# Hard Exclusive Processes: Theoretical Status 

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We review the descriptions of hard exclusive processes based on QCD factorization.

## 1 Introduction

Since a decade, there have been much developments in hard exclusive processes, based on collinear factorization. This was initiated by form factors studies more than 30 years ago, leading to the concept of Distribution Amplitudes (DA) [1], which describes the partonic content of a hadron facing an elastic scattering off a hard photonic probe. These DAs were then extended to Generalized Distribution Amplitudes (GDA) [2, 3, 4] in which two or more hadrons are produced. Independently, starting from inclusive DIS which relates Parton Distribution Functions (PDFs) to the discontinuity of the forward $\gamma^{*} p \rightarrow \gamma^{*} p$ amplitude, it was shown [5, 3] that the partonic interpretation remains valid for the Deep Virtual Compton Scattering (DVCS) amplitude $\gamma^{*} p \rightarrow \gamma p$ itself, leading to the concept of Generalized Parton Distributions (GPDs), and to more general exclusive processes studies. In a parallel way, tremendous progresses in experimental facilities (high luminosity beams, improvement of detectors...) opened the way to studies and measures with increasing precision of these non forward matrix elements $[6,7,8,9]$.

## 2 The illuminating example of $\rho$-electroproduction

## DVCS and GPD [10]

The factorization of the DVCS amplitude in the large $Q^{2}$ limit follows two steps. First, one should factorize it in momentum space. This can be set up more easily when using the Sudakov decomposition (introducing two light-cone vectors $p_{1(2)}\left(+(-)\right.$ directions) with $2 p_{1} \cdot p_{2}=s$ )

$$
\begin{gather*}
\left.k=\begin{array}{c}
\alpha p_{1} \\
+ \\
\\
+
\end{array} \underset{\perp}{\beta p_{2}}+\begin{array}{c}
k_{\perp} \\
\perp
\end{array}\right) \tag{1}
\end{gather*}
$$

In the limit $Q^{2} \rightarrow \infty$, the only component of the momentum $k$ to be kept in the hard blob $H$ is $k_{-}$. In particular, this means that the quark-antiquark pair entering $H$ can be considered as being collinear, flying in the direction of the $p_{2}$ momentum. Therefore, the amplitude reads

$$
\int d^{4} k S(k, k+\Delta) H(q, k, k+\Delta)=\int d k^{-} \int d k^{+} d^{2} k_{\perp} S(k, k+\Delta) H\left(q, k^{-}, k^{-}+\Delta^{-}\right),
$$

as illustrated in Fig.1. The Fierz identity in spinor and color space then shows that the DVCS amplitude completely factorizes, and reads symbolically: $\mathcal{M}=\mathrm{GPD} \otimes$ Hard part.


Figure 1: Factorization of the DVCS amplitude in the hard regime. The signs + and - indicate corresponding flows of large momentum components.
$\rho$-meson production: from the wave function to the $D A$


Figure 2: Factorization of the amplitude of hard $\rho$-electroproduction.
We now replace the produced photon by a $\rho-$ meson, described in QCD by its wave function $\Psi$ which reduces in hard processes to its Distribution Amplitude. As for DVCS, in the limit $Q^{2} \rightarrow \infty$, the amplitude of diffractive electroproduction of a $\rho-$ meson can be written as
$\int d^{4} \ell M\left(q, \ell, \ell-p_{\rho}\right) \Psi\left(\ell, \ell-p_{\rho}\right)=\int d \ell^{+} M\left(q, \ell^{+}, \ell^{+}-p_{\rho}^{+}\right) \int d \ell^{-} \int^{\left|\ell_{\perp}^{2}\right|<\mu_{F}^{2}} d^{2} \ell_{\perp} \Psi\left(\ell, \ell-p_{\rho}\right)$
(see Fig.2). This factorization involves the $\rho$-wave function integrated over $\ell_{\perp}$ (and $\ell^{-}$), which is the DA already involved in the partonic description of the hard meson form factor [1].
$\rho-$ meson production: factorization with a GPD and a $D A$ [11]
Combining the previous factorizations, one can describe the hard electroproduction of a $\rho-$ meson in a fully factorized form involving a GPD and a DA, as illustrated in Fig.3. It reads

$$
\begin{equation*}
\int d^{4} k d^{4} \ell S(k, k+\Delta) H(q, k, k+\Delta) \Psi\left(\ell, \ell-p_{\rho}\right)=\int d k^{-} d \ell^{+} \tag{3}
\end{equation*}
$$

$$
\begin{gathered}
\times \int d k^{+} \int^{\left|k_{\perp}^{2}\right|<\mu_{F_{2}}^{2}} d^{2} k_{\perp} S(k, k+\Delta) H\left(q ; k^{-}, k^{-}+\Delta^{-} ; \ell^{+}, \ell^{+}-p_{\rho}^{+}\right) \int d \ell^{-} \int^{\left|\ell_{\perp}^{2}\right|<\mu_{F_{1}}^{2}} d^{2} \ell_{\perp} \Psi\left(\ell, \ell-p_{\rho}\right) . \\
\operatorname{GPD} F\left(x, \xi, t, \mu_{F_{2}}^{2}\right) \quad \text { Hard part } T\left(x / \xi, u, \mu_{F_{1}}^{2}, \mu_{F_{2}}^{2}\right) \quad \text { DA } \Phi\left(u, \mu_{F_{1}}^{2}\right)
\end{gathered}
$$

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Figure 3: Full factorization of the amplitude of hard electroproduction of a $\rho-$ meson.

## Chiral-even DA

As discussed above, DAs are obtained from wave functions through $\int d \ell^{-} \int d^{2} \ell_{\perp}$ integration, and thus related to non-local correlators between fields separated by a light-like distance $z$ (along $p_{2}$, conjugated to the + direction by Fourier transformation). The vector correlator reads

$$
\begin{aligned}
& \langle 0| \bar{u}(z) \gamma_{\mu} d(-z)\left|\rho^{-}(P, \lambda)\right\rangle=f_{\rho} m_{\rho}\left[p_{\mu} \frac{e^{(\lambda)} \cdot z}{p \cdot z} \int_{0}^{1} d u e^{i(u-\bar{u}) p \cdot z} \phi_{\|}\left(u, \mu_{F}^{2}\right)\right. \\
& \left.\quad+\quad e_{\perp \mu}^{(\lambda)} \int_{0}^{1} d u e^{i(u-\bar{u}) p \cdot z} g_{\perp}^{(v)}\left(u, \mu_{F}^{2}\right)-\frac{1}{2} z_{\mu} \frac{e^{(\lambda)} \cdot z}{(p \cdot z)^{2}} m_{\rho}^{2} \int_{0}^{1} d u e^{i(u-\bar{u}) p \cdot z} g_{3}\left(u, \mu_{F}^{2}\right)\right]
\end{aligned}
$$

where $\phi_{\|}, g_{\perp}^{(v)}, g_{3}$ are DAs respectively of twist 2,3 and 4 , with $p=p_{1}, P=p_{\rho}$. Correspondingly, the axial correlator calls for the introduction of a twist 3 DA , as
$\langle 0| \bar{u}(z) \gamma_{\mu} \gamma_{5} d(-z)\left|\rho^{-}(P, \lambda)\right\rangle=\frac{1}{2}\left[f_{\rho}-f_{\rho}^{T} \frac{m_{u}+m_{d}}{m_{\rho}}\right] m_{\rho} \epsilon_{\mu}{ }^{\nu \alpha \beta} e_{\perp \nu}^{(\lambda)} p_{\alpha} z_{\beta} \int_{0}^{1} d u e^{i \xi p \cdot z} g_{\perp}^{(a)}\left(u, \mu_{F}^{2}\right)$.

## Selection rules and factorization status

Since for massless particle chirality $=+$ (resp. -) helicity for a (anti)particule and based on the fact that QED and QCD vertices are chiral even (no chirality flip during the interaction), one deduces that the total helicity of a $q \bar{q}$ pair produced by a $\gamma^{*}$ should be 0 . Therefore, the helicity of the $\gamma^{*}$ equals $L_{z}^{q \bar{q}}$ ( $z$ projection of the $q \bar{q}$ angular momentum). In the pure collinear limit (i.e. twist 2), $L_{z}^{q \bar{q}}=0$, and thus the $\gamma^{*}$ is longitudinally polarized. Additionaly, at $t=0$ there is no source of orbital momentum from the proton coupling, which implies that the helicity of the meson and of the photon should be identical. In the collinear factorization approach, the extension to $t \neq 0$ changes nothing from the hard side, the only dependence with respect to $t$ being encoded in the non-perturbative correlator which defines the GPDs. This implies that the above selection rule remains true. Thus, only 2 transitions are possible (this is called $s$-channel helicity conservation (SCHC)): $\gamma_{L}^{*} \rightarrow \rho_{L}$, for which QCD factorization holds at $\mathrm{t}=2$ at any order (i.e. LL, NLL, etc...) [11] and $\gamma_{T}^{*} \rightarrow \rho_{T}$, corresponding to twist $t=3$ at the amplitude level, for which QCD factorization is not proven, an explicit computation [12] at leading order showing in fact that the hard part has end-point singularities like $\int_{0}^{1} d u / u$.

## Some solutions to factorization breaking?

In order to extend the factorization theorem at higher twist, several solutions have been discussed. First, one may add contributions of 3-parton DAs [13] for $\rho_{T}[14,15]$ (of dominant twist equal 3 for $\rho_{T}$ ). This in fact does not solve the problem, while reducing the level of divergency, but is needed for consistency. Next, it was suggested to keep a transverse $\ell_{\perp}$ dependency in the $q, \bar{q}$ momenta, used to regulate end-point singularities, leading to the Improved Collinear Approximation (ICA). Soft and collinear gluon exchange between the valence quarks are responsible for large double-logarithmic effects which exponentiate. This is made easier when using the impact parameter space $b_{\perp}$ conjugated to $\ell_{\perp}$, leading to Sudakov factor

$$
\exp [-S(u, b, Q)]
$$

$S$ diverges when $b_{\perp} \sim O\left(1 / \Lambda_{Q C D}\right)$ (large transverse separation, i.e. small transverse momenta) or $u \sim O\left(\Lambda_{Q C D} / Q\right)[16]$. This regularizes end-point singularities for $\pi \rightarrow \pi \gamma^{*}$ and $\gamma \gamma^{*} \rightarrow \pi^{0}$ form factors [17]. This perturbative resummation tail effect combined with an ad-hoc nonperturbative gaussian ansatz for the DAs

$$
\exp \left[-a^{2}\left|k_{\perp}^{2}\right| /(u \bar{u})\right]
$$

which gives back the usual asymptotic DA $6 u \bar{u}$ when integrating over $k_{\perp}$, provides practical tools for the phenomenology of meson electroproduction [18].

## Chiral-odd sector

The $\pm$ chiralities are defined by the decomposition

$$
q_{ \pm}(z) \equiv \frac{1}{2}\left(1 \pm \gamma^{5}\right) q(z) \quad \text { with } \quad q(z)=q_{+}(z)+q_{-}(z)
$$

implying that $\bar{q}_{ \pm}(z) \gamma^{\mu} q_{ \pm}(-z)$ or $\bar{q}_{ \pm}(z) \gamma^{\mu} \gamma^{5} q_{ \pm}(-z)$ conserve chirality (chiral-even) while $\bar{q}_{ \pm}(z)$. $1 \cdot q_{\mp}(-z), \bar{q}_{ \pm}(z) \cdot \gamma^{5} \cdot q_{\mp}(-z)$ or $\bar{q}_{ \pm}(z)\left[\gamma^{\mu}, \gamma^{\nu}\right] q_{\mp}(-z)$ change chirality (chiral-odd). In the specific case of $\rho$, the chiral odd sector involves DAs of twists 2 and 4 for $\rho_{T}$ and DAs of twist 3 for $\rho_{L}$. Correspondingly, chiral-odd 3-partons DAs are of dominant twist equal to 3 for $\rho_{L}$ [13].

Since QED and QCD are chiral even, chiral-odd objects can only appear in pairs. While the amplitude of $\rho_{T}$ electroproduction on linearly polarized $N$ vanishes at leading twist 2 (a single gluon exchange between hard lines is not enough to prevent the vanishing of Dirac traces) [19], this vanishing can be avoided [20], for example in the electroproduction of a $\pi^{+}$and $\rho_{T}^{0}$ pair on a nucleon $N[21]$, the hard scale being provided by the $p_{T}$ of the produced mesons.

## 3 Generic results for DAs

## Gauge invariance

The non-local correlators $\langle 0| \bar{\Psi}(z) \gamma_{\mu} \Psi(-z)|\rho\rangle$ are gauge invariant since they should be understood as $\langle 0| \bar{\Psi}(z) \gamma_{\mu}[z,-z] \Psi(-z)|\rho\rangle$ where [,] is a Wilson line along $p_{2}$. This implies that even at twist 2 , gluons are there, although hidden. The Taylor expansion with respect to $z$ involves the covariant derivative $\overleftrightarrow{D_{\mu}}$. This can be used for studying hard electroproduction of exotic (non $q \bar{q}$ quantum numbers) hybrids mesons $|q \bar{q} g\rangle$ with $J^{P C}=1^{-+}$which cannot be described by the quark model. Thus, $\gamma^{*} p \rightarrow H^{0} p$ is not suppressed: it is twist 2 . The expected

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order of magnitude of the cross-section is comparable with $\rho$-electroproduction [22], with possible tests at JLab and Compass. The same conclusion applies for the process $\gamma \gamma^{*} \rightarrow H^{0}$ with the advantage of avoiding the mixing with GPDs [23]. Tests should be possible at BaBar, BELLE, BEPC-II. A possible candidate for the neutral hybrid $H^{0}$ could be the $\pi_{1}(1400)$.

## Equations of motion

The Dirac equation leads to $\left\langle i(\overrightarrow{D D}(0) \psi(0))_{\alpha} \bar{\psi}_{\beta}(z)\right\rangle=0$ which, after applying the Fierz decomposition to 2 and 3-parton correlators, implies Equations Of Motion (EOM) relating the various 2 and 3-body DAs.

## Renormalization group equations

Back to the factorization (2) or (3) of the process in term of a DA, which symbolically reads

$$
\begin{equation*}
\mathcal{M}\left(Q^{2}\right)=\Phi^{*}\left(x, \mu_{F}^{2}\right) \otimes T_{H}\left(x, Q^{2}, \mu_{F}^{2}\right) \tag{4}
\end{equation*}
$$

the arbitrariness of the factorization scale $\mu_{F}^{2}$ leads to the Efremov-Radyushkin, Brodsky-Lepage equation [24] for the $\operatorname{DA} \Phi\left(u, \mu_{F}^{2}\right)$ :

$$
\mu_{F}^{2} \frac{\partial}{\partial \mu_{F}^{2}} \Phi\left(x, \mu_{F}^{2}\right)=V\left(x, u, \mu_{F}^{2}\right) \otimes \Phi\left(u, \mu_{F}^{2}\right) .
$$

## Collinear conformal invariance [25]

The full conformal group $S O(4,2)$ is defined as transformations which only change the scale of the metric. In the limit $Q^{2} \rightarrow \infty$, hadron states are replaced by a bunch of partons that are collinear to $p_{1}$, which thus lives along $p_{2}$, implying that $z$ is the only remaining variable. The transformations which map the light-ray in the $p_{2}$ direction into itself is the collinear subgroup of the full conformal group $S O(4,2)$, that is $S L(2, \mathbb{R})$, made of translations $z \rightarrow z+c$, dilatations $z \rightarrow \lambda z$ and special conformal transformations $z \rightarrow z^{\prime}=z /(1+2 a z)$. The Lie algebra of $S L(2, \mathbb{R})$ is $O(2,1)$. One remaining additional generator commutes with the 3 previous ones: the collinear-twist operator. Interestingly, the light-cone operators which enter the definition of DAs can be expressed in terms of a basis of conformal operators. Since conformal transformations commute with exact EOM (they are not renormalized), EOM can be solved exactly (with an expansion in terms of the conformal spin $n+2$ ). For example the twist 2 DA for $\rho_{L}$ can be expressed, for unpredicted $a_{n}^{\|}(\mu)$, as [26]

$$
\phi_{\|}\left(u, \mu_{0}\right)=6 u \bar{u} \sum_{n=0}^{\infty} a_{n}^{\|}(\mu) C_{n}^{3 / 2}(u-\bar{u}) \quad C_{n}^{3 / 2}=\text { Gegenbauer polynomial } .
$$

Since the Leading Order renormalization of the conformal operators is diagonal in the conformal spin (counterterms are tree level at this accuracy and they thus respect the conformal symetry of the classical theory), this implies that

$$
\phi_{\|}(u, \mu)=6 u \bar{u} \sum_{n=0}^{\infty} a_{n}^{\|}\left(\mu_{0}\right)\left(\frac{\alpha_{s}(\mu)}{\alpha_{s}\left(\mu_{0}\right)}\right)^{\gamma_{n}^{(0)} / \beta_{0}} C_{n}^{3 / 2}(u-\bar{u}) \xrightarrow{\mu \rightarrow \infty} 6 u \bar{u} \text { asymptotic DA }
$$

with the anomalous dimensions

$$
\gamma_{n}^{(0)}=C_{F}\left(1-\frac{2}{(n+1)(n+2)}+4 \sum_{m=2}^{n+1} \frac{1}{m}\right)
$$



Figure 4: $k_{T}$-factorization in the case of $\gamma^{*} \gamma^{*} \rightarrow \rho \rho$.

At Next to Leading Order conformal symetry is broken; studying conformal anomalies provides the NLO anomalous dimensions and the corresponding ERBL kernels [27].

## 4 The specific case of QCD at large $s$

## Theoretical motivations and $k_{T}$-factorization

The dynamics of QCD in the perturbative Regge limit is governed by gluons (dominance of spin 1 exchange in $t$ channel). The BFKL Pomeron (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity in any diffractive or inclusive process. In this regime, the key tool is the $k_{T}$-factorization, shown in Fig. 4 in the case of $\gamma^{*} \gamma^{*} \rightarrow \rho \rho$. Using the Sudakov decomposition (1) for which $d^{4} k=\frac{s}{2} d \alpha d \beta d^{2} k_{\perp}$ and noting that $t$-channel gluons with non-sense polarizations $\left(\epsilon_{N S}^{u p / d o w n}=\frac{2}{s} p_{2 / 1}\right)$ dominate at large $s$, and then rearranging the $k$-integration (see Fig.4) leads to the impact representation

$$
\begin{equation*}
\mathcal{M}=i s \int \frac{d^{2} \underline{k}}{(2 \pi)^{2} \underline{k}^{2}(\underline{r}-\underline{k})^{2}} \Phi^{\gamma^{*}\left(q_{1}\right) \rightarrow \rho\left(p_{1}^{\rho}\right)}(\underline{k}, \underline{r}-\underline{k}) \Phi^{\gamma^{*}\left(q_{2}\right) \rightarrow \rho\left(p_{2}^{\rho}\right)}(-\underline{k},-\underline{r}+\underline{k}) \tag{5}
\end{equation*}
$$

where $\Phi^{\gamma^{*}(q) \rightarrow \rho\left(p^{\rho}\right)}$ is the $\gamma_{L, T}^{*}(q) g\left(k_{1}\right) \rightarrow \rho_{L, T}\left(p^{\rho}\right) g\left(k_{2}\right)$ impact factor (with $k_{\perp}^{2}=-\underline{k}^{2}$ ). QCD gauge invariance implies, for colorless probes, that the impact factor should vanish when $\underline{k} \rightarrow 0$ or $\underline{r}-\underline{k} \rightarrow 0$. In particular, at twist 3 level (for $\gamma_{T}^{*} \rightarrow \rho_{T}$ transition), gauge invariance is a nontrivial statement which requires 2- and 3-parton correlators. Recently, HERA provided data for vector mesons with detailled polarization studies, in particular for our example $\gamma_{L, T}^{*}+p \rightarrow$ $\rho_{L, T}+p$ [9]. It exhibits a total cross-section which strongly decreases with $Q^{2}$, with a dramatic increase with $W^{2}=s_{\gamma^{*} P}$. The transition from soft to hard regime is governed by $Q^{2}$. The transitions $\gamma_{L}^{*} \rightarrow \rho_{L}, \gamma_{T(-)}^{*} \rightarrow \rho_{T(-)}$ and $\gamma_{T(+)}^{*} \rightarrow \rho_{T(+)}$ dominate with respect to any other possible transition, as expected from SCHC discussed above. In particular at $t=t_{\text {min }}$ one can experimentally distinguish two transitions: $\gamma_{L}^{*} \rightarrow \rho_{L}$ which dominate (twist 2 dominance) and the $\gamma_{T( \pm)}^{*} \rightarrow \rho_{T}( \pm)$ which is sizable, although of twist 3 . This calls for detailled studies beyond the applicability of the collinear factorization theorem.

## Phenomenological applications: meson production at HERA

The production of mesons in diffraction-type experiments at HERA has been studied extensively in various situations $[9,8]$. In the safe case, like $J / \Psi$ photoproduction, collinear factor-

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ization holds and combined with $k_{T}$-factorization a consistent description of H 1 and ZEUS data was obtained $[28,29]$. In the more intricate case of exclusive light vector meson $(\rho, \phi)$ photoproduction at large $t$, relying on $k_{T}$-factorization, one can describe H1 and ZEUS data, which seem to favor BFKL $[29,30]$. One needs to regularize end-point singularities for $\rho_{T}$, using for example a quark mass $m=m_{\rho} / 2$, and a rather poor understanding of the whole spin density matrix has been achieved. The exclusive vector meson electroproduction $\gamma_{L, T}^{*}+p \rightarrow \rho_{L, T}+p$ has been described [18] based on the ICA for DA coupling and collinear factorization with GPDs, as explained above, without any use of $k_{T}$ factorization. However, it turns out that at moderate value of $s$, HERMES [8] measured the interference phase between $L \rightarrow L$ and $T \rightarrow T$ transitions which cannot be described within perturbative QCD at the moment.

A full twist 3 treatment of $\rho$-electroproduction in $k_{T}$-factorisation is possible [31]. It relies on the computation of the $\gamma_{T}^{*} \rightarrow \rho_{T}$ impact factor at twist 3 including consistently all twist 3 contributions, i.e. 2-parton and 3-parton correlators. This gives a gauge invariant impact factor, and an amplitude which is free of end-point singularities due to the presence of $k_{T}$.
Exclusive processes at Tevatron, RHIC, LHC, ILC
Exclusive $\gamma^{(*)} \gamma^{(*)}$ processes are golden places for testing QCD at large $s$, in particular at Tevatron, RHIC, LHC and ILC. Several proposals in order to test perturbative QCD in the large $s$ limit ( $t$-structure of the hard Pomeron, saturation, Odderon...) have been made, including $\gamma^{(*)}(q)+\gamma^{(*)}\left(q^{\prime}\right) \rightarrow J / \Psi J / \Psi[32]$, or $\gamma_{L, T}^{*}(q)+\gamma_{L, T}^{*}\left(q^{\prime}\right) \rightarrow \rho_{L}\left(p_{1}\right)+\rho_{L}\left(p_{2}\right)$ process in $e^{+} e^{-} \rightarrow e^{+} e^{-} \rho_{L}\left(p_{1}\right)+\rho_{L}\left(p_{2}\right)$ with double tagged lepton at ILC [33, 34]. This could be feasible at ILC (high energy and high luminosity), with an expected BFKL NLL enhancement with respect to Born and DGLAP. The elusive Odderon ( $C$-parity of Odderon $=-1$ ) is hard to reveal directly when entering in the amplitude of a process. When considering processes where it enters linearly, through interference with the Pomeron, the signal becomes more favorable [35], as in $\gamma+\gamma \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$: a $\pi^{+} \pi^{-}$pair has no fixed $C$-parity, allowing for both Odderon and Pomeron exchange, which can interfere in the charge asymmetry [36]. More generally exclusive ultraperipheral processes are very promising.

## 5 Light-Cone Collinear Factorization



Figure 5: Factorization of 2-parton contributions in the example of the $\gamma^{*} \rightarrow \rho$ impact factor.
There are basically two ways of dealing with collinear factorization when including higher twist corrections. The Light-Cone Collinear Factorization developped first for polarized DIS [37], is self-consistent for exclusive processes [38, 39], while non-covariant, and very efficient for practical computations [31]. Using the Sudakov decomposition ( $p=p_{1}, n=2 p_{2} / s$ thus $p \cdot n=1$ )

$$
\begin{array}{cccc}
\ell_{\mu} & =u p_{\mu}+\ell_{\mu}^{\perp}+(\ell \cdot p) n_{\mu}, & u=\ell \cdot n  \tag{6}\\
\text { scaling: } & 1 & 1 / Q & 1 / Q^{2}
\end{array}
$$

one decomposes $H(k)$ around the $p$ direction:

$$
H(\ell)=H(u p)+\left.\frac{\partial H(\ell)}{\partial \ell_{\alpha}}\right|_{\ell=u p}(\ell-u p)_{\alpha}+\ldots \quad \text { with } \quad(\ell-u p)_{\alpha} \approx \ell_{\alpha}^{\perp}
$$

from which the twist 3 term $l_{\alpha}^{\perp}$ turns after Fourier transform into the derivative of the soft term $\int d^{4} z e^{-i \ell \cdot z}\langle\rho(p)| \psi(0) i \overleftrightarrow{\partial_{\alpha^{\perp}}} \bar{\psi}(z)|0\rangle$. Using the Fierz transformation, this gives finally a factorized expression up to twist 3, as illustrated in Fig. 5 for 2-parton contributions in the example of the $\gamma^{*} g \rightarrow \rho g$ impact factor. This requires the parametrization of matrix elements of non-local correlators defined along the light-like prefered direction $z=\lambda n$ conjugated to $p$. In the case of the $\rho$-electroproduction, 7 DAs at twist 3 (2- and 3 -parton DAs) are needed. Their number is reduced to a minimal set of 3 DAs when combining the 2 equations of motions and the $n$-independency condition $[37,38,39,14]$ of the full factorized amplitude (which provides 2 process-independent equations). A second approach, the Covariant Collinear Factorization [13], fully covariant but less convenient when practically computing coefficient functions, can equivalently be used. The dictionary and equivalence between the two approaches has recently been obtained, and explicitly checked for the $\gamma_{T}^{*} \rightarrow \rho_{T}$ impact factor at twist 3 [31].

## 6 Conclusion

Since a decade, there has been much progress in the understanding of hard exclusive processes: at moderate energies, combined with GPDs, there is now a framework starting from first principles to describe a huge number of processes; at high energy, the impact representation is a powerful tool for describing exclusive processes in diffractive experiments, which are and will be essential for studying QCD in the hard Regge limit (Pomeron, Odderon, saturation...). Still, some problems remain: proofs of factorization have been optained only for a very few processes (ex.: $\gamma^{*} p \rightarrow \gamma p, \gamma_{L}^{*} p \rightarrow \rho_{L} p, \gamma^{*} p \rightarrow J / \Psi p$ ). For some other processes factorization is highly plausible, but not fully demonstrated at any order (ex.: processes involving Transition Distribution Amplitudes [40]) while some processes explicitly show signs of breaking of factorization (ex.: $\gamma_{T}^{*} p \rightarrow \rho_{T} p$ which has end-point singularities at Leading Order). Models and results from the lattice for the non-perturbative correlators entering GPDs, DAs, GDAs, TDAs are needed, even at a qualitative level. The effect of QCD evolution and renormalization/factorization scale, as well as studies at the full NLL order, might be relevant with the increasing precision of data in the near future. Finally, let us insist on the fact that links between theoretical and experimental communities are very fruitful, in particular at HERA, Jlab, Compass. It is now time to use the potential of high luminosity $e^{+} e^{-}$machine like BaBar, BELLE, BEPC-II which are golden places for hard exclusive processes studies in $\gamma^{*} \gamma^{(*)}$ channels.

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