1} dx \left(\frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right) A^{\pi^0}_q(x,\xi,t) = \tag{7}$$

$$\frac{1}{\sqrt{2}} \sum_{q=u,d} |Q_q| \Bigg[\int_{-1}^{1} dx \, \frac{A^{\pi^0}_q(x,\xi,t) - A^{\pi^0}_q(\xi,\xi,t)}{x-\xi} + A^{\pi^0}_q(\xi,\xi,t)(\log\left(\frac{1-\xi}{1+\xi}\right) - i\pi) +$$

$$\int_{-1}^{1} dx \, \frac{A^{\pi^0}_q(x,\xi,t) - A^{\pi^0}_q(-\xi,\xi,t)}{x+\xi} + A^{\pi^0}_q(-\xi,\xi,t)(\log\left(\frac{1+\xi}{1-\xi}\right) + i\pi) \Bigg].$$

The scaling law for the amplitude is

$$\mathcal{M}^{TDA}_{\gamma^\star\gamma}(Q^2,\xi,t) \sim \frac{\alpha_s \sqrt{-t}}{Q} \,, \tag{8}$$

up to logarithmic corrections due to the anomalous dimension of the TDA and the running of α_s.

3.2 Remarks on available models

Lacking any non-perturbative calculations of matrix element defining TDAs, we have initially built a toy model [4] based on double distributions [18] to get estimates for the cross sections, to be compared with experimental data. In [4], we compared the rate obtained with this model with the one from the model built in [19]. Subsequently, a model based on quark spectral representation was developed in [20], another based on NJL model was studied in [21, 22] and lastly the $\pi \to \gamma$ TDAs were studied in a non-local chiral quark model [23]. All the models (see e.g. [24]) used so far for the pion GPDs could be extended to the construction of $\pi \to \gamma$ TDAs. We refer to the different references for details. For illustration, we show here on Fig. 2 the TDA $A(x,\xi,t)$ obtained in Ref. [4] in arbitrary unit; its normalisation would be eventually fixed by the experimental data.

For the purpose of this note, we only need a rough evaluation of the order of magnitude of the cross section and will only use the Model 1 of Ref. [4]. When a dedicated experimental analysis is being carried out, a careful survey of the cross sections obtained from the different theoretical models will be in order. Hence, based on a first experimental study of the ξ dependence and after having checked the scaling in Q^2, we shall be in position to see which model describes best the physics involved. For this best model, we could then obtain by sum rules relations a first measurement of the axial π^0 form factor.

For the following, we shall show results for $\langle \pi^0 | \bar{d} \mathcal{O}_A d | \gamma \rangle = -1/2 \langle \pi^0 | \bar{u} \mathcal{O}_A u | \gamma \rangle$ expected from the different charges of the u and d quarks and using (from isospin arguments)

$$\langle \pi^+ | \bar{d} \mathcal{O}_A u | \gamma \rangle = \langle \pi^0 | \bar{d} \mathcal{O}_A d | \gamma \rangle - \langle \pi^0 | \bar{u} \mathcal{O}_A u | \gamma \rangle. \tag{9}$$

This would give $A^{\pi^0}_d = 1/3 A^{\pi^+}$ and $A^{\pi^0}_u = -2/3 A^{\pi^+}$. Note that more realistic models may give significantly larger rates.

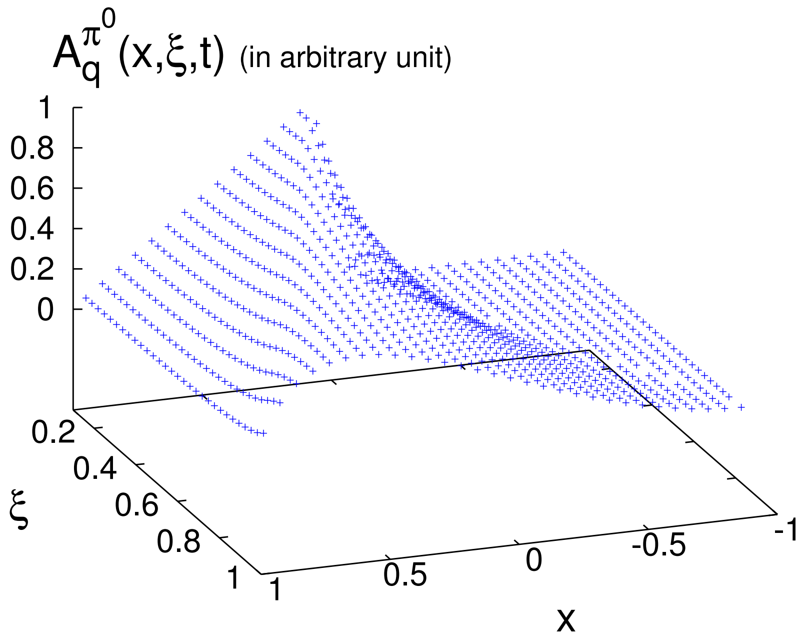

Figure 2: The $\gamma \to \pi^0$ axial transition distribution amplitude $A_q^{\pi 0}(x,\xi,t)$ in Model 1 of Ref. [4] (for $t = -0.5$ GeV^2) in arbitrary unit.

3.3 Cross section

Taking into account the contribution from the fermionic line for the emission by the electron of a longitudinal photon, averaging over the real photon polarisation and integrating over φ thanks to the φ-independence of the TDA process, we eventually obtain the differential cross section[1]:

$$\frac{d\sigma^{TDA}_{e\gamma\to e\pi^0\pi^0}}{dQ^2dtd\xi} = \frac{64\pi\alpha_{em}^3\alpha_s^2 2\pi}{9(\xi+1)^4Q^8}(-2\xi t)(1-\xi-(1+\xi)\frac{W^2_{\pi\pi}}{s_{e\gamma}})(\mathrm{Re}^2(\mathcal{I}^A_x)+\mathrm{Im}^2(\mathcal{I}^A_x)). \quad (10)$$

For the hypothesis discussed above, the resulting cross section is roughly one sixth of the one obtained in [4] for the charged case. The evolution as function of ξ is displayed on Fig. 3. Note that for small ξ (particularly $W^2_{\pi\pi} \to Q^2$), the cross section shows a peak.

The Q^2-behaviour is model independent and thus constitutes a crucial test of the validity of our approach.

[1] A factor 1/4 is missing in Eq.(23) of [4].

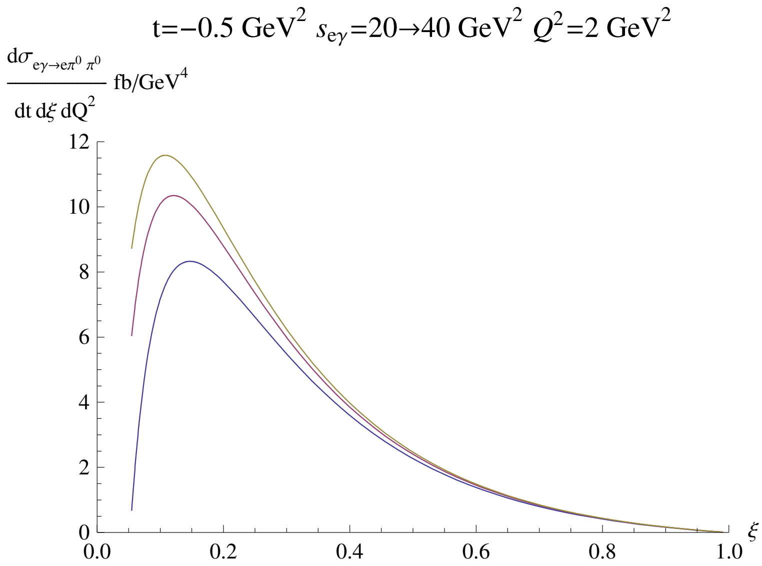

Figure 3: Differential cross sections $\frac{d\sigma_{e\gamma\to e\pi^0\pi^0}}{dQ^2dtd\xi}$ for the TDA subprocess as a function of ξ for $Q^2 = 2$ GeV^2, $t = -0.5$ GeV^2 and 3 values of $s_{e\gamma}$: 20, 30 and 40 GeV^2 (from bottom to top).

Conclusion

We believe that our models for the photon to meson transition distribution amplitudes are sufficiently constrained to give reasonable orders of magnitude for the estimated cross sections. Cross sections are large enough for quantitative studies to be performed at high luminosity e^+e^- colliders. After verifying the scaling and the φ independence of the cross section, one should be able to measure these new hadronic matrix elements, and thus open a new gate to the understanding of the hadronic structure. In particular, we argued here that the study of $\gamma^\star\gamma \to \pi^0\pi^0$ in the TDA regime could provide with a unique experimental measurement of the π^0 axial form factor.

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