Minicharges, Monopoles, and Magnetic Mixing

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Minicharged particles (MCPs) arise naturally in extensions of the Standard Model with hidden sector gauge groups. Many such extensions also contain magnetic monopoles. For models containing both monopoles and MCPs, we clarify the role of the Dirac charge quantization condition in restricting the possible charges. We also show that monopoles of the hidden sector may manifest themselves as MCPs, by a generalization of the Witten effect, which we call *magnetic mixing*.

1 Introduction

Many extensions of the Standard Model contain additional U(1) gauge factors as part of a hidden sector. These may arise, for instance, directly from some string compactification, or from a non-abelian gauge factor spontaneously broken to U(1). For a single extra U(1), the most general low-energy Lagrangian for the abelian gauge fields is then

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + 2\,\chi\,F_{\mu\nu}G^{\mu\nu}\right)\\ &-\frac{1}{32\pi^2}\left(\theta_F\,F_{\mu\nu}\tilde{F}^{\mu\nu} + \theta_G\,G_{\mu\nu}\tilde{G}^{\mu\nu} + 2\,\theta_{FG}\,F_{\mu\nu}\tilde{G}^{\mu\nu}\right).\end{aligned} \tag{1}$$

Here $F_{\mu\nu}$ is the field strength of electromagnetism, $G_{\mu\nu}$ is the field strength of the hidden sector U(1), $\tilde{F}_{\mu\nu}$ and $\tilde{G}_{\mu\nu}$ are the respective dual field strengths, and χ, θ_F, θ_G, θ_{FG} are constants.

The first line represents the ordinary kinetic Lagrangian, including a kinetic mixing term $2\,\chi\,F_{\mu\nu}G^{\mu\nu}$. If there are massive fields charged under both $F_{\mu\nu}$ and $G_{\mu\nu}$, kinetic mixing is generically induced radiatively [1]. Fields which were charged under $G_{\mu\nu}$ only will then pick up effective electromagnetic charges $\sim\chi$, and will show up as minicharged particles (MCPs).[1]

The θ-terms in the second line are usually ignored in abelian theories. They do not affect the equations of motion and carry no topological charges. However, they do become important in the presence of magnetic monopoles: By the Witten effect [3], a $\theta_F\,F_{\mu\nu}\tilde{F}^{\mu\nu}$ term causes a monopole to pick up an electric charge $\sim\theta_F$. Similarly, as we will argue, a *magnetic mixing* term $\theta_{FG}\,F_{\mu\nu}\tilde{G}^{\mu\nu}$ will cause a hidden sector monopole to pick up a visible electric minicharge.

In the following we will consider models with both kinetic and magnetic mixing terms and with magnetic monopoles. We will show that the Dirac quantization condition for electric charges must be suitably modified in the presence of kinetic mixing, in order not to lead to a contradiction between charge quantization and the appearance of MCPs [4]. We will also demonstrate how magnetic mixing terms may give electric minicharges to hidden sector monopoles

[1] For some concrete models in field theory and string theory, see e.g. [2].

[5]. Our considerations will in particular apply to the case where the hidden sector U(1) is the remainder of a spontaneously broken non-abelian gauge group, in which magnetic monopoles appear naturally as 't Hooft–Polyakov monopoles [6].

2 Kinetic mixing and charge quantization

Let us start by ignoring the θ-terms for now and consider a model with a kinetic mixing term. For a U(1)×U(1) gauge theory with field strengths $F = dA$ and $G = dB$, the Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{\chi}{2}F^{\mu\nu}G_{\mu\nu} - ej_\mu A^\mu - e'j'_\mu B^\mu.$$

Diagonalizing the gauge kinetic terms, by defining $C \equiv B + \chi A$ and $H \equiv dC = G + \chi F$ and eliminating B and G in favour of C and H, gives a coupling of the current j' to the gauge field A with charge $-\chi e'$. Identifying A with the photon of electromagnetism, hidden sector charged matter fields have picked up ordinary electric charges $\sim \chi$, thus becoming MCPs. If the MCPs are light, these induced charges must in fact be tiny to evade experimental bounds [7].

In the presence of magnetic monopoles, electric charges should be quantized [8]. This can most easily be seen as follows: Consider a static system consisting of an electron and a monopole. This system carries angular momentum, which semi-classically should be quantized [9]:

$$\mathbf{L} = \int d^3x\, \mathbf{x} \times (\mathbf{E} \times \mathbf{B}) = \frac{eg}{4\pi}\,\mathbf{n}.$$

Here e is the electron charge, g is the monopole charge, and $\mathbf{n}$ is a unit vector pointing from one towards the other. Requiring $|\mathbf{L}|$ to be half-integral gives the Dirac quantization condition

$$eg \in 2\pi\mathbb{Z}. \tag{2}$$

It follows that the ratio of any two electric charges e_i and e_j should be rational, $e_i/e_j \in \mathbb{Q}$.

In models with a visible and a hidden sector U(1), and with kinetic mixing between the two, this leads to a problem because the induced minicharges are proportional to χ, which is an arbitrary and generally irrational number. The problem is solved, however, if we only allow for monopoles which carry a suitable magnetic charge also under the hidden U(1). For example, consider a model with an electron, an MCP, and a magnetic monopole with the following electric and magnetic charges:

Particle	q_{vis}	q_{hid}	g_{vis}	g_{hid}
electron	e	0	0	0
MCP	$-\chi e'$	e'	0	0
monopole	0	0	g	g'

The total angular momentum of the combined hidden and ordinary electromagnetic fields is, in a basis where the gauge-kinetic Lagrangian is diagonal,

$$\mathbf{L} = \int d^3x\, \mathbf{x} \times (\mathbf{E}_{\mathrm{vis}} \times \mathbf{B}_{\mathrm{vis}} + \mathbf{E}_{\mathrm{hid}} \times \mathbf{B}_{\mathrm{hid}}), \qquad |\mathbf{L}| = \frac{q_{\mathrm{vis}}\, g}{4\pi} + \frac{q_{\mathrm{hid}}\, g'}{4\pi}.$$

It is quantized if the monopole charges are $(g, g') = \left(0, \frac{2\pi n}{e'}\right)$ or $(g, g') = \left(\frac{2\pi m}{e}, \frac{2\pi\chi m}{e}\right)$ or any linear combination of these, with $n, m \in \mathbb{Z}$. Only monopoles with these quantum numbers can be consistently included in the model. This condition on monopole charges is in fact a special case of the Schwinger–Zwanziger dyon charge quantization condition [10] with multiple U(1)s.

In models with fundamental U(1)s, one may or may not choose to include magnetic monopoles. By contrast, in many models where one of the U(1)s is the remnant of a spontaneously broken non-abelian gauge group, magnetic monopoles necessarily appear as topologically non-trivial field configurations. As a simple example consider a model where the hidden sector gauge group is SU(2), spontaneously broken to U(1) by an adjoint scalar ϕ^a (the 't Hooft–Polyakov model [6]). The Lagrangian for the scalar and the hidden sector gauge field with field strength $G^a_{\mu\nu}$ is

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} - \frac{1}{2}(D_\mu \phi)^a (D^\mu \phi)^a + m^2 \phi^a \phi^a - \lambda(\phi^a \phi^a)^2.$$

A field configuration which represents a monopole at the origin, $r = 0$, is given by $\langle \phi^a \rangle = r^a\, f(r)$ with $f(r)$ a certain function. It breaks SU(2) $\rightarrow$ U(1) at large r. We can couple this model to a visible sector U(1) with field strength $F_{\mu\nu}$ by adding the terms

$$\Delta\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2M} \phi^a G^{\mu\nu\, a} F_{\mu\nu}.$$

The last term represents kinetic mixing between the surviving hidden U(1) and the visible U(1). It is generated by integrating out heavy states which are charged under both the visible and the hidden sector gauge group. We have checked [4] in this setting that the 't Hooft–Polyakov monopole carries a monopole charge of precisely the allowed kind, as by the above discussion.

3 Magnetic mixing

In a model with a single U(1), a θ-term $\sim F^{\mu\nu} \tilde{F}_{\mu\nu}$ in the Lagrangian density gives electric charges to magnetic monopoles [3]. This can be seen as follows: Consider a magnetic monopole background with monopole charge g at $r = 0$, superimposed with some static gauge potential $(A^\mu) = (A^0, \mathbf{A})$. The θ-term can be written in terms of electric and magnetic fields as

$$-\frac{\theta}{32\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} = \frac{\theta}{8\pi^2} \mathbf{E} \cdot \mathbf{B} = \frac{\theta}{8\pi^2} \left(\nabla A^0\right) \cdot \left(\nabla \times \mathbf{A} + \frac{g}{4\pi} \frac{\mathbf{r}}{r^3}\right).$$

By integrating by parts, the Lagrangian contains a piece

$$L = \int d^3r\, \mathcal{L} \supset -\frac{\theta}{8\pi^2} \int d^3r\, A^0\, \nabla \cdot \frac{g\,\mathbf{r}}{4\pi\, r^3} = -\frac{\theta g}{8\pi^2} \int d^3r\, A^0 \delta^3(\mathbf{r}).$$

This is a coupling of an electric point charge $-\theta g/(8\pi^2)$, located at at $\mathbf{r} = 0$, to the electrostatic potential A^0. In other words, the monopole has acquired an electric charge.

In a model with a visible U(1) and a hidden U(1), and a magnetic mixing term

$$\mathcal{L} \supset -\frac{\theta_{FG}}{32\pi^2} F_{\mu\nu} \tilde{G}^{\mu\nu}$$

such as in Eq. (1), an analogous calculation [5] shows that hidden magnetic monopoles acquire visible electric charges. This is potentially very interesting with regard to phenomenology: Magnetic monopoles of ordinary electromagnetism are expected to be much heavier than M_{GUT} and therefore undetectable. Hidden sector monopoles, on the other hand, could very well be relatively light. If they carry electromagnetic charges from magnetic mixing, they could be detected in the same way as MCPs.

For instance, if the monopole is again a 't Hooft–Polyakov monopole of a spontaneously broken non-abelian gauge group in the hidden sector, its mass is semi-classically given by the breaking scale, divided by the gauge coupling. For models with an arbitrarily low breaking scale, the monopole could be arbitrarily light. Alternatively, if a high breaking scale is preferred for naturalness reasons, one might speculate that the hidden sector gauge group could be strongly coupled, such that the semi-classical approximation is invalid and monopoles could still be light. In fact, with the Seiberg–Witten model [11] there exists even a calculable example of a gauge theory with 't Hooft–Polyakov monopoles becoming arbitrarily light in some strong-coupling region of moduli space.

Despite the fact that there is no non-abelian analogue of kinetic or magnetic mixing terms, they may be generated radiatively [5] once a non-abelian gauge group is broken to U(1) (which is precisely the situation in which 't Hooft–Polyakov monopoles appear). For inducing these terms, the model should contain matter fields charged under both the visible and the hidden sector. Magnetic mixing can only be induced if these fields possess CP-violating couplings, since a magnetic mixing term itself violates CP.

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