Leading Theoretical Uncertainties in the Muon $g-2$

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A brief overview of the standard model (SM) prediction for the muon magnetic anomaly is given with main emphasis in the leading order (LO) hadronic contribution which provides at present the main source of uncertainty. Combining data on the 2π spectral functions from τ decays at Belle with previous measurements and using new calculations of the isospin breaking (IB) corrections we give a new determination of the LO term, which is closer to the one based on e^+e^- data. Further progress in understanding IB effects combined with more precise data on hadron production at e^+e^- machines can produce an accurate determination of the LO terms as required by future measurements of the muon magnetic anomaly.

1 Introduction

For a particle of charge e and mass m, its intrinsic magnetic dipole moment and spin vectors are related by $\vec{\mu} = g(e/2m)\vec{s}$. In a quantum field theory description, an elementary fermion has a gyromagnetic ratio $g = 2$; the quantum corrections of the self-interacting fermion naturally generates a magnetic anomaly $a \equiv (g-2)/2 \neq 0$. In the following we will be concerned with the magnetic anomaly of the muon which will be denoted by a_μ (for some comprehensive recent articles, see for example [1]).

The most precise measurements of the muon magnetic anomaly have been achieved by the BNL-E821 Collaboration in recent years (an account of previous measurements can be found in [1]). The current world average from positive and negative muons is $a_\mu^{exp} = 116592080(63) \times 10^{-11}$ [2], an impressive accuracy of 0.54 ppm. This experimental accuracy has prompted improved theoretical calculations in recent years, which include the effects of the three standard model interactions beyond the one-loop level. It becomes also sensitive to the effects expected from New Physics contributions. In view of current proposals aiming to reduce the experimental uncertainty up to $\pm 15 \times 10^{-11}$ [3], improved theoretical calculations are required to make a meaningful test of the SM and, eventually, to establish the existence of physics beyond it [3].

In this contribution we give an overview of the present SM calculation in the determination of the muon magnetic anomaly, where current uncertainties stems mainly from hadronic contributions. We focus on some recent progress in the evaluation of the LO hadronic contribution based on tau lepton and electron-positron data.

2 Brief summary of the SM prediction

It is customary to separate the theoretical calculation according to the contributions of the three SM interactions: $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$ [1]. The QED and electroweak EW contributions can be calculated from the first principles of perturbative field theory, whereas the hadronic contributions rely either on input data or tools to deal with strong interactions in the non-perturbative regime.

The calculation of the QED contribution involve only loops of leptons and photons. The result is known in analytic form up to the third order in α, in a numerical form at the fourth order and even the leading-logs of order α^5 have been evaluated (references to original works can be found in [1]). By using the most precise evaluation of the fine structure constant $\alpha^{-1} = 137.035999710(96)$ from the measured value of the electron anomalous magnetic moment [4] one obtains the following QED prediction (see for instance the most recent reviews in [1]):

$$a_\mu^{QED} = (116584718.09 \pm 0.14 \pm 0.04) \times 10^{-11} \quad (1)$$

where the first and second uncertainties stems from the error estimate of the order α^5 corrections and from the experimental uncertainty in the measured value of α, respectively.

The EW corrections involve loops with at least one weakly interacting boson. The one-loop corrections were calculated long ago and the two-loop corrections were completed until recently in Refs. [5]. Even some estimates of the leading-logs of third order have been reported in the literature [6]. The final numerical results reads:

$$a_\mu^{EW} = (154 \pm 2 \pm 1) \times 10^{-11} \ , \quad (2)$$

where the first error bar includes uncertainties in the Higgs boson and top quark masses and third order loop effects, while the second includes hadronic uncertainties associated to triangle graphs [5]. It is clear that the uncertainties from QED and EW corrections are very small and do not pose a problem for present and even future comparisons of theory and experiment.

The most uncertain contribution in the SM calculation does arise from corrections involving hadronic loops. There are three kinds of such hadronic corrections:

$$a_\mu^{had} = a_\mu^{had,LO} + a_\mu^{had,HO} + a_\mu^{had,LBL} \ , \quad (3)$$

where superscripts refers to leading order (LO, at $O(\alpha^2)$), higher order (HO, at $O(\alpha^3)$) and light by light (LBL, at $O(\alpha^3)$) effects.

The different contributions in Eq.(3) cannot be calculated with arbitrary high accuracy as they involve strong interactions in the non-perturbative regime. Inputs from experimental data or models of hadronic interactions at low energies are useful in this case to come to a reliable result. By far, the largest contribution in Eq. (3) is the leading order correction, which needs to be calculated with a precision below the 1 % level and will be discussed in further detail in the next section. The accuracy required in the higher order HO and LBL corrections to match the experimental precision is much lower. The second term in Eq. (3) can be calculated using the same input data as the one used to compute the LO contribution. We just reproduce here the value obtained using electron-positron data (the numerical value obtained using τ lepton data is approximately 3% larger) [7]:

$$a_\mu^{had,HO} = (-98 \pm 1) \times 10^{-11} \ , \quad (4)$$

where the uncertainty stems mainly from input experimental data.

The hadronic light-by-light contribution is by far the most difficult to compute and the least precise ingredient of a_μ. There is not a direct connection to measurable quantities as in the case of the other two terms in Eq. (3). For the purposes of comparison with experiment, we use the results of a recent calculation [8]:

$$a_\mu^{had,LBL} = (105 \pm 26) \times 10^{-11} \; . \tag{5}$$

More details about recent improvements leading to Eq. (5) and a comparison with results of previous calculations are described in the accompanying paper by J. Prades [9].

3 The hadronic contribution at leading order

The hadronic contribution at leading order is obtained by inserting one loop of quarks or hadrons in the photonic propagator as indicated in Figure 1. The LO hadronic contribution

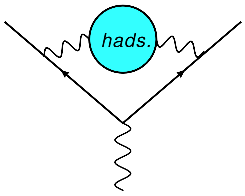

Figure 1: Hadronic contribution at leading order.

can be evaluated by means of the dispersion integral [10, 11]

$$a_\mu^{had,LO} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} \frac{\sigma^0(e^+e^- \to \text{hadrons})}{\sigma^{\text{point}}(e^+e^- \to \mu^+\mu^-)} \; , \tag{6}$$

where the superscript '0' denotes the cross section for hadron production with photonic corrections to the initial state and vacuum polarization effects removed but with final state photonic corrections included. The QED kernel $K(s)/s \sim s^{-2}$ [12] gives a large weight to low energy hadronic cross sections which makes the dominant contributions fall into the non-perturbative domain of strong interactions. Thus for instance, the 2π channel below 1.8 GeV contributes approximately 73% of the integral (6) and 82% of its total uncertainty, while the total contribution above 5.0 GeV is only 1.4%. The small contributions from large values of s, typically $\sqrt{s} \geq 5$ GeV, can be reliably obtained from perturbative QCD.

In this section we present an update of the 2π contribution to the muon magnetic anomaly taking into account recent data from the KLOE collaboration in the case of e^+e^- [13]. We also take advantage of a high-statistics study of the 2π hadronic spectrum from Belle [14] and recent calculations of the IB effects in the τ lepton case to re-evaluate the LO from τ data. For further details we refer the reader to our recent paper [15].

3.1 2π contribution using electron-positron data

The most relevant measurements of the 2π cross section in electron-positron collisions have been reported by the CMD2 [16, 17], SND [18] and, very recently, the KLOE [13] collaborations. The accuracy of these data sets around the $\rho(770)$ resonance peak is below the 1% level, as required for theoretical predictions to match the experimental accuracy. The recent data published by KLOE [13] have reduced uncertainties and are slightly closer to CMD2 and SND results than before [19]. The Babar Collaboration [20] has reported preliminary measurements of this channel for center of mass energies between 0.5 and 3.0 GeV with an accuracy below the 0.6% level, which looks promissing for future improved analysis.

Previous evaluations of the 2π contribution to the muon magnetic anomaly using CMD2, SND and older KLOE measurements, can be found in [21, 22, 23, 24, 25]. In Ref. [15] we have presented an updated evaluation of the two-pion contribution by using the published data from CMD2, SND and KLOE. The evaluations of $a_\mu^{2\pi,LO}$ have been compared for the energy regions where data from different experiments overlap. As in previous analysis [23, 24], a good agreement is found between results based on CMD2 and SND data, while KLOE gives a lower result. Thus, we quote two results (either by including or excluding KLOE data) from combined data sets in the energy region where they overlap. As in previous analysis [21, 23], the evaluation of $a_\mu^{2\pi,LO}$ in the low energy region (chosen as $\sqrt{s} = 2m_\pi - 0.36$ GeV in Ref. [15]) is done by using a cubic expansion in s for the pion form factor as described in [21]. Our updated evaluation for the two-pion contribution gives [15]:

$$a_\mu^{2\pi,LO}(e^+e^-) = \begin{cases} (5027.7 \pm 30.1 \pm 11.1) \times 10^{-11}, & \text{including KLOE} \\ (5038.3 \pm 37.9 \pm 16.5) \times 10^{-11}, & \text{excluding KLOE} \end{cases} \tag{7}$$

where the first uncertainty stems from experimental input data and the second from the procedure used to extrapolate between data points in the dispersion integral [15]. As expected, using KLOE data [13] leads to a more precise but a slightly lower result.

3.2 2π contribution using tau decay data

The conserved vector current (CVC) hypothesis allows to replace data on the production of an even number of pions in e^+e^- collisions via the $I=1$ current by the isospin analogous final state produced in τ lepton decays [26] ($\mathcal{B}_X$ denotes the branching fraction for final state X in τ decays):

$$\sigma(e^+e^- \to X_0^{I=1}) = \left(\frac{4\pi\alpha^2}{s}\right) \frac{m_\tau^2}{6|V_{ud}|^2} \frac{\mathcal{B}_{X_-}}{\mathcal{B}_e} \left(\frac{1}{N_X}\frac{dN_X}{ds}\right) \left(1 - \frac{s}{m_\tau^2}\right)^{-2} \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \frac{R_{IB}(s)}{S_{EW}} \,. \tag{8}$$

In our numerical evaluations we use [15]: $|V_{ud}| = 0.97418 \pm 0.00019$ [27], $\mathcal{B}_e = (17.818 \pm 0.032)\%$ [28], $S_{EW} = 1.0235 \pm 0.0003$ for the short-distance electroweak corrections [29]. We denote $(1/N_X)dN_X/ds$ as the normalized hadronic mass distribution in $\tau \to X^-\nu$ decays. In the case of the two-pion final state we use $\mathcal{B}_{2\pi} = (25.42 \pm 0.10)\%$, which corresponds to the weighted average of different measurements [15].

The s-dependent factor $R_{IB}(s)$ encodes the information about IB corrections that must be applied to the hadronic spectrum in τ decays in order to be used in the dispersion integral ($R_{IB}(s) = 1$ in the absence of IB effects or in the limit of exact CVC). It is defined as:

$$R_{IB}(s) = \frac{FSR}{G_{EM}(s)} \cdot \left(\frac{\beta_0(s)}{\beta_-(s)}\right)^3 \left|\frac{F_0(s)}{F_-(s)}\right|^2 \,. \tag{9}$$

The factor FSR [30] refers to the final state photonic corrections to the $e^+e^- \to \pi^+\pi^-$ cross section, $G_{EM}(s)$ denotes the long-distance radiative corrections to $\tau \to \pi\pi\nu$ decays [31],

$(\beta_0/\beta_-)^3$ is the ratio of pion velocities in their center of mass frame and $F_{0,-}(s)$ refers to the pion form factors (the subscripts $0,-$ refer to the electric charge of the 2π system). In Figure 2 we plot the s-dependent IB correction factors that enter the definition of $R_{IB}(s)$. Note that the first two factors in Eq. (9) have an important effect close to threshold $s = 4m_\pi^2$, while the IB effects in the ratio of pion form factors are more important around the ρ resonance region.

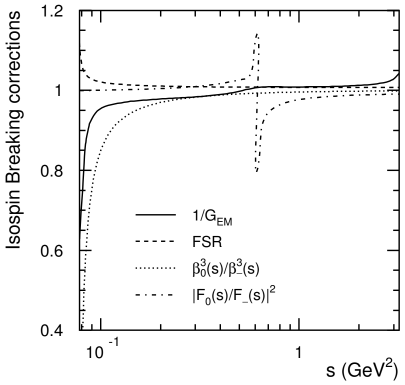

Figure 2: s dependence of IB correction factors defined in Eq. (9).

Some of the most recent evaluations of the 2π contribution using τ data from ALEPH [33], OPAL [34] and CLEO [35] Collaborations were reported in [21, 23]. Very recently, the Belle collaboration has reported a high-statistics study of the hadronic spectrum in $\tau \to \pi\pi\nu$ decay [14]. In addition, new calculations of the $G_{EM}(s)$ corrections to this decay [32] and of the width difference of $\rho^0 - \rho^\pm$ vector mesons [36] have become available. Particularly important is the role that plays the width difference $\Delta\Gamma_\rho$ given the wide resonance shape of the 2π cross section in the ρ meson region. The new calculation of $\Delta\Gamma_\rho$ takes into account the full radiative corrections to the dominant $\rho \to \pi\pi$ decay modes [36] including the effects of hard photons. All these new ingredients have prompted the analysis undertaken in Ref. [15].

In Table I we summarize the effects produced in $a_\mu^{\pi\pi,LO}(\tau)$ by the different sources of IB corrections that enter Eq. (9). Other numerical values of the IB parameters required in the

Source	$\Delta a_\mu^{\pi\pi,LO}(\tau)[10^{-11}]$
S_{EW}	-121.9 ± 1.5
G_{EM}	-18.6 ± 8.8
FSR	$+46.4 \pm 4.6$
$\rho - \omega$ interference	$+24.0 \pm 3.5$
$m_{\pi^\pm} - m_{\pi^0}$ in cross section	-77.1
$m_{\pi^\pm} - m_{\pi^0}$ in ρ widths	$+41.1 \pm 4.0$
$m_{\rho^\pm} - m_{\rho^0}$	-0.8 ± 3.5
$\pi\pi\gamma$ em decays	-59.4 ± 5.9
total	-165.5 ± 15.5

Table 1: Contributions to $\Delta a_\mu^{\pi\pi,LO}$ from isospin-breaking corrections.

form factors can be found in Ref. [15]. The most important changes with respect to previous evaluations [21, 23] come from the width difference of ρ mesons. The largest uncertainty in Table 1 comes from the difference in the IB corrections from $G_{EM}(s)$ as calculated in [32] and that from Ref. [31]. We have attributed a 10% uncertainty to the IB effects that arise from FSR and $\Delta\Gamma_\rho$ due to neglected effects induced by the electromagnetic structure of pions in virtual corrections. Finally, the uncertainties quoted in the fourth, sixth and seventh rows of Table 1 arise from taking the difference between results obtained using the Gounaris-Sakurai (GS) and Kuhn-Santamaria parametrizations of the pion form factors (see [15]). Despite the conservative estimate of errors in Table 1, the total uncertainty becomes smaller than before [21].

Using the combined $\pi^-\pi^0$ mass spectrum of ALEPH, CLEO, OPAL and Belle collaborations, and applying the IB corrections discussed above, the dispersion integral Eq. (6) together with (8) and (9) yields [15]:

$$a_\mu^{\pi\pi,LO}(\tau) = (5143 \pm 12 \pm 22 \pm 16) \times 10^{-11} \ , \tag{10}$$

where the quoted errors arise, respectively, from uncertainties in the measured hadronic spectrum, in the $\pi^-\pi^0$ branching ratio and in the IB corrections.

A comparison of Eqs. (7) and (10), yields the following difference in the $\pi\pi$ channel,

$$\delta a_\mu^{\pi\pi,LO} = a_\mu^{\pi\pi,LO}(\tau) - a_\mu^{\pi\pi,LO}(e^+e^-) = \begin{cases} (115.3 \pm 43.8) \times 10^{-11}, & \text{incl. KLOE} \\ (114.7 \pm 50.1) \times 10^{-11}, & \text{excl. KLOE} \end{cases} \ , \tag{11}$$

which should be compared with previously obtained $\delta a_\mu^{\pi\pi,LO} = (154 \pm 49) \times 10^{-11}$ [23]. These results makes explicit the impact of new data and the new calculation of IB effects.

CVC can be used to predict other contributions to $a_\mu^{had,LO}$ that involve an even number of pions. Currently, data on the 4π mass spectrum in τ decays yields [23] (in units of 10^{-11}): $214 \pm 25 \pm 6_{IB}$ (for $\pi^+\pi^-2\pi^0$)) and $123 \pm 10 \pm 4_{IB}$ (for $2\pi^+2\pi^-$). Including other hadronic contributions from e^+e^- data (see Ref. [23]) we get the following results for the LO hadronic contribution [15]:

$$a_\mu^{had,LO} = \begin{cases} (6901 \pm 44 \pm 19_{rad} \pm 7_{QCD}) \times 10^{-11}, & e^+e^- \text{ excl. KLOE} \\ (6891 \pm 38 \pm 19_{rad} \pm 7_{QCD}) \times 10^{-11}, & e^+e^- \text{ incl. KLOE} \\ (7044 \pm 35 \pm 7_{rad} \pm 18_{IB}) \times 10^{-11}, & \tau \ \text{ decay data} \end{cases} \tag{12}$$

The discrepancy between the predictions of the LO terms based on τ and e^+e^- data is at the 2.2σ (2.5σ) level obtained by excluding (including) KLOE data.

4 Updated SM prediction for a_μ

The SM prediction is obtained by adding up results in Eqs. (1), (2), (4), (5) and (12):

$$a_\mu^{SM} = \begin{cases} (116591780.1 \pm 55.0) \times 10^{-11}, & \text{from } e^+e^- \text{ (excl. KLOE)} \\ (116591770.1 \pm 50.3) \times 10^{-11}, & \text{from } e^+e^- \text{ (incl. KLOE)} \\ (116591923.1 \pm 47.7) \times 10^{-11}, & \text{from } \tau \ \text{ data} \end{cases} \tag{13}$$

All error bars were added in quadrature. They are dominated by the hadronic uncertainties from LO and light-by-light contributions.

In Figure 3 we plot the deviations of the SM predictions from the experimental value of the muon magnetic anomaly. The predictions of other recent analysis [21, 23, 24, 25] that include recent CMD2 and SND results and the previous SM prediction based on τ decay data are also shown for comparison.

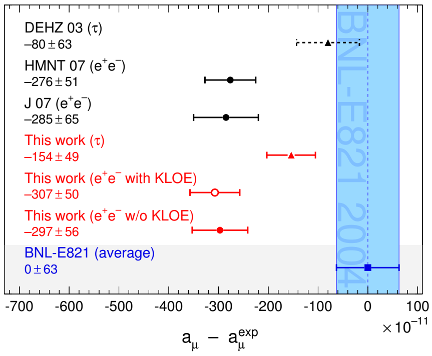

Figure 3: Deviations of SM predictions from experiment for the muon anomaly.

5 CVC prediction for the branching fraction

An independent consistency test of the 2π spectral functions is provided by comparing the measured branching fraction of τ decays with its prediction based on e^+e^- data using CVC. The formula relating these quantities is given by [15]:

$$B(\tau \to \pi\pi\nu) = \frac{3}{2}\frac{B_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{min}}^{m_\tau^2} ds\sigma^0_{\pi^+\pi^-}(s)\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\frac{S_{EW}}{R_{IB}(s)} \, . \tag{14}$$

In the limit of isospin symmetry, $S_{EW} = 1$ and $R_{IB}(s) = 1$. A previous evaluation of this branching ratio [21, 23] exhibited a discrepancy at the 4.5σ level when compared to the average of direct measurements. Including the new calculation of the isospin breaking effects increases the CVC-based prediction by the amount $(+0.73 \pm 0.19)\%$ [15]. This error bar is dominated by the uncertainties in the long-distance radiative corrections to $\tau \to \pi\pi\nu$ decays due to neglected pion form factor effects in virtual corrections [15].

In Figure 4 we compare the direct measurements of the branching ratio for τ decays, with the predictions based on e^+e^- data from CMD2, SND and KLOE corrected by IB effects according to Eq. (14). The difference between direct measurements of the branching fraction and predictions based on IB corrected e^+e^- data are now $(0.61 \pm 0.27)\%$ and $(0.47 \pm 30)\%$, respectively, by including and excluding KLOE data. This shows a smaller discrepancy than in previous results [21] and gives further support to the use of τ decay data to predict the muon magnetic anomaly.

6 Conclusions

Recent progress in calculations of the hadronic leading order and light-by-light contributions, allows to predict the muon magnetic anomaly a_μ with a better accuracy that its measured value. The prediction of a_μ based on e^+e^- data persistently shows a disagreement with the experimental value, currently at the 3.5σ level. Taking advantage of the high-statistics measurements of the 2π hadronic spectrum from Belle [14], and using new calculations of the IB effects, we have found [15] a SM prediction based on τ data which is closer to the one based on e^+e data; the discrepancy between both predictions still remains large given their smaller

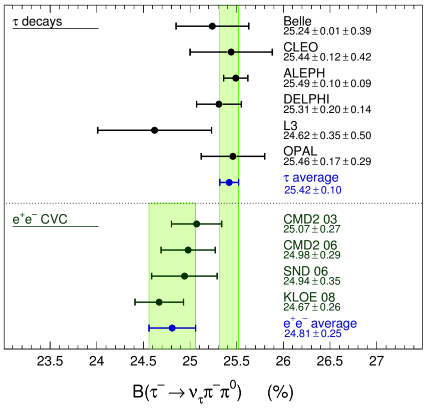

Figure 4: Branching fraction of τ decay compared to predictions based on CVC.

current uncertainties. On the other hand, this brings further support to the use of tau decay data in the search of an improved prediction of a_μ as required by future measurements.

An even better accuracy of the SM prediction can be achieved with more precise measurement of the $e^+e^- \to$ hadrons cross sections at CMD2, SND, KLOE and BaBar. Although some advances have been done [15] in understanding IB effects in the 2π spectral functions when comparing τ and e^+e- data, further work is required to solve completely the discrepancy between both sets of data. Solving this discrepancy will help to reach a precise prediction for a_μ as required by the E969 experiment.

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