

# The Hadronic Light-by-Light Contribution to Muon g-2: A Short Review

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I review the recent calculations and current status of the hadronic light-by-light scattering contribution to muon g-2. In particular, I discuss the main results obtained in a recent work together with Eduardo de Rafael and Arkady Vainshtein where we came to the estimate  $a_\mu^{\text{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}$ . How the two-photon physics program of low energy facilities can help to reduce the present model dependence is also emphasized.

## 1 Introduction

One momenta configuration out of the six possible ones contributing to the hadronic light-by-light to muon g-2 is depicted in Fig. 1 and described by the vertex function

$$\begin{aligned} \Gamma^\mu(p_2, p_1) &= -e^6 \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \frac{\Pi^{\mu\nu\rho\sigma}(q, k_1, k_2, k_3)}{k_1^2 k_2^2 k_3^2} \\ &\times \gamma_\nu(\not{p}_2 + \not{k}_2 - m)^{-1} \gamma_\rho(\not{p}_1 - \not{k}_1 - m)^{-1} \gamma_\sigma \end{aligned} \quad (1)$$

where  $q \rightarrow 0$  is the momentum of the photon that couples to the external magnetic source,  $q = p_2 - p_1 = -k_1 - k_2 - k_3$  and  $m$  is the muon mass.

The dominant contribution to the hadronic four-point function

$$\begin{aligned} \Pi^{\rho\nu\alpha\beta}(q, k_1, k_3, k_2) &= \\ i^3 \int d^4 x \int d^4 y \int d^4 z e^{i(-k_1 \cdot x + k_3 \cdot y + k_2 \cdot z)} &\langle 0 | T [V^\mu(0) V^\nu(x) V^\rho(y) V^\sigma(z)] | 0 \rangle \end{aligned} \quad (2)$$

comes from the three light quark ( $q = u, d, s$ ) components in the electromagnetic current  $V^\mu(x) = [\bar{q} \hat{Q} \gamma^\mu q](x)$  where  $\hat{Q} \equiv \text{diag}(2, -1, -1)/3$  denotes the quark electric charge matrix. We are interested in the limit  $q \rightarrow 0$  where current conservation implies

$$\Gamma^\mu(p_2, p_1) = -\frac{a^{\text{HLbL}}}{4m} [\gamma^\mu, \gamma^\nu] q_\nu. \quad (3)$$

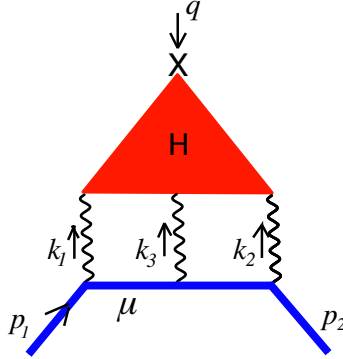


Figure 1: Hadronic light-by-light scattering contribution.

Therefore, the muon anomaly can then be extracted as

$$a^{\text{HLbL}} = \frac{e^6}{48m} \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \frac{1}{k_1^2 k_2^2 k_3^2} \left[ \frac{\partial}{\partial q^\mu} \Pi^{\lambda\nu\rho\sigma}(q, k_1, k_3, k_2) \right]_{q=0} \times \text{tr} \{ (\not{p} + m)[\gamma_\mu, \gamma_\lambda](\not{p} + m)\gamma_\nu(\not{p} + \not{k}_2 - m)^{-1}\gamma_\rho(\not{p} - \not{k}_1 - m)^{-1}\gamma_\sigma \}. \quad (4)$$

Here I discuss the results of [1] and [2]. Previous work can be found in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and recent reviews are in [13, 14, 15, 16].

The hadronic four-point function  $\Pi^{\mu\nu\rho\sigma}(q, k_1, k_3, k_2)$  is an extremely difficult object involving many scales and no full first principle calculation of it has been reported yet –even in the simpler large numbers of colors  $N_c$  limit of QCD. Notice that we need it with momenta  $k_1$ ,  $k_2$  and  $k_3$  varying from 0 to  $\infty$ . Unfortunately, unlike the hadronic vacuum polarization, there is neither a direct connection of  $a^{\text{HLbL}}$  to a measurable quantity. Two lattice groups have started exploratory calculations [17, 18] but the final uncertainty that these calculations can reach is not clear yet.

Attending to a combined large number of colors of QCD and chiral perturbation theory (CHPT) counting, one can distinguish four types of contributions [19]. Notice that the CHPT counting is only for organization of the contributions and refers to the lowest order term contributing in each case. In fact, Ref. [1] shows that there are chiral enhancement factors that demand more than Nambu-Goldstone bosons in the CHPT expansion in the light-by-light contribution to the muon anomaly. See more comments on this afterwards.

The four different types of contributions are:

- Nambu-Goldstone boson exchanges contribution are  $\mathcal{O}(N_c)$  and start at  $\mathcal{O}(p^6)$  in CHPT.
- One-meson irreducible vertex contribution and non-Goldstone boson exchanges contribute also at  $\mathcal{O}(N_c)$  but start contributing at  $\mathcal{O}(p^8)$  in CHPT.
- One-loop of Goldstone bosons contribution are  $\mathcal{O}(1/N_c)$  and start at  $\mathcal{O}(p^4)$  in CHPT.
- One-loop of non-Goldstone boson contributions which are  $\mathcal{O}(1/N_c)$  but start contributing at  $\mathcal{O}(p^8)$  in CHPT.

Based on the counting above there are two full calculations [3, 4, 6] and [5, 7]. There is also a detailed study of the  $\pi^0$  exchange contribution [8] putting emphasis in obtaining analytical expressions for this part.

Recently, two new calculations of the pion exchange using also the organization above have been made. In Ref. [10], the pion pole term exchange is evaluated within an effective chiral model. These authors also study the box diagram one-meson irreducible vertex contribution. The results are numerically very similar to the ones found in the literature as can be seen in Table 1. In Ref. [11], the author uses a large  $N_c$  model  $\pi^0\gamma^*\gamma^*$  form factor with the pion also off-shell. This has to be considered as a first step and more work has to be done in order to have the full light-by-light within this approach. In particular, it would be very interesting to calculate the contribution of one-meson irreducible vertex contribution within this model.

Using operator product expansion (OPE) in QCD, the authors of [12] pointed out a new short-distance constraint of the reduced full four-point Green function

$$\langle 0|T[V^\nu(k_1)V^\rho(k_3)V^\sigma(-(k_1+k_2+q))|\gamma(q)\rangle \quad (5)$$

when  $q \rightarrow 0$  and in the special momenta configuration  $-k_1^2 \simeq -k_3^2 \gg -(k_1+k_3)^2$  Euclidean and large. In that kinematical region,

$$T[V^\nu(k_1)V^\rho(k_3)] \sim \frac{1}{\hat{k}^2} \varepsilon^{\nu\rho\alpha\beta} \hat{k}_\alpha \left[ \bar{q} \hat{Q}^2 \gamma_{\beta\gamma\delta} q \right] \quad (6)$$

with  $\hat{k} = (k_1 - k_3)/2 \simeq k_1 \simeq -k_3$ . See also [20]. This short distance constraint was not explicitly imposed in previous to [12] calculations.

## 2 Leading in $1/N_c$ Results

Using effective field theory techniques, the authors of [9] shown that leading large  $N_c$  contribution to  $a^{\text{HLbL}}$  contains an enhanced term at low energy by  $\log^2(M_\rho/m_\pi)$  where the rho mass  $M_\rho$  acts as an ultraviolet scale and the pion mass  $m_\pi$  provides the infrared scale.

$$a^{\text{HLbL}}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c \frac{m^2 N_c}{48\pi^2 f_\pi^2} \left[ \ln^2 \frac{M_\rho}{m_\pi} + \mathcal{O}\left(\ln \frac{M_\rho}{m_\pi}\right) + \mathcal{O}(1) \right] \quad (7)$$

This leading logarithm is generated by the Goldstone boson exchange contributions and is fixed by the Wess–Zumino–Witten (WZW) vertex  $\pi^0\gamma\gamma$ . In the chiral limit where quark masses are neglected and at large  $N_c$ , the coefficient of this double logarithm is model independent and has been calculated and shown to be positive in [9]. All the calculations we discuss here agree with these leading behaviour and its coefficient including the sign. A global sign mistake in the  $\pi^0$  exchange in [3, 4, 5] was found by [8, 9] and confirmed by [6, 7] and by others [21, 22]. The subleading ultraviolet scale  $\mu$ -dependent terms [9], namely,  $\log(\mu/m_\pi)$  and a non-logarithmic term  $\kappa(\mu)$ , are model dependent and calculations of them are implicit in the results presented in [3, 4, 5, 7, 12]. In particular,  $\kappa(\mu)$  contains the large  $N_c$  contributions from one-meson irreducible vertex and non-Goldstone boson exchanges. In the next section we review the recent model calculations of the full leading in the  $1/N_c$  expansion contributions.

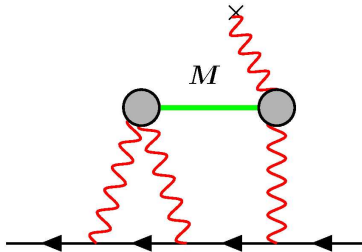


Figure 2: A generic meson exchange contribution to the hadronic light-by-light part of the muon  $g-2$ .

Table 1: Results for the  $\pi^0$ ,  $\eta$  and  $\eta'$  exchange contributions.

Reference	$10^{10} \times a$	
	$\pi^0$ only	$\pi^0, \eta$ and $\eta'$
[3, 4, 6]	5.7	$8.3 \pm 0.6$
[5, 7]	5.6	$8.5 \pm 1.3$
[8] with $h_2 = 0$	5.8	$8.3 \pm 1.2$
[8] with $h_2 = -10 \text{ GeV}^2$	6.3	
[10]	$6.3 \sim 6.7$	
[11]	7.2	$9.9 \pm 1.6$
[12]	7.65	$11.4 \pm 1.0$

## 2.1 Model Calculations

The pseudo-scalar exchange is the dominant numerical contribution and was saturated in [3, 4, 5, 6, 7, 8, 10, 11] by Nambu-Goldstone boson's exchange. This contribution is depicted in Fig. 2 with  $M = \pi^0, \eta, \eta'$ . The relevant four-point function was obtained in terms of the off-shell  $\pi^0 \gamma^*(k_1) \gamma^*(k_3)$  form factor  $\mathcal{F}(k_1^2, k_3^2)$  and the off-shell  $\pi^0 \gamma^*(k_2) \gamma(q=0)$  form factor  $\mathcal{F}(k_2^2, 0)$  modulating each one of the two WZW  $\pi^0 \gamma \gamma$  vertex.

In all cases several short-distance QCD constraints were imposed on these form-factors. In particular, they all have the correct QCD short-distance behaviour

$$\mathcal{F}(Q^2, Q^2) \rightarrow \frac{A}{Q^2} \quad \text{and} \quad \mathcal{F}(Q^2, 0) \rightarrow \frac{B}{Q^2} \quad (8)$$

when  $Q^2$  is Euclidean and large and are in agreement with  $\pi^0 \gamma^* \gamma$  low-energy data<sup>1</sup>. They differ slightly in shape due to the different model assumptions (VMD, ENJL, Large  $N_c$ , N $\chi$ QM) but they produce small numerical differences always compatible within quoted uncertainty  $\sim 1.3 \times 10^{-10}$  –see Table 1.

Within the models used in [3, 4, 5, 6, 7, 8, 10, 11], to get the full contribution at leading in  $1/N_c$  one needs to add the one-meson irreducible vertex contribution and the non-Goldstone

<sup>1</sup>See however the new measurement of the  $\gamma \gamma^* \rightarrow \pi^0$  transition form factor by BaBar [23] at energies between 4 and 40  $\text{GeV}^2$

Table 2: Sum of the short- and long-distance quark loop contributions [5] as a function of the matching scale  $\Lambda$ .

$\Lambda$ [GeV]	0.7	1.0	2.0	4.0
$10^{10} \times a^{\text{HLbL}}$	2.2	2.0	1.9	2.0

Table 3: Results for the axial-vector exchange contributions from [3, 4, 6] and [5, 7].

References	$10^{10} \times a^{\text{HLbL}}$
[3, 4, 6]	$0.17 \pm 0.10$
[5, 7]	$0.25 \pm 0.10$

boson exchanges. In particular, below some scale  $\Lambda$ , the one-meson irreducible vertex contribution was identified in [5, 7] with the ENJL quark box contribution with four dressed photon legs. While to mimic the contribution of short-distance QCD quarks above  $\Lambda$ , a loop of bare massive heavy quark with mass  $\Lambda$  and QCD vertices was used. The results are in Table 2 where one can see a very nice stability region when  $\Lambda$  is in the interval [0.7, 4.0] GeV. Similar results for the quark loop below  $\Lambda$  were obtained in [3, 4] though these authors didn't discuss the short-distance long-distance matching.

In [5, 7], non-Goldstone boson exchanges were saturated by the hadrons appearing in the model, i.e. the lowest scalar and pseudo-vector hadrons. Both states in nonet-symmetry –this symmetry is exact in the large  $N_c$  limit. Within the ENJL model, the one-meson irreducible vertex contribution is related through Ward identities to the scalar exchange which we discuss below and *both* have to be included [5, 7]. The result of the scalar exchange obtained in [5] is

$$a^{\text{HLbL}}(\text{Scalar}) = -(0.7 \pm 0.2) \times 10^{-10}. \quad (9)$$

The scalar exchange was not included in [3, 4, 6, 8]. The result of the axial-vector exchanges in [3, 4, 6] and [5, 7] can be found in Table 3.

Melnikov and Vainshtein used a model that saturates the hadronic four-point function in (2) at leading order in the  $1/N_c$  expansion by the exchange of the Nambu-Goldstone  $\pi^0, \eta, \eta'$  and the lowest axial-vector  $f_1$  states. In that model, the new OPE constraint of the reduced four-point function found in [12] mentioned above, forces the  $\pi^0 \gamma^*(q) \gamma(p_3 = 0)$  vertex to be point-like rather than including a  $\mathcal{F}(q^2, 0)$  form factor.

There are also OPE constraints for other momenta regions [24] which are not satisfied by the model in [12] though they argued that this made only a small numerical difference of the order of  $0.05 \times 10^{-10}$ . In fact, within the large  $N_c$  framework, it has been shown [25] that in general for other than two-point functions, to satisfy fully the QCD short-distance properties requires the inclusion of an infinite number of narrow states.

### 3 Next-to-leading in $1/N_c$ Results

For the next-to-leading in  $1/N_c$  contributions to the  $a^{\text{HLbL}}$  there is no model independent result at present and is possibly the most difficult component. Charged pion and kaon loops saturated this contribution in [3, 4, 5, 6, 7]. To dress the photon interacting with pions, a particular Hidden Gauge Symmetry (HGS) model was used in [3, 4, 6] while a full VMD was used in [5, 7]. The results obtained are  $-(0.45 \pm 0.85) \times 10^{-10}$  in [3] and  $-(1.9 \pm 0.5) \times 10^{-10}$

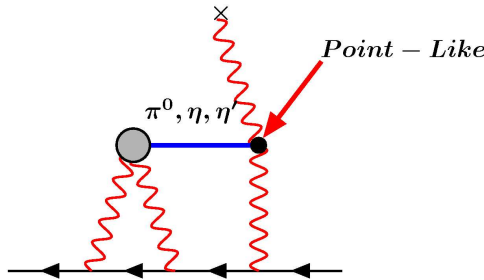


Figure 3: Goldstone boson exchange in the model in [12] contributing to the hadronic light-by-light.

Table 4: Results for the full hadronic light-by-light contribution to  $a^{\text{HLbL}}$ .

Full Hadronic Light-by-Light	$10^{10} \times a_\mu$
[3, 4, 6]	$8.9 \pm 1.7$
[5, 7]	$8.9 \pm 3.2$
[12]	$13.6 \pm 2.5$

in [5] while using a point-like vertex one gets  $-4.6 \times 10^{-10}$ . Both models satisfy the known constraints though start differing at  $\mathcal{O}(p^6)$  in CHPT. Some studies of the cut-off dependence of the pion loop using the full VMD model was done in [5] and showed that their final number comes from fairly low energies where the model dependence should be smaller. The authors of [12] analyzed the model used in [3, 4] and showed that there is a large cancellation between the first three terms of an expansion in powers of  $(m_\pi/M_\rho)^2$  and with large higher order corrections when expanded in CHPT orders but the same applies to the  $\pi^0$  exchange as can be seen from Table 6 in the first reference in [2] by comparing the WZW column with the others. The authors of [12] took  $(0 \pm 1) \times 10^{-10}$  as a guess estimate of the total NLO in  $1/N_c$  contribution. This seems too simple and certainly with underestimated uncertainty.

## 4 Comparing Different Calculations

The comparison of individual contributions in [3, 4, 5, 6, 7, 8, 10, 11, 12] has to be done with care because they come from different model assumptions to construct the full relevant four-point function. In fact, the authors of [10] have shown that their constituent quark loop provides the correct asymptotics and in particular the new OPE found in [12]. It has more sense to compare results for  $a^{\text{HLbL}}$  either at leading order or at next-to-leading order in the  $1/N_c$  expansion.

The results for the final hadronic light-by-light contribution to  $a^{\text{HLbL}}$  quoted in [3, 4, 5, 6, 7, 12] are in Table 4. The apparent agreement between [3, 4, 6] and [5, 7] hides non-negligible differences which numerically almost compensate between the quark-loop and charged pion and [12] are in Table 4. Notice also that [3, 4, 6] didn't include the scalar exchange. Comparing the results of [5, 7] and [12], as discussed above, we have found several differences of order  $1.5 \times 10^{-10}$  which are not related to the new short-distance constraint used in [12]. The different

axial-vector mass mixing accounts for  $-1.5 \times 10^{-10}$ , the absence of the scalar exchange in [12] accounts for  $-0.7 \times 10^{-10}$  and the use of a vanishing NLO in  $1/N_c$  contribution in [12] accounts for  $-1.9 \times 10^{-10}$ . These model dependent differences add up to  $-4.1 \times 10^{-10}$  out of the final  $-5.3 \times 10^{-10}$  difference between [5, 7] and [12]. Clearly, the new OPE constraint used in [12] accounts only for a small part of the large numerical final difference.

## 5 Conclusions and Prospects

To give a result at present for the hadronic light-by-light contribution to the muon anomalous magnetic moment, the authors of [1], from the above considerations, concluded that it is fair to proceed as follows

*Contribution to  $a^{\text{HLbL}}$  from  $\pi^0$ ,  $\eta$  and  $\eta'$  exchanges*

Because of the effect of the OPE constraint discussed above, we suggested to take as central value the result of Ref. [12] with, however, the largest error quoted in Refs. [5, 7]:

$$a^{\text{HLbL}}(\pi, \eta, \eta') = (11.4 \pm 1.3) \times 10^{-10}. \quad (10)$$

Recall that this central value is quite close to the one in the ENJL model when the short-distance quark loop contribution is added there.

*Contribution to  $a^{\text{HLbL}}$  from pseudo-vector exchanges*

The analysis made in Ref. [12] suggests that the errors in the first and second entries of Table 2 are likely to be underestimates. Raising their  $\pm 0.10$  errors to  $\pm 1$  puts the three numbers in agreement within one sigma. We suggested then as the best estimate at present

$$a^{\text{HLbL}}(\text{pseudo} - \text{vectors}) = (1.5 \pm 1) \times 10^{-10}. \quad (11)$$

*Contribution to  $a^{\text{HLbL}}$  from scalar exchanges*

The ENJL-model should give a good estimate for these contributions. We kept, therefore, the result of Ref. [5, 7] with, however, a larger error which covers the effect of other unaccounted meson exchanges,

$$a^{\text{HLbL}}(\text{scalars}) = -(0.7 \pm 0.7) \times 10^{-10}. \quad (12)$$

*Contribution to  $a^{\text{HLbL}}$  from a dressed pion loop*

Because of the instability of the results for the charged pion loop and unaccounted loops of other mesons, we suggested using the central value of the ENJL result but with a larger error:

$$a^{\text{HLbL}}(\pi\text{-dressed loop}) = -(1.9 \pm 1.9) \times 10^{-10}. \quad (13)$$

From these considerations, adding the errors in quadrature, as well as the small charm contribution  $0.23 \times 10^{-10}$ , we get

$$a^{\text{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}, \quad (14)$$

as our final estimate.

The proposed new  $g_\mu - 2$  experiment accuracy goal of  $1.6 \times 10^{-10}$  calls for a considerable improvement in the present calculations. The use of further theoretical and experimental constraints could result in reaching such accuracy soon enough. In particular, imposing as many as

possible short-distance QCD constraints [3, 4, 5, 6, 7, 8, 11] has result in a better understanding of the numerically dominant  $\pi^0$  exchange. At present, none of the light-by-light hadronic parametrization satisfy fully all short distance QCD constraints. In particular, this requires the inclusion of infinite number of narrow states for other than two-point functions and two-point functions with soft insertions [25]. A numerical dominance of certain momenta configuration can help to minimize the effects of short distance QCD constraints not satisfied, as in the model in [12].

More experimental information on the decays  $\pi^0 \rightarrow \gamma\gamma^*$ ,  $\pi^0 \rightarrow \gamma^*\gamma^*$  and  $\pi^0 \rightarrow e^+e^-$  (with radiative corrections included [22, 26, 27]) can also help to confirm some of the neutral pion exchange results. A better understanding of other smaller contributions but with comparable uncertainties needs both more theoretical work and experimental information. This refers in particular to pseudo-vector exchanges. Experimental data on radiative decays and two-photon production of these and other C-even resonances can be useful in that respect.

New approaches to the pion dressed loop contribution, together with experimental information on the vertex  $\pi^+\pi^-\gamma^*\gamma^*$  in the intermediate energy region (0.5 – 1.5 GeV) would also be very welcome. Measurements of two-photon processes like  $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$  can be useful to give information on that vertex and again could reduce the model dependence. The two-gamma physics program low energy facilities like the experiment KLOE-2 at DAΦNE will be very useful and well suited in the processes mentioned above which information can help to decrease the present model dependence of  $a_\mu^{\text{HLbL}}$ .

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