

Photon 2009: Summary of Theory Talks

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The conference *Photon 2009* on the structure and the interactions of the photon included sessions on photon-photon collisions and a future high-energy photon linear collider. This summary of theoretical contributions to the conference therefore has two parts. I will discuss the physics potential of photon colliders with an emphasis on the study of electroweak physics and the search for physics beyond the standard model. Secondly, I will describe a few highlights in recent progress in the understanding of the properties and the interaction of the photon, comprising the production of prompt photons, the photon structure and exclusive hadron production, small- x and total cross sections of deep inelastic scattering. Finally, I will review the status of the comparison of measurement and theory for the muon anomalous magnetic moment $g - 2$.

1 Introduction

The photon, the leitmotif of the conference *Photon 2009*, has appeared in the various talks in two different roles. First, the photon is an ideal tool to probe new physics in high-energy photon-photon or photon-electron collisions. During recent years a lot of work has been devoted to the study of the prospects of photon colliders in the search for physics beyond the standard model or to perform precision measurements of standard model phenomena. It was also suggested that a photon collider would be the suitable place to study the properties of a Higgs boson, if it had a mass of about 120 GeV. In particular, the possibility to build a photon-collider as a precursor of an electron-positron linear collider has been discussed, but was not supported by the International Linear Collider Steering Committee [1] and the construction of such a facility will have to wait for its turn, maybe as an extension of an e^+e^- linear collider. Corresponding plans will have a chance to be revived only, if results of forthcoming experiments at the LHC would require new theoretical ideas whose further scrutiny at a photon collider can be expected to considerably improve our knowledge. Till then, it may be worthwhile to study the feasibility of experimentation with $\gamma\gamma$ and γp collisions at the LHC.

Secondly, as the main and traditional subject of this series of conferences, the photon has appeared as a research object on its own right. The study of the photon and its properties, described in terms of structure functions that are measured first of all in virtual-photon scattering (i.e., in deep-inelastic lepton-nucleon scattering), and its generalizations needed for a description of exclusive processes, opens a wide field of research topics in QCD, located at the border-line of perturbative and non-perturbative phenomena. There is a wealth of experimental data not yet understood at a quantitative level and more theoretical work is needed. The photon as a theory laboratory, as an object to gain a deeper understanding of theoretical concepts,

as for example factorization in exclusive processes, is often helpful. Recent progress in this field of research is highlighted in a second part of this summary.

A topic of special importance where aspects of both electroweak and strong interactions play a crucial role, is the measurement of the anomalous magnetic moment of the muon and the comparison with theoretical predictions. A short status review is presented at the end of this article.

This printed version of the summary talk includes only a tiny portion of the figures that have been shown at the conference. The text should therefore be read together with the slides [2].

2 Physics potential of a photon collider

2.1 Motivation

A first motivation for the study of scattering processes with one or two photons in the initial state comes from a naive evaluation of typical cross sections for standard model processes. These cross sections are often comparable, in some cases even larger, than for corresponding processes in e^+e^- annihilation. From a quick look at Fig. 1 it should be obvious that detailed studies including experimental conditions are worthwhile being performed. In particular one may expect that through studies of the W^+W^- final state one can obtain information on 3- and 4-boson interactions. A crucial question is, however, whether photon collisions can be realized with high enough luminosity and present technical design studies indicate that this would indeed be the main limitation. It is therefore important to identify possible measurements at a photon collider that provide information complementary to what can be found at an e^+e^- linear collider. It was argued that in particular with respect to the determination of properties of standard model or supersymmetric Higgs bosons a photon collider may be advantageous in comparison with e^+e^- collisions¹.

For a meaningful assessment of the possible reach of high energy experiments in the search for new phenomena, it is an important prerequisite to know the constraints on model parameters imposed by present day's experiments, including those at low energies. A new tool has been presented [5] that allows to obtain allowed parameter ranges for supersymmetric models and

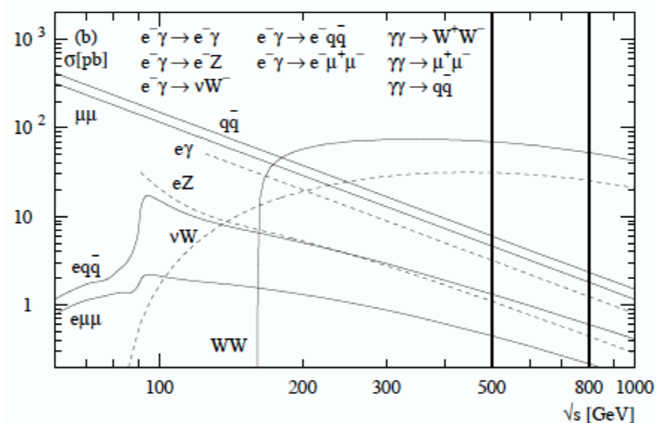


Figure 1: Typical cross sections at $e\gamma$ and $\gamma\gamma$ colliders [3].

¹For more details and for phenomenological aspects of QCD and hadron physics at photon colliders, see Refs. [3, 4].

general 2-Higgs-doublet models. Flavor physics observables like rare B decays are viewed as most promising for this purpose.

2.2 Higgs bosons at a photon collider

It is a common belief that a Higgs boson, if it exists, cannot be missed at the forthcoming experiments at the LHC. The question whether an observed Higgs boson fits into a supersymmetric model will then be of utmost importance. In the minimal supersymmetric standard model (MSSM), there is a large region in parameter space where the LHC will not be able to distinguish a SM from a MSSM Higgs boson. This 'blind wedge' covers values above $m_A \simeq 200$ GeV, the mass of the CP -odd Higgs particle, and a region around $\tan\beta \simeq 5$ increasing in size with increasing m_A . Precise measurements of the decay branching ratios will then be needed. In particular if CP -violating interactions are present in the Higgs sector, a potential photon collider will provide additional help to identify the correct underlying theory.

In $\gamma\gamma$ collisions, s -channel production of the electrically neutral SM Higgs boson proceeds via a triangle loop of the heaviest SM particles, mainly top and W . A measurement of the production cross section will provide a determination of the 2-photon decay width $\Gamma(H \rightarrow \gamma\gamma)$ [6]. Realistic studies have shown that a statistical precision of roughly 2% can be reached for a light Higgs boson in the interesting mass range of $M_H = 120$ GeV with a total luminosity of 410 fb^{-1} in the decay channel $H \rightarrow b\bar{b}$. For a heavy CP -even or CP -odd MSSM Higgs boson, the precision is worse, ranging between 10 and 20% [7]. If the energy of $\gamma\gamma$ collisions is high enough, the production of two Higgs bosons will be observable and cross section measurements will allow one to determine also the values of the Higgs boson self couplings. Corresponding information will be crucial to an analysis of the Higgs sector. At an e^+e^- linear collider, a precision between 10 and 20% for a measurement of the three-Higgs

coupling λ_{HHH} can be reached, provided the center-of-mass energy is $\sqrt{s} = 1$ TeV and M_H in the range between 100 and 200 GeV. At lower $\sqrt{s} = 500$ GeV, $\Delta\lambda_{HHH}/\lambda_{HHH}$ will degrade quickly, in particular for larger M_H . For larger values of the Higgs mass up to $M_H \simeq 200$ GeV, measurements of the process $\gamma\gamma \rightarrow HH$ are expected to provide a determination of $\Delta\lambda_{HHH}/\lambda_{HHH}$ with a statistical precision of about 20%, even in the energy range between $\sqrt{s} \simeq 520 - 650$ GeV, assuming 100% tagging efficiency (see Fig. 2).

In models which implement CP violation in the Higgs sector, a photon collider will play a crucial role [9]. In the presence of CP violation, the three neutral states in the SUSY Higgs sector may mix with no fixed CP property. The coupling to the Z boson, relevant for

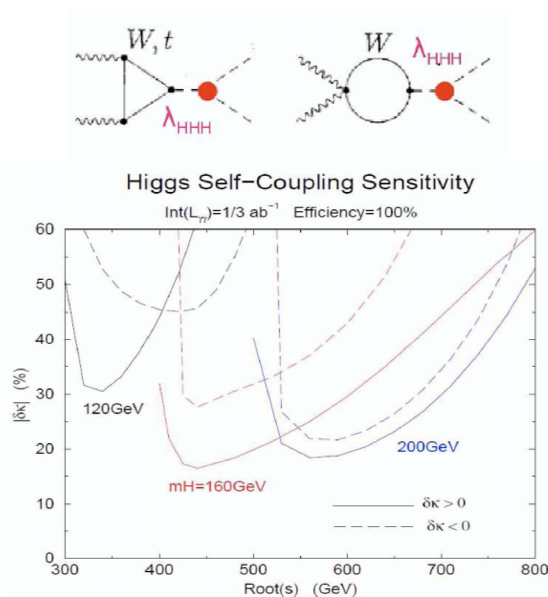


Figure 2: Higgs boson self coupling: sensitivity at a $\gamma\gamma$ collider [8].

the dominating discovery channel $e^+e^- \rightarrow Z\phi_1$ may be reduced for the lightest scalar mass eigenstate ϕ_1 . In fact, the search at LEP may have missed such Higgs bosons and existing limits, for example from OPAL, cannot exclude the full range of $\tan\beta$ even for masses in the range below 50 GeV. It may also be missed at the Tevatron and at the LHC if the coupling to top quarks is also reduced. A photon collider would be the place for a discovery since there these states can always be produced. The possibility to control the polarization of the back-scattered photons will be an important prerequisite for a study of the CP properties of Higgs bosons; an analysis of WW and ZZ final states in combination with information obtained from top (and τ) polarization observables may then provide access to regions in the parameter space of CP -violating scenarios which are unreachable by the LHC. Also mixed polarization-charge asymmetries in $e\gamma$ collisions with a polarized electron beam have been studied as a means to enhance the signal of heavy CP -even or CP -odd Higgs production [9].

In the MSSM or in general 2-Higgs-doublet models, violation of CP can occur via different mechanisms. As is well-known in the case of CP violation in the quark sector of the standard model, a careful definition of parameters is needed, even more so in the Higgs sector in order to identify the specific mechanism. Reparametrization invariants, similar to the Jarlskog determinant, can be defined and allow to distinguish CP violation through mixing of states or through direct CP -violating interactions at tree level [10].

A characteristic feature of the MSSM is that there is decoupling of the heavy states in the Higgs sector, i.e. the MSSM becomes indistinguishable from the SM if the masses of the heavier Higgs bosons (and of the superpartner particles, of course) become large. In a general 2-Higgs doublet model this is not necessarily the case. Even if the model parameters are chosen such that all tree-level couplings to fermions and gauge bosons are the same as in the standard model, differences may enter through one-loop contributions. The production of a pair of the lightest CP -even Higgs boson h^0 in $\gamma\gamma$ collisions proceeds both in the standard model and in its extensions with two Higgs doublets via one-loop diagrams and is therefore an ideal place to look for differences. An important contribution to this process comes from one-loop corrections to the $h^0h^0h^0$ vertex which is affected by non-decoupling effects: after renormalization, quartic mass terms of the heavier Higgs bosons remain and lead to enhancements with respect to the tree-level coupling. These non-decoupling effects may be visible in the production cross section $\sigma(\gamma\gamma \rightarrow h^0h^0)$. Also diagrams with the charged Higgs boson contribute here and depending on its mass m_{H^\pm} , cross sections can be orders of magnitudes above the (MS)SM prediction [11, 12]. More exotic extensions of the Higgs sector may predict the existence of doubly charged Higgs bosons. The production process $\gamma\gamma \rightarrow H^{++}H^{--}$ is enhanced by the square of the charge as compared to production in e^+e^- annihilation and provides another example of the complementarity in searches for physics beyond the standard model that a photon collider may provide.

2.3 Non-standard gauge boson couplings at photon colliders

One of the less well-studied properties of the SM are the gauge boson self-interactions. Precise measurements would require high enough energies to produce at least a pair of W or Z bosons with large cross sections. At a photon-photon collider such processes, and even three- and four-boson production, could be studied. It is conceivable that the processes $\gamma\gamma \rightarrow W^+W^-$ or $e\gamma \rightarrow \nu W$ could be measured with a precision high enough to be sensitive to two-loop contributions. From the theoretical point of view, a full understanding of these processes is difficult since it involves the yet unsolved problem of a consistent definition of scattering

amplitudes for unstable particles [13].

The main motivation for a study of multiple boson production consists in the possibility to observe deviations from standard model predictions. Many speculative models of physics beyond the standard model (from more conventional extensions of the underlying gauge group to models with extra dimensions) lead to effective three- and four-gauge boson vertices with couplings that deviate from the standard model prediction. There are various parametrizations of these couplings that are conventionally used to perform model-independent studies. As a minimal requirement, anomalous couplings have to be defined in such a way that the electromagnetic $U(1)$ gauge invariance is respected. However, for realistic underlying theories that are compatible with existing data, one should base studies on an effective Lagrangean that takes also $SU(2)$ invariance into account. Limits on anomalous gauge-boson couplings have been determined by the LEP experiments and could be considerably improved at a linear collider based on measurements of the process $e^+e^- \rightarrow W^+W^-$. Also the LHC is expected to contribute to this kind of analyses since there protons can be used as a source of photons and anomalous gauge boson couplings could be studied in processes like $pp \rightarrow pp + W^+W^-$ and $pp \rightarrow pp + ZZ$ (the latter is forbidden at tree-level in the standard model). For the LHC, studies taking into account a realistic detector environment have obtained limits that improve those obtained by the LEP experiments by a factor of up to 10^3 , but these studies assume independent variations of coupling parameters not respecting $SU(2)$ symmetry [14]. A study based on a $SU(2)$ -invariant effective Lagrangean [15] has revealed strong correlations and smaller sensitivities. Limits that could be obtained at a photon collider are comparable with those from an e^+e^- linear collider, but better than those from photon-photon collisions at the LHC by a factor of roughly 10^2 .

3 Direct photons

The study of direct photon production as a testing ground for QCD has a long tradition, also as a topic at this series of conferences. A considerable amount of experimental and theoretical work was reported already at *Photon 2007* [16]. A very good description by QCD predictions at NLO has been obtained for inclusive prompt photon production in hadronic collisions from the majority of experiments (i.e., with the exception of E706) for energies ranging from about 20 GeV up to 1.96 TeV and up to transverse momenta slightly above 200 GeV. Recent improved measurements of prompt photon + jet production by D0 show a slight disagreement of the predicted p_T -slope. Data for photoproduction from H1 and ZEUS are underestimated by present NLO calculations if one considers inclusive prompt photon production, even at the largest measured values of the photon transverse momentum of $p_T = 10$ GeV [17]. In the presence of an accompanying jet, however, there appears to be good agreement if the photon p_T is large, i.e. for $p_T > 7$ GeV. The presence of a second hard scale, provided by the jet- p_T , seems to make predictions of a perturbative calculation more reliable. The rapidity distribution is usually not described as good, but this is simply so because corresponding data are dominated by small values of p_T . There are now also data for prompt photon production in the deep-inelastic kinematic regime from both experiments at HERA. These data, however, can at present be compared with leading-order predictions only and it is not surprising that no agreement of data with theory can be obtained.

It is interesting to note that an alternative approach based on the so-called quasi-multi-Regge-kinematics [18] describes both inclusive photon production and the associated production of a photon and a jet in deep inelastic scattering at HERA. Moreover, it was successfully applied

to heavy quark production. In this approach, a class of corrections beyond the collinear parton model is resummed in the framework of an effective theory and only tree-level calculations are needed to obtain predictions. It remains to be studied how this approach could be extended to the next-to-leading order level, so that a reliable estimation of scale uncertainties can be obtained.

It would be very interesting if additional direct photon data for more exclusive measurements could be made available, as for example $\gamma + c/b$ -tagged jets, or γ plus a rapidity gap. Also in this case, more theoretical work is needed. The recent work on photon + jet associated production in pp collisions [19] has shown that forthcoming measurements at the LHC are promising and may provide sensitivity on parton distribution and fragmentation functions.

Prompt photon production in nuclear collisions at RHIC is believed to be a crucial measurement since it might provide a reference process to be compared with jet quenching. Since photons, once produced, do not interact strongly with the nuclear medium in heavy-ion collisions, a comparison of both processes should help in understanding the dynamics of the quark-gluon plasma. The essential object needed for reliable predictions of photon production is the gluon density in nuclei, but it is poorly constrained by present fixed-target data. This lack of precise knowledge leads to huge uncertainties at low x and low scales. In addition, effects like jet-photon conversion, medium-induced photon emission, or photon quenching may be important.

Data from PHENIX at RHIC on the nuclear production ratio R_{AA} for photons produced in $Au + Au$ collisions are consistent with various models predicting almost no suppression (see Fig. 3). Certainly, more precise data are needed; but it seems that also the observed problems in the description of other prompt photon data, in particular from photoproduction, have to be resolved before firm conclusions can be drawn with confidence.

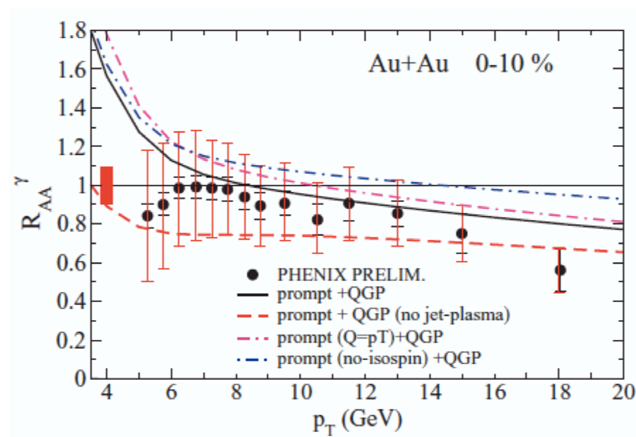


Figure 3: Prompt photon production in $Au + Au$ collisions at PHENIX [20].

4 Photon structure and exclusive hadron production

Since its first measurement by PLUTO in 1981, the analysis of the photon structure function F_2^γ has been a fruitful laboratory of perturbative QCD. The corresponding final LEP2 results published in 2005 have provided us with very precise data spanning two orders of magnitude in Q^2 , and more improved data may be expected from Belle and BaBar, or possibly from the ILC. Most of the data come from deep inelastic scattering off a photon target with negligibly small virtuality P^2 . At large P^2 , the photon structure function $F_2^\gamma(x, Q^2, P^2)$ is suppressed; however, in the kinematic range $Q^2 \gg P^2 \gg \Lambda_{\text{QCD}}^2$ perturbative QCD provides a definite prediction,

THEORY SUMMARY

both for the shape and the magnitude of $F_2^\gamma(x, Q^2, P^2)$ since no separate non-perturbative hadronic input is needed as for the case of $P^2 \simeq 0$. In [21], heavy quark effects at NLO and target mass corrections have been studied for the structure function of a highly virtual photon, with the conclusion that PLUTO data at $Q^2 = 5 \text{ GeV}^2$, $P^2 = 0.35 \text{ GeV}^2$ are better described with 3 massless quarks + massive charm than with 4 massless quarks. For L3 data at higher Q^2 and P^2 where b quarks contribute, the comparison is not conclusive, both due to the smaller b -quark charge and the less precise measurements.

The hadronic properties of the electromagnetic current measured in deep-inelastic scattering and described with the help of structure functions can be studied in the framework of perturbative QCD since the factorization theorem allows one to separate structure functions into hard parton scattering cross sections and non-perturbative hadronic matrix elements of certain operators constructed from quark and gluon fields. In the case of inclusive deep-inelastic scattering, these matrix elements correspond to the kinematics of γ^* -hadron forward scattering and are called parton distribution functions (PDF). In the case of non-forward scattering, as for virtual Compton scattering $\gamma^* + p \rightarrow \gamma + p$, the necessary matrix elements correspond to so-called generalized parton distribution (GPD) functions. If the photon in the final state of virtual Compton scattering is replaced by a hadron, so that exclusive forward-production of hadrons is described, one has to introduce in addition distribution amplitudes (DA) or, for backward-production after $t \rightarrow u$ -channel crossing, transition distribution amplitudes (TDA). Finally, $s \rightarrow t$ crossing leads from GPDs to generalized distribution amplitudes (GDA) needed to describe hadron production in two-photon processes. The diagrams shown in Figure 4 illustrate how the various hard exclusive processes and the objects required for their description are related to each other.

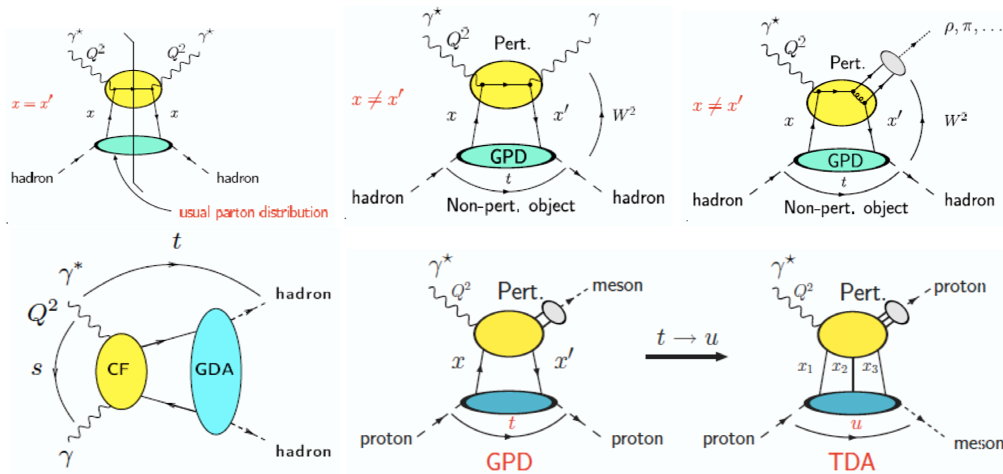


Figure 4: Basics of hard exclusive processes [22].

Measurements of exclusive processes like deeply virtual Compton scattering and meson production, also for experiments with polarized beams, are being performed at moderate and high energies and the amount and precision of corresponding data require corresponding efforts towards an improved theoretical understanding. Considerable work in this direction has been done in the recent years and a framework has been developed to describe a large number

of processes. However, factorization has been shown to work only for very few cases (e.g. $\gamma^*p \rightarrow \gamma p$), for others a prove at any order is not yet available, but factorization is plausible (e.g. for processes involving TDAs). Some studies have revealed explicit factorization breaking (e.g. $\gamma_T^*p \rightarrow \rho_T p$ with a transversely polarized photon and ρ) [22].

Corresponding calculations are complicated and the study of simplified situations may often be helpful. For example, the process $\gamma\gamma \rightarrow \gamma\gamma$ is considered as a theory playground to study factorization in GPDs close to threshold kinematics [23]. No phenomenological application for this process is visible, but apart from insight into the way how factorization works, one may also obtain hints for a reasonable choice of parametrizations of GPDs in more realistic cases. Similarly, calculations for $\gamma\gamma^* \rightarrow \pi\pi$ in the framework of a euclidean ϕ_E^3 model have been used to study the duality between factorization into GDAs and TDAs [24]. In the overlap region, when both s and t are small compared to Q^2 , there is an ambiguity since both factorization into GDAs and TDAs can operate and it is not clear whether the two descriptions are equivalent to each other on a quantitative level, or whether the predictions based on both mechanisms should be added. In the scalar ϕ_E^3 model, duality between the two mechanisms has been demonstrated. If this property also holds for the case of QCD, then it would pose strong restrictions on the allowed non-perturbative ingredients needed for the description of various exclusive processes.

The framework for backward production of mesons was described in [25]. The kinematics with small u forces one to consider matrix elements that describe the exchange of three quarks and factorization leads to TDAs, i.e. probability amplitudes to find a meson inside a nucleon. The non-perturbative TDAs have to be modeled by a comparison with measurements, e.g. with existing data for $\gamma^*p \rightarrow \pi p$ or $\gamma^*p \rightarrow \eta p$, or based on measurements of related processes like $p\bar{p} \rightarrow \gamma^*\pi^0 \rightarrow \ell^+\ell^-\pi^0$ or $\gamma^*p \rightarrow J/\Psi p$, which may be accessible by future experiments, for example at GSI/FAIR, at JLAB or at B-factories.

GDAs are needed to describe hadron-pair production in 2-photon processes and can be used also to describe the production of two pairs of mesons, e.g. in $\gamma\gamma \rightarrow \pi^+\pi^-\pi^+\pi^-$. This process was identified as a candidate for the observation of the perturbative odderon. In the language of perturbative QCD, the odderon is described by the exchange of three gluons in the color singlet state. A suitably defined angular asymmetry for 2π -pair production is sensitive to the interference of the odderon and the Pomeron. The 2π GDAs are unknown and have to be modeled; reasonable choices for them predict asymmetries that rise above the level of 10% only at very low or very large 2π invariant masses. The event rates in 2-photon scattering at the LHC may be large enough, but background from hadronic interactions will most likely prevent a corresponding analysis [26].

Similarly to conventional parton distribution functions, also DAs and their generalizations, the transverse-momentum unintegrated light-cone wave functions obey simple evolution equations derived in perturbative QCD, at leading order so-called Efremov-Radyushkin-Brodsky-Lepage evolution equations. The DAs reach an ultraviolet fixed-point at large scales and are, consequently, uniquely defined by perturbative QCD; however, scales that are accessible by present experiments are low and corresponding predictions are sensitive to the non-perturbative initial conditions. Therefore, a comparison with data is mainly a test of various model assumptions used to obtain these initial conditions, as for example models based on QCD sum rules, relativistic quark models, instanton liquid models, or effective chiral quark models. The case of pion or photon DAs is particularly interesting. From lattice QCD there are results for the pion DAs available. Experimental information is obtained indirectly from hard di-jet production by incident pions or real photons as well as from the electromagnetic pion form factor and from the $\gamma\pi$ transition form factor. In the case of pions, the DAs (and corresponding generalized form

factors) obtained from a chiral quark model and evolved at leading order [27] are in reasonable overall agreement with the available data and results from lattice simulations.

The classical area for the study of GPDs is deeply virtual Compton scattering (DVCS), i.e. the process $e^\pm N \rightarrow e^\pm N \gamma$. Other related processes are the production of lepton pairs, e.g. $e^\pm N \rightarrow e^\pm N \mu^+ \mu^-$ or $\gamma N \rightarrow N' e^+ e^-$ and hard exclusive meson production, e.g. $ep \rightarrow ep \pi^0$, $ep \rightarrow ep \rho$, $ep \rightarrow en \pi^+$, $ep \rightarrow en \rho^+$. There is a considerable amount of data available now, for example from H1, ZEUS and HERMES at HERA, or from the CEBAF Large Acceptance Spectrometer (CLAS) and Hall A (CEBAF Experiment 91-010) collaborations at JLAB. Moreover, measurements are expected to provide more data from COMPASS, or from experiments at JLAB with a future 12 GeV beam upgrade.

Before fitting GPDs to data, their functional form has to be carefully modeled. GPDs are intricate functions with a non-trivial interplay of the dependence of various kinematic variables and subject to constraints in a number of limiting cases. For example, they have to reduce to the conventional PDFs in the limit of forward scattering, there are various sum rules, and at LO they have to obey a positivity constraint. Moreover, their evolution at next-to-leading order has to be described properly. Because of their relation with conventional PDFs, it seems reasonable to perform a simultaneous fit of inclusive DIS and DVCS data. If based on a flexible ansatz for the GPDs, good fits to DVCS data can be obtained [28]. The complicated structure of GPDs make them a promising tool to reveal the transverse distribution of partons in a nucleon, or to address the spin content of the nucleon. It will be interesting to perform at more detail comparisons with non-perturbative methods, as for example lattice simulations.

5 Small- x and total cross sections

At small Bjorken- x , the behavior of structure functions in deep inelastic scattering is most conveniently studied in terms of color dipoles, the pair of color charges carried by quark-anti-quark pairs into which the virtual photon splits during its interaction with a nucleon. At high energies, these color charges propagate along straight lines separated by a transverse distance that remains unchanged, and the associated color dipole is described by a two-Wilson-line operator. The low- x evolution of the structure functions is then governed by the rapidity evolution of the color dipoles. At leading order in the leading logarithmic approximation, the rapidity evolution can be determined by the non-linear Balitsky-Kovchegov (BK) equation. The BK equation extends the BFKL equation by a non-linear term which describes parton annihilation and predicts saturation at low x . The leading-order BK equation is conformally invariant, however, at next-to-leading order, conformal invariance is broken in QCD by the running of the strong coupling constant.

It should be instructive to consider the evolution of color dipoles in a $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. The $\mathcal{N} = 4$ SYM theory has received a lot of interest recently as it is conjectured to be dual, at strong coupling, to a type IIB string theory in 5-dimensional Anti-de-Sitter space-time. Its β -function vanishes and one can hope that its conformal invariance provides strong restrictions on the correct effective action of QCD at high energies. At LO, the color dipole evolution equation has the same form as in QCD and it has been demonstrated [29] that with suitably defined "composite conformal dipole operators", the conformally invariant analytic part of the QCD evolution equation at NLO can be obtained in that theory. It has also been shown that the resulting evolution equation agrees with the forward BFKL equation at next-to-leading order.

The complicated physics of the deep inelastic structure functions results from the cooperation of weak coupling due to the asymptotic freedom of QCD at high energies and the high density of gluons inside the nucleus. The gluon density increases towards lower Bjorken- x since at weak coupling, soft and collinear gluon emission is favored. Ultimately, this would break unitarity. However, gluon bremsstrahlung is limited by recombination effects, an effect which restores unitarity and leads to saturation at a limit depending on x and Q^2 given by $Q_s^2(x) \propto 1/x^\lambda$ with $\lambda \simeq 0.2 - 0.3$.

The situation may change dramatically in heavy-ion collisions where data indicate the presence of strong coupling effects. According to the famous conjecture by Maldacena, there is a correspondence between a strongly coupled gauge theory and a string theory at weak coupling. If this conjecture is right, it would be possible to infer from the study of the classical dynamics of a black hole in AdS₅-supergravity properties of photon interactions with a strongly coupled quark-gluon plasma. Corresponding investigations [30] lead to the conclusion that saturation should set in faster and the behavior of $Q_s^2(x)$ would be changed to $Q_s^2(x) \propto 1/x$. As an important consequence, since strong coupling would lead to unlimited, quasi-democratic parton branchings, one would expect to observe no collimated jets in e^+e^- annihilation, and no large- x partons in the hadron wave functions, i.e. no jets in the forward or backward direction of hadron-hadron collisions. It has to be seen whether future data from RHIC can contribute to a clarification of these concepts.

6 Vacuum polarization and $g - 2$

The comparison of experimental data with theoretical predictions for the anomalous magnetic moment of the muon has received considerable interest in the past years, after the experiment E821 at Brookhaven has published the most precise measurement of $a_\mu^{\text{exp}} = (g_\mu - 2)/2 = (11659208.0 \pm 6.3) \times 10^{-10}$. Since then a discrepancy of about 3σ has been observed. It is an open question whether this is a sign of new physics, or of an incomplete understanding of photon-hadron interactions contributing to a_μ .

At present, the precision of theoretical calculations nicely match the size of the experimental uncertainty, but for a conclusive comparison of theory with results of future experiments aiming at an accuracy of $\pm 1.5 \times 10^{-10}$, considerable improvements in the theoretical understanding will be necessary. The present theoretical uncertainty is dominated by the precision with which hadronic contributions to a_μ can be calculated.

Hadronic contributions enter at the two-loop level via the hadronic vacuum polarization (the two-point correlator of the electromagnetic current) and at the three-loop level via light-by-light scattering (a certain component of the four-point correlator of the electromagnetic current). The

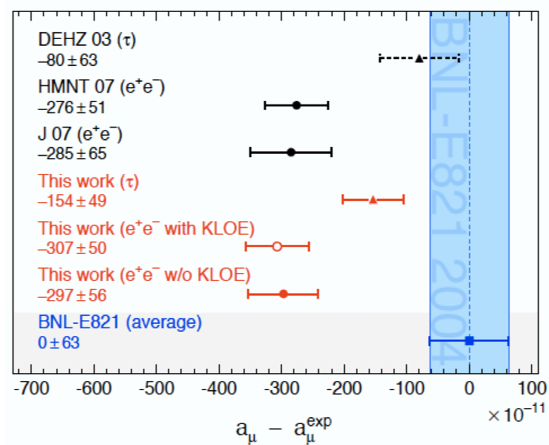


Figure 5: Status of $g - 2$ [31].

THEORY SUMMARY

former contribution, denoted $a_\mu^{\text{had,LO}}$, was traditionally calculated, by using a dispersion relation, from data of the total hadronic cross section for e^+e^- annihilation and its accuracy is limited by those data. Improvements in the measurement of e^+e^- cross section data by the CMD-2, SND and KLOE collaborations have reduced the accuracy of $a_\mu^{\text{had,LO}}$ to $\pm 5.0 \times 10^{-10}$ [31]. On the other hand, with the increasing precision of LEP data for the decay spectrum of the τ , it became possible to use, instead of e^+e^- data, the τ spectral functions in the calculation of $a_\mu^{\text{had,LO}}$. The most recent analyses [31] based on τ data reports a precision of $\pm 4.9 \times 10^{-10}$, very similar to the e^+e^- -based calculations. The new analysis takes into account recent high-precision data from the Belle collaboration on the decay $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and is based on a new analysis of isospin-violating corrections. Interestingly, this analysis results in a better agreement with e^+e^- -based calculations than previously, but is shifted away from a_μ^{exp} by -15.4×10^{-10} . The long-standing discrepancy between spectral functions for $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ is reduced, but still present, in particular for data from KLOE. It will be interesting to see whether the inconclusive situation persists when preliminary data from BaBar on $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ using the ISR-method are finalized so that they can be used for an updated calculation of $a_\mu^{\text{had,LO}}$.

The hadronic contribution to a_μ from light-by-light scattering, a_μ^{hLbL} , cannot be directly related to experimental data and there is at present no complete calculation from first principles. A counting of various contributions can be based on the $1/N_c$ expansion and power-counting of chiral perturbation theory. At order $O(N_c)$, one-particle-reducible exchange diagrams involving Goldstone bosons (at $O(p^6)$) and non-Goldstone bosons (at $O(p^8)$) dominate, whereas loop-diagrams start at order $O(1/N_c)$. Various model calculations are in fair agreement with each other, resulting in the dominating contribution $a_\mu^{\text{hLbL}}(\pi^0, \eta, \eta') = (11.4 \pm 1.3) \times 10^{-10}$ [32] from Goldstone-boson exchange. Contributions due to the exchange of pseudo-vector and scalar resonances, heavy quark loops and π loops are less well-known and cancel each other to some extent. A conservative estimate of their precision, but adding theoretical errors from different contributions in quadrature, results in $a_\mu^{\text{hLbL}} = (10.5 \pm 2.6) \times 10^{-10}$ [32]. Improvements will be difficult to achieve, but may be expected by a more refined consideration of short-distance QCD constraints and by utilizing experimental information on the properties of the off-shell $\pi\gamma\gamma$ and $\pi\pi\gamma\gamma$ form factors. Data from BaBar, KLOE-2 and DAΦNE on radiative decays of pions and pseudo-vector resonances and other 2-photon processes will play an important role in this respect.

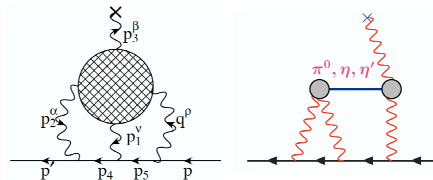


Figure 6: Hadronic light-by-light scattering and the dominating π^0, η, η' exchange diagram [32].

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