Strong-field effects for lepton and photon production in collisions of relativistic heavy nuclei at RHIC and LHC

Ulrich D. Jentschura[1] *and Valery G. Serbo*[2]

[1]Dept. of Physics, Missouri Univ. of Science and Technology, Rolla, Missouri 65409-0640, USA
[2]Novosibirsk State University, Pirogova 2, 630090 Novosibirsk, Russia,
corresponding author: serbo@math.nsc.ru

DOI: http://dx.doi.org/10.3204/DESY-PROC-2009-03/Serbo

We consider effects of strong electromagnetic fields in collisions of ultra-relativistic nuclei at the RHIC and LHC colliders. Since the parameter $Z\alpha$ is not small ($Z\alpha \approx 0.6$ for Au-Au and Pb-Pb collisions), the whole series in $Z\alpha$ has to be summed in order to obtain the cross section with sufficient accuracy. For the production of lepton pairs we present new results related to the Coulomb corrections (corresponding to multi-photon exchange of the produced leptons with the nuclei) and to the unitarity corrections (corresponding to the exchange of light-by-light blocks between nuclei). For nuclear bremsstrahlung we calculated the unitarity corrections and a special case related to virtual Delbrück scattering.

1 Introduction

Recently, electromagnetic processes in ultra-relativistic nuclear collisions were discussed in numerous papers (see [1, 2, 3] for a review and references therein) which are connected mainly with operation of the RHIC collider and the future LHC lead-lead option. For these colliders, the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given in Table 1, together with indicative values for relevant lepton pair production cross sections.

Collider	Z	γ	$\sigma_{\rm Born}^{e^+e^-}$ [kb]	$\sigma_{\rm Born}^{\mu^+\mu^-}$ [b]
RHIC, Au-Au	79	108	36.0	0.213
LHC, Pb-Pb	82	3000	227	2.49

Table 1: Colliders and cross sections for lepton pair production.

Strictly speaking, only a few electromagnetic processes with the production of leptons or photons are related to fundamental physics. Nevertheless, many of electromagnetic processes are of great importance for two reasons: either they are dangerous or they are useful for experiments at the RHIC and LHC colliders. Since the Born cross section $\sigma_{\rm Born}^{e^+e^-}$ is huge (see

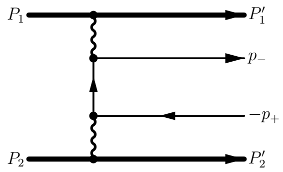

Figure 1: Feynman diagram for lepton pair production in the Born approximation.

Table 1), e^+e^- pair production can be a serious background for many experiments. It is also an important issue for the beam lifetime and luminosity of these colliders [4]. This means that various corrections to the Born cross section are of great importance. The subject is notoriously problematic, and a few controversies have been discussed and clarified in Refs. [1, 5, 6, 7, 8, 9].

A primary reason for the enhanced interest in the nuclear collisions is due to the fact that the typical Lorentz-boosted electric fields of nuclei are immensely strong; they are of the order of

$$\mathcal{E} \sim \frac{Ze}{\rho^2}\gamma = \gamma\, Z\alpha\, \mathcal{E}_{\rm Schwinger} \;\text{ with }\; \rho = \frac{\hbar}{m_e c}\,,\;\; \mathcal{E}_{\rm Schwinger} = \frac{m_e^2 c^3}{e\hbar} = 1.3\cdot 10^{16}\;\frac{\rm V}{\rm cm}\,; \tag{1}$$

therefore,

$$\mathcal{E}/\mathcal{E}_{\rm Schwinger} \;\sim 60 \;\text{ for RHIC and }\; \sim 1800 \;\text{ for the LHC}\,, \tag{2}$$

but the interaction times are very short, so that only a very small four-dimensional space-time volume is available for pair production. As a result, one can still use perturbation theory in terms of the nuclear interaction, but the perturbation parameter $Z\alpha \approx 0.6$ approaches unity for Au-Au and Pb-Pb collisions.

Throughout the paper we use a system of units in which $c = 1$, $\hbar = 1$, $\alpha = e^2/(\hbar c) \approx 1/137$, and denote the electron, muon and nuclear mass as m, μ and M and $L = \ln(\gamma^2)$.

2 Strong-field effects in the e^+e^- pair production

The cross section of one pair production in the Born approximation (described by the Feynman diagram of Fig. 1) was obtained many years ago [10]. Since the parameter $Z\alpha$ is not small the whole series in $Z\alpha$ has to be summed to obtain the cross section with sufficient accuracy. Fortunately, there is an important small parameter

$$\frac{1}{L} < 0.11\,, \qquad L = \ln(\gamma^2)\,, \tag{3}$$

and therefore, in some cases it is sufficient to calculate the corrections in the leading logarithmic approximation (LLA) only.

2.1 Summary of available theoretical results

The exact cross section for one pair production σ_1 can be written in the form

$$\sigma_1 = \sigma_{\rm Born} + \sigma_{\rm Coul} + \sigma_{\rm unit}\,, \tag{4}$$

where two different types of strong-field corrections have been distinguished. We start our discussion with the production of the lightest lepton pairs, electrons and positrons. The *Coulomb*

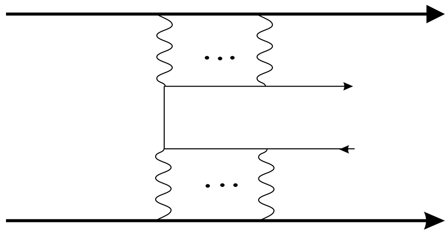

Figure 2: Feynman diagram for the Coulomb correction to the lepton pair production.

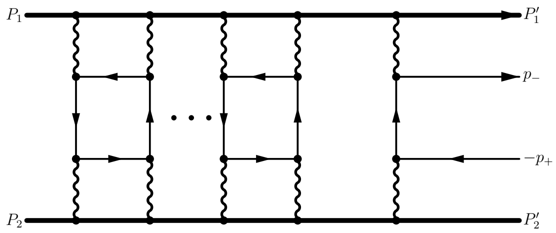

Figure 3: Feynman diagram for the unitarity correction to the lepton pair production

corrections $\sigma_{\rm Coul}$ to electron-positron pair production correspond to multi-photon exchanges of the produced $e^\pm$ with the nuclei (Fig. 2),

$$\sigma_{\rm Coul} = -A(Z\alpha)\,[L^2 - B(Z\alpha)\,L]\,\frac{28}{27\pi}\,\frac{(Z\alpha)^4}{m^2}\,, \tag{5}$$

where the leading coefficient

$$A(Z\alpha) = 6f(Z\alpha) = 6\,(Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)} \approx 1.9 \tag{6}$$

was calculated ten years ago in Ref. [5] and next-to-leading coefficient

$$B(Z\alpha) \approx 5.5 \tag{7}$$

was calculated only recently [9]. It was also shown in Ref. [5] that the Coulomb corrections disappear for large transverse momenta of the produced leptons, at $p_{\pm\perp} \gg m$.

Collider	$\frac{\sigma_{\rm Coul}}{\sigma_{\rm Born}}$	$\frac{\sigma_{\rm unit}}{\sigma_{\rm Born}}$	$\frac{\sigma_{\rm Coul}}{\sigma_{\rm Born}}$ (Ref. [11])
RHIC, Au-Au	-10%	-5.0%	-17%
LHC, Pb-Pb	-9.4%	-4.0%	-11%

Table 2: Coulomb and unitarity corrections to the e^+e^- pair production.

The unitarity corrections $\sigma_{\rm unit}$ which are due to the unitarity requirement of the scattering matrix, correspond to the exchange of the virtual light-by-light blocks between the nuclei (Fig. 3). They were calculated in Ref. [7] and updated in Ref. [8].

It was found that the Coulomb corrections are about 10 % while the unitarity corrections are about two times smaller (see Table 2). In the last column of Table 2 we display the result of Baltz [11] obtained by numerical calculations using a formula for the cross section resulting from "exact solution of the semiclassical Dirac equations". In fact, this formula allows one to calculate the Coulomb correction in the leading logarithmic approximation only which is insufficient in this very case.

2.2 Probabilities for e^+e^- pair production at a given impact parameter between nuclei

Due to the fact that $Z_1Z_2\alpha \gg 1$ for modern heavy-ion colliders, it is possible for $\gamma \gg 1$ to treat the nuclei as sources of the external field and calculate the probability of n-pair production $P_n(\rho)$ in the collision of two nuclei at a given impact parameter ρ. The cross section is then found to be

$$\sigma_n = \int P_n(\rho)\, d^2\rho\,. \tag{8}$$

What do we know about the function $P_n(\rho)$?

It was realized many years ago that in the Born approximation

$$P_1(\rho) \sim (Z\alpha)^4 L \quad \text{at } \rho \sim 1/m \tag{9}$$

and, therefore, this probability can be greater than 1 (see Ref. [12]). It means *(i)* that one should take into account the unitarity corrections, which come from the unitarity requirement for the S-matrix and *(ii)* that the cross section for multiple pair production should be quite large.

It was argued in [13] that the factorization of the multiple pair production probability is described with good accuracy by the Poisson distribution:

$$P_n(\rho) = \frac{[\bar{n}_e(\rho)]^n}{n!} \mathrm{e}^{-\bar{n}_e(\rho)}\,, \tag{10}$$

where $\bar{n}_e(\rho)$ is *the average number of produced* e^+e^- *pairs*. It is evident that the unitarity requirement is fulfilled by the Poisson distribution, whose sum over n gives one.

The probability for producing one pair, given in perturbation theory by $\bar{n}_e(\rho)$, should be modified to read $\bar{n}_e(\rho)\,\exp[-\bar{n}_e(\rho)]$. For the one-pair production it corresponds to replacement:

$$\sigma_{e^+e^-} = \int \bar{n}_e(\rho)\, d^2\rho \quad \to \quad \sigma_{e^+e^-} + \sigma^{\rm unit}_{e^+e^-} = \int \bar{n}_e(\rho)\, \mathrm{e}^{-\bar{n}_e(\rho)}\, d^2\rho\,, \tag{11}$$

where

$$\sigma^{\rm unit}_{e^+e^-} = -\int \bar{n}_e(\rho)\, \left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right]\, d^2\rho \tag{12}$$

is the unitarity correction. It should be noted that the main contribution to $\sigma_{e^+e^-}$ comes from $\rho \gg 1/m$, while the main contribution to $\sigma^{\rm unit}_{e^+e^-}$ comes from $\rho \sim 1/m$.

The function $\bar{n}_e(\rho)$ is a very important quantity for the evaluation of unitarity corrections. It was found for $\gamma \gg 1$ in closed form (taken into account $(Z\alpha)^n$ terms exactly) in Ref. [14]

and the problem of its proper regularization was solved in [6]. But the obtained close form for $\bar{n}_e(\rho)$ is, in fact, a nine-fold integral and its calculation is very laborious.

A simpler approximate expression for $\bar{n}_e(\rho)$ is very desirable. The functional form of this function in the region of interest reads

$$\bar{n}_e(\rho,\gamma,Z) = (Z\alpha)^4\, F(x,Z)\ [L - G(x,Z)]\ , \quad L = \ln(\gamma^2)\,,\ x = m\,\rho\,. \tag{13}$$

The simple analytical expressions for functions $F(x,Z)$ and $G(x,Z)$ is obtained in [7] only at large values of the impact parameters, $\rho \gg 1/m$. On the other hand, for the calculation of the unitarity corrections we need $F(x,Z)$ and $G(x,Z)$ in the range $\rho \sim 1/m$.

In the paper [15] the authors gave a detailed consideration of the function $F(x,Z)$ including tables and compact integral presentation in the form of an "only" five-dimensional integral. Using some numerical calculations for the function $\bar{n}_e(\rho,\gamma,Z)$, a simple approximation for $G(x,Z)$ has been found in [8]:

$$G(x,Z) \approx 1.5\ \ln(x+1.4) + 1.9\,. \tag{14}$$

As a result, the approximate expression

$$\bar{n}_e(\rho,\gamma,Z) = (Z\alpha)^4\, F(x,Z)\ [L - 1.5\ \ln(x+1.4) - 1.9]\ ,\ \ L = \ln(\gamma^2)\,, \quad x = m\,\rho \tag{15}$$

with the function $F(x,Z)$ from [15] can be used for calculation of unitarity corrections with an accuracy on the order of 5 %.

3 Strong-field effects in $\mu^+\mu^-$ pair production

The motivation for the consideration of processes involving heavier lepton pairs is given by the fact that they may be easier to observe experimentally than e^+e^- pair production. This process was recently considered in detail in Refs. [16, 8]. It was found that: *(i)* the Coulomb corrections are small, while unitarity corrections are large; *(ii)* the exclusive cross section differs considerable from its Born value, but its experimental observation is difficult; *(iii)* the inclusive cross section coincides with the Born cross section; *(iv)* the Born contribution can be easily calculated using the equivalent photon approximation (EPA) which has a very good accuracy in this particular case.

3.1 Born cross section for $\mu^+\mu^-$ pair production

Let us consider the production of a $\mu^+\mu^-$ pair,

$$Z_1 + Z_2 \to Z_1 + Z_2 + \mu^+\mu^-\,, \tag{16}$$

using the EPA, but taking into account nuclear electromagnetic form factors. The Born differential cross section $d\sigma_{\rm B}$ for the considered process is related to the cross section $\sigma_{\gamma\gamma}$ for the real $\gamma\gamma \to \mu^+\mu^-$ process by the equation

$$d\sigma_{\rm B} = dn_1 dn_2\, d\sigma_{\gamma\gamma}\,, \tag{17}$$

where dn_i is the number of equivalent photons. A further simple integration leads to the result shown in Table 1. The accuracy of this calculation is of the order of a few percent.

In the impact parameter representation, the probability of muon pair production $P_{\rm B}(\rho)$ in the Born approximation is given as follows (in the LLA),

$$P_{\rm B}(\rho) = \int dn_1 dn_2\, \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 - \boldsymbol{\rho})\, \sigma_{\gamma\gamma} = \frac{28}{9\pi^2}\, \frac{(Z_1\alpha Z_2\alpha)^2}{(\mu\rho)^2}\, \Phi(\rho)\,, \tag{18}$$

where μ is the muon mass. There are two scales in dependence of function $\Phi(\rho)$ on ρ:

$$\Phi(\rho) = \left(4\ln\frac{\gamma}{\mu\rho} + \ln\frac{\rho}{R}\right)\ln\frac{\rho}{R} \quad \text{at} \quad R \ll \rho \le \gamma/\mu\,,$$

$$\Phi(\rho) = \left(\ln\frac{\gamma^2}{\mu^2\rho R}\right)^2 \quad \text{at} \quad \gamma/\mu \le \rho \ll \gamma^2/(\mu^2 R)$$

(here R is the nuclear radius). Reassuringly, when we compare the expression for $\Phi(\rho)$ with numerical calculations based on the exact matrix element, we find good agreement for Pb-Pb collisions: the discrepancy is less then 10 % for $\mu\rho > 10$, so that $\Phi(\rho)$ can be used with good effect for the calculation of unitarity corrections below.

3.2 Coulomb and unitarity corrections

For the Coulomb correction, the following simple estimate can be obtained. Due to the nuclear form factor, there is restriction of transverse momenta of additional exchange photons on the level of $1/R$ and the effective parameter of the perturbation series is not $(Z\alpha)^2$, but $(Z\alpha)^2/(R\mu)^2$. Besides, there is an additional logarithmic suppression because the Coulomb corrections lack the large Weizsäcker–Williams logarithm L. As a consequence, the real suppression parameter is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L}\,, \quad L = \ln\left(\gamma^2\right), \tag{19}$$

which corresponds to the Coulomb correction $\sim 1\,\%$. We have recently carried out a calculation, based on the asymptotic limit of the Coulomb corrections for the muon pair photo-prodution by Ivanov and Melnikov [5]. The results of this investigation, whose details will be published

Collider	$\frac{\sigma_{\rm Coul}}{\sigma_{\rm Born}}$	$\frac{\sigma_{\rm Coul}}{\sigma_{\rm Born}}$ (Ref. [17])
RHIC, Au-Au	-3.7%	-22%
LHC, Pb-Pb	-1.3%	-14%

Table 3: Coulomb corrections to the $\mu^+\mu^-$ pair production.

elsewhere, are in agreement with (19) and they are given in Table 3. In the last column we display the recent results of Baltz [17], which are in strong disagreement with the results in the second column obtained by us. Because the simple estimate (19) corresponds to a decrease of the Coulomb corrections with the growth of the lepton mass, it seems questionable that the

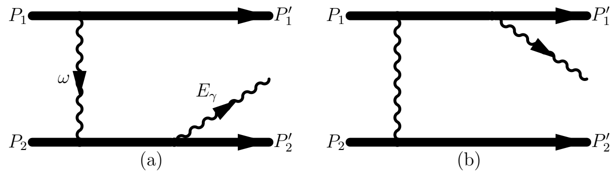

Figure 4: Ordinary nuclear bremsstrahlung via a virtual Compton scattering.

Coulomb corrections for the muon pair production, according to Ref. [17], might be larger that those for the e^+e^- pair production.

The unitarity correction σ_{unit} to muon pair production is described by the exchange of blocks, corresponding to light-by-light scattering via a virtual *electron* loop, between the nuclei (see Fig. 3). As usual,

$$\sigma_{\text{B}} = \int_{2R}^{\infty} P_{\text{B}}(\rho)\ d^2\rho \to \sigma_{\text{B}} + \sigma_{\text{unit}} = \int_{2R}^{\infty} P_{\text{B}}(\rho)\,\mathrm{e}^{-\bar{n}_e(\rho)}\ d^2\rho \tag{20}$$

and

$$\sigma_{\text{unit}} = -\int_{2R}^{\infty} \left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right] P_{\text{B}}(\rho)\, d^2\rho \tag{21}$$

is the unitarity correction for the exclusive production of one muon pair. In LLA we find

$$\delta_{\text{unit}} = \frac{\sigma_{\text{unit}}}{\sigma_{\text{B}}} = -49\ \%\ \text{for the Pb-Pb collisions at LHC.} \tag{22}$$

It is seen that unitarity corrections are large, in other words, the exclusive production of one muon pair differs considerable from its Born value.

However, the experimental study of the exclusive muon pair production seems to be a very difficult task. Indeed, this process requires that the muon pair should be registered without any electron–positron pair production, including $e^\pm$ emitted at very small angles. Otherwise, the corresponding inclusive cross section will be inclusive and close to the Born cross section (for details see Ref. [8]).

4 Strong-field effects for the nuclear bremsstrahlung

Ordinary nuclear bremsstrahlung without excitation of the final nuclei is given by the Feynman diagrams of Fig. 4 and was known in detail many years ago. It can be described as the Compton scattering of the equivalent photon off the incoming nucleus:

$$\mathrm{d}\sigma_{\text{br}} = \mathrm{d}\sigma_{\text{br}}^{a} + \mathrm{d}\sigma_{\text{br}}^{b}\,, \tag{23}$$

and

$$\mathrm{d}\sigma_{\text{br}}^{a} = \mathrm{d}n_1(\omega)\ \mathrm{d}\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2)\,. \tag{24}$$

Here, $\mathrm{d}n_1$ is the number of equivalent photons emitted by the nucleus Z_1 and $\mathrm{d}\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2)$ is the differential cross section for the Compton scattering off nucleus Z_2. We can rewrite these equations as

$$\mathrm{d}\sigma_{\text{br}}^{a} = \mathrm{d}P_a(\rho)\,\mathrm{d}^2\rho\,, \tag{25}$$

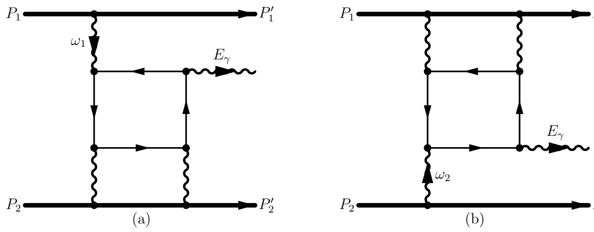

Figure 5: Nuclear bremsstrahlung via the virtual Delbrück scattering.

where the differential probability $\mathrm{d}P_a(\rho)$ assumes the form

$$\mathrm{d}P_a(\rho) = \frac{Z_1^2\alpha}{\pi^2}\,\frac{\sigma_\mathrm{T}(Z_2)}{\rho^2}\left(1 - x_\gamma + \frac{3}{4}x_\gamma^2\right)\frac{\mathrm{d}E_\gamma}{E_\gamma}\,, \quad x_\gamma = \frac{E_\gamma}{E_2} \tag{26}$$

with the Thomson cross section (M is the mass of nucleus)

$$\sigma_\mathrm{T}(Z_2) = \frac{8\pi}{3}\,\frac{Z_2^4\alpha^2}{M^2}\,. \tag{27}$$

Now we calculate the unitarity correction [8].

$$\delta_\mathrm{unit} = \frac{\mathrm{d}\sigma^a_\mathrm{unit}}{\mathrm{d}\sigma^a_\mathrm{br}}\,. \tag{28}$$

In our case it reads

$$\delta_\mathrm{unit} = -\frac{1}{L_\gamma}\int_{2R}^{\infty}\frac{\mathrm{d}\rho}{\rho}\left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right]\,, \quad L_\gamma = \ln\left(\frac{2\gamma_1\,\gamma_2^2\,(1-x_\gamma)}{R\,E_\gamma}\right). \tag{29}$$

An evaluation of this integral gives the following result for a photon energy $E_\gamma = 1$ GeV:

$$\delta_\mathrm{unit} = -19\,\% \text{ for the RHIC}\,, \quad \delta_\mathrm{unit} = -15\,\% \text{ for the LHC}\,. \tag{30}$$

5 Large contribution of the virtual Delbrück scattering into nuclear bremsstrahlung

Recently, in Ref. [18], we have considered the emission of photons not via the virtual Compton subprocess, but via another one – the virtual Delbrück scattering subprocess of Fig. 5 (the first results were presented at the PHOTON-2007 conference in Paris). Nuclear bremsstrahlung via virtual Delbrück scattering in the lowest order of quantum electrodynamics is described by Feynman diagram of Fig. 5. A first note about this process was given in Ref. [19].

At first sight, this is a process of a very small cross section since $\sigma \propto \alpha^7$. But at second sight, we should add a very large factor $Z^6 \sim 10^{11}$ and take into account that the cross section scale $1/m^2$ is determined not by the nucleon mass, but the electron mass. And last, but not least, we found that this cross section has an additional logarithmic enhancement of the order of

$$L^2 \gtrsim 100\,, \quad L = \ln\left(\gamma^2\right). \tag{31}$$

Thus, the estimate is

$$\sigma \sim \frac{(Z\alpha)^6\,\alpha}{m^2}\,L^2\,. \tag{32}$$

Our analytical result (see for detail Refs. [18]) is

$$\sigma = C\,\frac{(Z\alpha)^6\,\alpha}{m^2}\,L^2 \tag{33}$$

with $C \approx 0.4$. This cross sections is considerably larger than that for ordinary nuclear bremsstrahlung in the photon energy range:

$$m \ll E_\gamma \ll m\,\gamma\,. \tag{34}$$

Thus, the discussed cross section is

$$\sigma = 14 \text{ barn for RHIC}, \quad \sigma = 50 \text{ barn for LHC}\,. \tag{35}$$

That is quite a serious number! Note for comparison, that the last cross section is 6 times larger than the total hadronic/nuclear cross section in Pb–Pb collisions, which is roughly 8 barn. The energy and angular distribution of photons is also calculated in [18].

We conclude this brief report by emphasizing that Coulomb and unitarity corrections, and loop effects (virtual Delbrück scattering) are essential for an accurate quantitative understanding of photon and lepton production cross sections in ultrarelativistic heavy-ion collisions. The extremely strong fields encountered in these processes lead to a physical situation not encountered anywhere else in nature, and thus, surprising effects like loop-dominance over the tree-level graphs for photon production or a 50 % decrease of an exclusive over an inclusive muon pair production cross section due to unitarity represent testimonies of the extreme state of matter and radiation at the RHIC and LHC colliders.

Acknowledgments

U.D.J. acknowledges support from the National Science Foundation and from the Missouri Research Board. V.G.S. is supported by grants of RFBR 08-02-00334-a and NSh-1027.2008.2 and acknowledges support from GSI Darmstadt (project HD–JENT).

References

[1] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky and Yu. Kharlov, Phys. Rep. **364** 359 (2002).

[2] G. Baur, K. Hencken and D. Trautmann, Phys. Rep. **453** 1 (2007).

[3] A. Baltz et al., Phys. Rep. **458** 1 (2008).

[4] S.R. Klein, Nucl. Instrum. Meth. **A459** 51 (2001).

[5] D.Yu. Ivanov, A. Schiller and V.G. Serbo, Phys. Lett. **B454** 155 (1999).

[6] R.N. Lee and A.I. Milstein, Phys. Rev. **A61** 032103 (2000).

[7] R.N. Lee, A.I. Milstein and V.G. Serbo, Phys. Rev. **A65** 022102 (2002).

[8] U.D. Jentschura, K. Hencken and V.G. Serbo, Eur. Phys. J. **C58** 281 (2008).

[9] R.N. Lee and A.I. Milstein, arXiv:hep-ph/0903.0235 (2009).

[10] L.D. Landauand E.M. Lifshitz, Phys. Z. Sowjet. **6** 244 (1934);
G. Racah, Nuovo Cimento **14** 93 (1937).

[11] A. Baltz, Phys. Rev. **C71** 024901 (2005) and arXive:nucl-th/0409044 (2004).

[12] G. Baur, Phys. Rev. **A42** 5736 (1990).

[13] G. Baur, Phys. Rev. **D41** 3535 (1990);
M. J. Rhoades-Brown and J. Weneser, Phys. Rev. **A44** 330 (1991);
C. Best, W. Greiner and G. Soff, Phys. Rev **A46** 261 (1992);
K. Henken, D. Trautman and G. Baur, Phys. Rev. **A51** 998 (1995);
E. Bartoš, S.G. Gevorkyan, E.A. Kuraev and N.N. Nikolaev, Phys. Lett. **B538** 45 (2002).

[14] A. Baltz and L. McLerran, Phys. Rev. **C58** 1679 (1998);
B. Segev and J.C. Wells, Phys. Rev. **A57** 1849 (1998);
A. Baltz, F. Gelis, L. McLerran and A. Peshier, Nucl. Phys. **A695** 395 (2001).

[15] R. N. Lee and A. I. Milstein, arXive:nucl-th/0610008 (2006).

[16] K. Hencken, E. A. Kuraev and V. G. Serbo. Phys. Rev. **C75** 034903 (2007).

[17] A. Baltz, arXiv:0901.0891 [nucl-th] (2009).

[18] I. F. Ginzburg, U. D. Jentschura and V. G. Serbo, Phys. Lett. **B658** 125 (2008);
I. F. Ginzburg, U. D. Jentschura and V. G. Serbo, Eur. Phys. J. **C54** 267 (2008).

[19] G. Baur and C. Bertulany, Z. f. Phys. **A330** 77 (1988).