From Diphoton GDAs and Photon GPDs to the Chiral Odd Photon DA

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The photon is a very interesting object for QCD studies since it has both a pointlike coupling to quarks, which yields a perturbative part of photons wave function, and a non-perturbative coupling related to the magnetic susceptibility of the QCD vacuum and which builds its chiral-odd twist-2 distribution amplitude. The first feature allows us to compute the photon anomalous generalized parton distributions (GPD) and the diphoton generalized distribution amplitudes. The second feature allows us to use a transverse spin asymmetry to probe the chiral odd distribution amplitude of the photon.

1 Photon GPDs and diphoton GDAs

The factorization of the amplitude for the deeply virtual Compton scattering (DVCS) process $\gamma^*(Q)\gamma \to \gamma\gamma$ at high Q^2 is demonstrated in two distinct kinematical domains, allowing to define the photon generalized parton distributions and the diphoton generalized distribution amplitudes. Both these quantities exhibit an anomalous scaling behaviour and obey new inhomogeneous QCD evolution equations. The parton content of the photon has been the subject of many studies since the seminal paper by Witten [1] which allowed to define the anomalous quark and gluon distribution functions. Recent progress in exclusive hard reactions focuses on generalized parton distributions (GPDs), which are defined as Fourier transforms of matrix elements between different states, such as $\langle N'(p',s')|\bar{\psi}(-\lambda n)\gamma.n\psi(\lambda n)|N(p,s)\rangle$ and their crossed versions, the generalized distribution amplitudes (GDAs) which describe the exclusive hadronization of a $\bar{q}q$ or gg pair in a pair of hadrons, see Fig. 1. In the photon case, these quantities are perturbatively calculable [2, 3] at leading order in α_{em} and leading logarithmic order in Q^2. They constitute an interesting theoretical laboratory for the non-perturbative hadronic objects that hadronic GPDs and GDAs are.

1.1 The diphoton generalized distribution amplitudes

Defining the momenta as $q = p - \frac{Q^2 n}{s}$, $q' = \frac{Q^2 n}{s}$, $p_1 = \zeta p$, $p_2 = (1-\zeta)p$, where p and n are two light-cone Sudakov vectors and $2p \cdot n = s$, the amplitude of the process

$$\gamma^*(Q,\epsilon)\gamma(q',\epsilon') \to \gamma(p_1,\epsilon_1)\gamma(p_2,\epsilon_2) \tag{1}$$
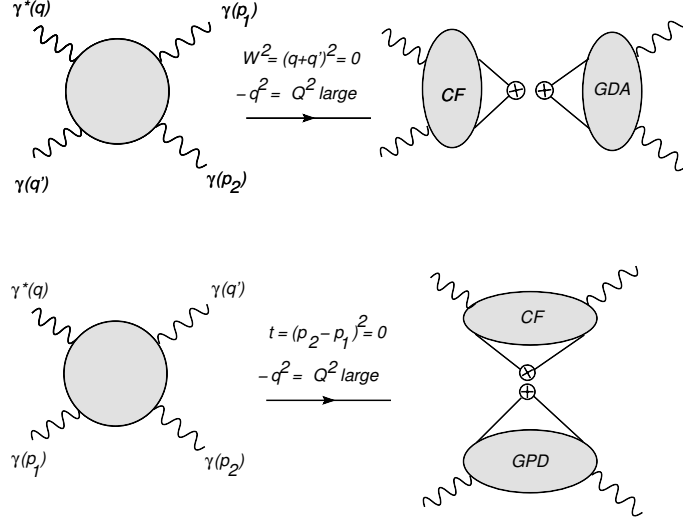
Figure 1: Factorizations of the DVCS process on the photon.

may be written as $A = \epsilon_\mu \epsilon'_\nu \epsilon^*_{1\alpha} \epsilon^*_{2\beta} T^{\mu\nu\alpha\beta}$. In forward kinematics where $(q + q')^2 = 0$, the tensorial decomposition of $T^{\mu\nu\alpha\beta}$ reads (see [3])

$$\frac{1}{4} g_T^{\mu\nu} g_T^{\alpha\beta} W_1^q + \frac{1}{8} \left(g_T^{\mu\alpha} g_T^{\nu\beta} + g_T^{\nu\alpha} g_T^{\mu\beta} - g_T^{\mu\nu} g_T^{\alpha\beta} \right) W_2^q + \frac{1}{4} \left(g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\mu\beta} g_T^{\alpha\nu} \right) W_3^q . \quad (2)$$

At leading order, the three scalar functions W_i^q can be written in a factorized form which is particularly simple when the factorization scale M_F equals the photon virtuality Q. W_1^q is then the convolution $W_1^q = \int_0^1 dz \, C_V^q(z) \, \Phi_1^q(z, \zeta, 0)$ of the coefficient function $C_V^q = e_q^2 \left(\frac{1}{z} - \frac{1}{1-z} \right)$ with the *anomalous* vector GDA $(\bar{z} = 1 - z, \bar{\zeta} = 1 - \zeta)$:

$$\Phi_1^q(z, \zeta, 0) = \frac{N_C \, e_q^2}{2\pi^2} \log \frac{Q^2}{m^2} \left[\frac{\bar{z}(2z - \zeta)}{\bar{\zeta}} \theta(z - \zeta) + \frac{\bar{z}(2z - \bar{\zeta})}{\zeta} \theta(z - \bar{\zeta}) \right.$$
$$\left. + \frac{z(2z - 1 - \zeta)}{\zeta} \theta(\zeta - z) + \frac{z(2z - 1 - \bar{\zeta})}{\bar{\zeta}} \theta(\bar{\zeta} - z) \right]. \quad (3)$$

Conversely, W_3^q is the convolution of the function $C_A^q = e_q^2 \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)$ with the axial GDA :

$$\Phi_3^q(z, \zeta, 0) = \frac{N_C \, e_q^2}{2\pi^2} \log \frac{Q^2}{m^2} \left[\frac{\bar{z}\zeta}{\bar{\zeta}} \theta(z - \zeta) - \frac{\bar{z}\bar{\zeta}}{\zeta} \theta(z - \bar{\zeta}) - \frac{z\bar{\zeta}}{\zeta} \theta(\zeta - z) + \frac{z\zeta}{\bar{\zeta}} \theta(\bar{\zeta} - z) \right] \quad (4)$$

and $W_2^q = 0$. Note that these GDAs are not continuous at the points $z = \pm \zeta$. The anomalous nature of Φ_1^q and Φ_3^q comes from their proportionality to $\log \frac{Q^2}{m^2}$, which reminds us of the anomalous photon structure functions. A consequence is that $\frac{d}{d \ln Q^2} \Phi_i^q \neq 0$; consequently the QCD evolution equations of the diphoton GDAs obtained with the help of the ERBL kernel are non-homogeneous ones.

1.2 The photon generalized parton distributions

We now look at the same process in different kinematics, namely $q = -2\xi p + n$, $q' = (1 + \xi)p$, $p_1 = n$, $p_2 = p_1 + \Delta = (1 - \xi)p$, where $W^2 = \frac{1-\xi}{2\xi}Q^2$ and $t = 0$. The tensor $T^{\mu\nu\alpha\beta}$ is now decomposed on different tensors with the help of three functions $\mathcal{W}_i^q$ as (see [2]):

$$\frac{1}{4}g_T^{\mu\alpha}g_T^{\nu\beta}\mathcal{W}_1^q + \frac{1}{8}\left(g_T^{\mu\nu}g_T^{\alpha\beta} + g_T^{\alpha\nu}g_T^{\mu\beta} - g_T^{\mu\alpha}g_T^{\nu\beta}\right)\mathcal{W}_2^q + \frac{1}{4}\left(g_T^{\mu\nu}g_T^{\alpha\beta} - g_T^{\mu\beta}g_T^{\nu\alpha}\right)\mathcal{W}_3^q \qquad (5)$$

These functions can also be written in factorized forms which have direct parton model interpretations when the factorization scale M_F is equal to Q: $\mathcal{W}_{1/3}^q = \int\limits_{-1}^{1} dx C_{V/A}^q(x)H_{1/3}^q(x,\xi,0)$, $\mathcal{W}_2 = 0$. The coefficient functions are $C_{V/A}^q = -2e_q^2\left(\frac{1}{x-\xi+i\eta} \pm \frac{1}{x+\xi-i\eta}\right)$ and the unpolarized H_1^q and polarized H_3^q anomalous GPDs of quarks inside a real photon read :

$$H_1^q(x,\xi,0) = \frac{N_C\,e_q^2}{4\pi^2}\left[\theta(x-\xi)\frac{x^2+(1-x)^2-\xi^2}{1-\xi^2}\right.$$
$$\left. + \theta(\xi-x)\theta(\xi+x)\frac{x(1-\xi)}{\xi(1+\xi)} - \theta(-x-\xi)\frac{x^2+(1+x)^2-\xi^2}{1-\xi^2}\right]\ln\frac{Q^2}{m^2}\,, \qquad (6)$$

$$H_3^q(x,\xi,0) = \frac{N_C\,e_q^2}{4\pi^2}\left[\theta(x-\xi)\frac{x^2-(1-x)^2-\xi^2}{1-\xi^2}\right.$$
$$\left. - \theta(\xi-x)\theta(\xi+x)\frac{1-\xi}{1+\xi} + \theta(-x-\xi)\frac{x^2-(1+x)^2-\xi^2}{1-\xi^2}\right]\ln\frac{Q^2}{m^2}\,. \qquad (7)$$

Similarly as in the GDA case, the anomalous generalized parton distributions H_i^q are proportional to $\ln\frac{Q^2}{m^2}$, which violates the scaling. Consequently, the anomalous terms H_i^q supply to the usual homogeneous DGLAP-ERBL evolution equations of GPDs a non-homogeneous term which changes them into non-homogeneous evolution equations.

We do not anticipate a rich phenomenology of these photon GPDs, but in the case of a high luminosity electron - photon collider which is not realistic in the near future. However, the fact that one gets explicit expressions for these GPDs may help to understand the meaning of general theorems such as the polynomiality and positivity [4] constrains or the analyticity structure [5]. For instance, one sees that a D-term in needed when expressing the photon GPDs in terms of a double distribution. One also finds that, in the DGLAP region, $H_1(x,\xi)$ is smaller than its positivity bound by a sizeable and slowly varying factor, which is of the order of $0.7 - 0.8$ for $\xi \approx 0.3$.

2 Accessing the photon chiral-odd DA and the proton transversity

In Ref.[6], we describe a new way to access the photon distribution amplitude through the photoproduction of lepton pairs on a transversely polarized proton.
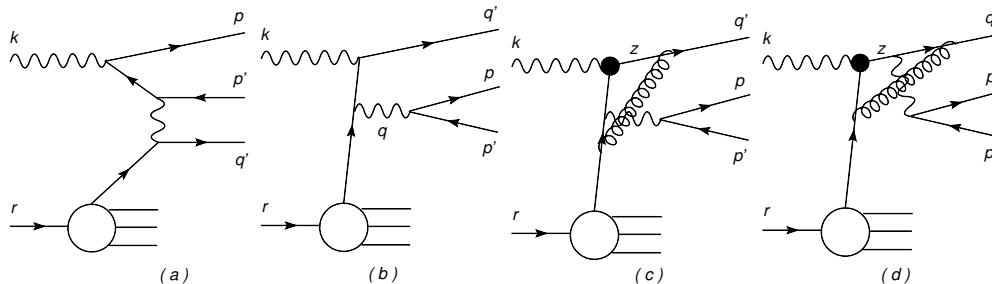
Figure 2: Some amplitudes contributing to lepton pair photoproduction. (a) : The Bethe-Heitler process. (b) : The Drell-Yan process with the photon pointlike coupling. (c) -(d) : The Drell-Yan process with the photon Distribution Amplitude.

The leading twist chiral-odd photon distribution amplitude $\phi_\gamma(u)$ reads [7]

$$\langle 0|\bar{q}(0)\sigma_{\alpha\beta}q(x)|\gamma^{(\lambda)}(k)\rangle = i\,e_q\,\chi\,\langle\bar{q}q\rangle\left(\epsilon_\alpha^{(\lambda)}k_\beta - \epsilon_\beta^{(\lambda)}k_\alpha\right)\int\limits_0^1 dz\,e^{-iz(kx)}\,\phi_\gamma(z)\,, \qquad (8)$$

where the normalization is chosen as $\int dz\,\phi_\gamma(z) = 1$, and z stands for the momentum fraction carried by the quark. The product of the quark condensate and of the magnetic susceptibility of the QCD vacuum $\chi\langle\bar{q}q\rangle$ has been estimated [8] with the help of the QCD sum rules techniques to be of the order of 50 MeV and a lattice estimate has recently been performed [9]. The distribution amplitude $\phi_\gamma(z)$ has a QCD evolution which drives it to an asymptotic form $\phi_\gamma^{as}(z) = 6z(1-z)$. Its z−dependence at non asymptotic scales is very model-dependent [10].

We consider the following process (s_T is the transverse polarization vector of the nucleon):

$$\gamma(k,\epsilon)N(r,s_T) \rightarrow l^-(p)l^+(p')X\,, \qquad (9)$$

with $q = p + p'$ in the kinematical region where $Q^2 = q^2$ is large and the transverse component $|\vec{Q}_\perp|$ of q is of the same order as Q. Such a process occurs either through a Bethe-Heitler amplitude (Fig. 2a) where the initial photon couples to a final lepton, or through Drell-Yan type amplitudes (Fig. 2b) where the final leptons originate from a virtual photon. Among these Drell-Yan processes, one must distinguish the cases where the real photon couples directly (through the QED coupling) to quarks or through its quark content. We thus consider the contributions where the photon couples to the strong interacting particles through its lowest twist-2 chiral odd distribution amplitude (Fig. 2c and 2d). We will call this amplitude $\mathcal{A}_\phi$.

One can easily see by inspection that interfering the amplitude $\mathcal{A}_\phi$ with a pointlike amplitude is the only way to get at the level of twist 2 (and with vanishing quark masses) a contribution to nucleon transverse spin dependent observables. Reaction (9) thus opens a natural access to the photon distribution amplitude [11], provided the amplitude $\mathcal{A}_\phi$ interferes with the Bethe-Heitler or a usual Drell-Yan process. Moreover, since this amplitude has an absorptive part, *single spin* effects do not vanish. The amplitude where the photon interacts through its distribution amplitude at lowest order (Fig. 2c and 2d) and in Feynman gauge, reads

$$\mathcal{A}_\phi(\gamma q \rightarrow l\bar{l}q) \qquad (10)$$
$$= 2i\frac{C_F}{4N_c}e_q^2 e 4\pi\alpha_s\chi\,\langle\bar{q}q\rangle\frac{1}{Q^2}\int dz\phi_\gamma(z)\bar{u}(q')[\frac{A_1}{x\bar{z}s(t_1+i\epsilon)} + \frac{A_2}{zu(t_2+i\epsilon)}]u(r)\bar{u}(p)\gamma^\mu v(p')\,,$$

with $t_1 = (zk - q)^2$ and $t_2 = (\bar{z}k - q)^2$ and

$$A_1 = x\,\hat{r}\,\hat{\epsilon}\,\hat{k}\,\gamma^\mu + \gamma^\mu\,\hat{k}\,\hat{\epsilon}\,\hat{q}, \qquad A_2 = \hat{\epsilon}\,\hat{q}\,\gamma^\mu\,\hat{k} + \hat{k}\,\gamma^\mu\,\hat{q}\,\hat{\epsilon}, \tag{11}$$

which do not depend on the light-cone fraction z. Most interesting is the analytic structure of this amplitude since the quark propagators may be on shell so that the amplitude $\mathcal{A}_\phi$ develops an absorptive part proportional to

$$\int dz\phi_\gamma(z)\bar{u}(q')[\frac{A_1}{x\bar{z}s}\delta(t_1) + \frac{A_2}{zu}\delta(t_2)]u(r)\bar{u}(p)\gamma^\mu v(p').$$

The $z-$integration, after using the $z - \bar{z}$ symmetry of the distribution amplitude, yields an absorptive part of the amplitude $\mathcal{A}_\phi$ proportional to $\phi_\gamma(\frac{\alpha Q^2}{Q^2 + \vec{Q}_\perp^2})$. This absorptive part may be measured in single spin asymmetries and thus scans the photon chiral-odd distribution amplitude.

The cross section for reaction (9) can be decomposed as

$$\frac{d\sigma}{d^4Q\,d\Omega} = \frac{d\sigma_{BH}}{d^4Q\,d\Omega} + \frac{d\sigma_{DY}}{d^4Q\,d\Omega} + \frac{d\sigma_\phi}{d^4Q\,d\Omega} + \frac{\Sigma d\sigma_{int}}{d^4Q\,d\Omega},$$

where $\Sigma d\sigma_{int}$ contains various interferences, while the transversity dependent differential cross section (we denote $\Delta_T \sigma = \sigma(s_T) - \sigma(-s_T)$) reads

$$\frac{d\Delta_T\sigma}{d^4Q\,d\Omega} = \frac{d\sigma_{\phi int}}{d^4Q\,d\Omega}, \tag{12}$$

where $d\sigma_{\phi int}$ contains only interferences between the amplitude $\mathcal{A}_\phi$ and the other amplitudes. Moreover, one may use the distinct charge conjugation property (with respect to the lepton part) of the Bethe Heitler amplitude to select the interference between $\mathcal{A}_\phi$ and the Bethe-Heitler amplitude :

$$\frac{d\Delta_T\sigma(l^-) - d\Delta_T\sigma(l^+)}{d^4Q\,d\Omega} = \frac{d\sigma_{\phi BH}}{d^4Q\,d\Omega}. \tag{13}$$

Conversely, one may use this charge asymmetry to cancel out the interference of $\mathcal{A}_\phi$ with the Bethe Heitler amplitude

$$\frac{d\Delta_T\sigma(l^-) + d\Delta_T\sigma(l^+)}{d^4Q\,d\Omega} \propto \frac{d\sigma_{\phi DY}}{d^4Q\,d\Omega}. \tag{14}$$

The simplest observable which contains all appealing features of our proposal is the interference of $\mathcal{A}_\phi$ and the Bethe-Heitler amplitudes, see Eq.13, in the unpolarized photon case. The polarization average of $d\sigma_{\phi BH}$ reads :

$$\frac{1}{2}\sum_\lambda d\sigma_{\phi BH}(\gamma(\lambda)p \to l^- l^+ X) \tag{15}$$

$$= \frac{(4\pi\alpha_{em})^3}{4s}\frac{C_F 4\pi\alpha_s}{2N_c}\frac{\chi\,\langle\bar{q}q\rangle}{\vec{Q}_\perp^2}\int dx\sum_q Q_l^3 Q_q^3 h_1^q(x)2\mathcal{R}e(\mathcal{I}_{\phi BH})\,dLIPS,$$

with the usual phase space factor dLips and

$$2\mathcal{R}e(\mathcal{I}_{\phi BH}) = \phi_\gamma \left[\frac{\alpha Q^2}{Q^2 + \vec{Q}_\perp^2}\right] \frac{32\pi\alpha^2\bar{\alpha}}{xs(\bar{\alpha}Q^2 + \vec{Q}_\perp^2)^2}(Q^2 + \vec{Q}_\perp^2)[\epsilon^{rk s_T Q_T}\mathcal{A} + \epsilon^{rk s_T l_T}\mathcal{B}], \qquad (16)$$

where $\mathcal{A}$ and $\mathcal{B}$ are algebraic functions [6]. Eqs. 15, 16 demonstate at the level of a highly differential cross section the existence of a non-vanishing observable proportional to the photon distribution amplitude $\Phi_\gamma(z = \frac{\alpha Q^2}{Q^2 + \vec{Q}_\perp^2})$ and the nucleon transversity h_1.

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