Photon Production From The Scattering of Axions Out of a Solenoidal Magnetic Field

Eduardo I. Guendelman[1], *Idan Shilon*[1], *Giovanni Cantatore*[2], *Konstantin Zioutas*[3]

[1]Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel
[2]Universitá and INFN Trieste, via valerio 2, 34127 Trieste, Italy
[3]University of Patras, Patras, Greece

DOI: http://dx.doi.org/10.3204/DESY-PROC-2009-05/shilon_idan

We calculate the total cross section for the production of photons from the scattering of axions by a strong inhomogeneous magnetic field in the form of a 2D delta function, a cylindrical step function and a 2D Gaussian distribution, which can be approximately produced by a solenoidal current. The theoretical result is used to estimate the axion-photon conversion probability which could be expected in a reasonable experimental situation. Finally, we have also considered scattering at a resonance $E_{axion} \sim m_{axion}$, which gives the most enhanced results.

In a series of recent publications by one of us [1], it was shown that an axion-photon system displays a continuous duality symmetry when an external magnetic field is present and when the axion mass is neglected. This allows one to analyze the behavior of axions and photons in external magnetic fields in terms of an axion-photon complex field. It is important to note here that the same duality symmetry exists also when considering massive photons, under the condition $m_\gamma = m_a$.

This new 2D formalism uses a duality symmetry between the axion field and the scattered component of the photon to define an axion-photon complex field as $\Psi = 1/\sqrt{2}(\phi + iA)$, where ϕ is the axion field and A is the photon component that takes part in the scattering process. We focus here on the case where an electromagnetic field with propagation along the x and y directions and a strong static magnetic field pointing in the z-direction are present. The magnetic field may have an arbitrary space dependence in x and y. For convenience let us neglect the axion mass so we can write the lagrangian in terms of the new canonical variables Ψ and its charge conjugate Ψ^*

$$\mathcal{L} = \partial_\mu \Psi^* \partial^\mu \Psi - \frac{i}{2}\beta(\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^*) \ , \tag{1}$$

where $\beta(x, y) = gB(x, y)$ with $B(x, y)$ being the external magnetic field and where g is the photon-axion coupling constant. To apply these results to some specific system with a magnetic field, we write separately the time and space dependence of the axion-photon field as $\Psi(\vec{r}, t) = \mathrm{e}^{-i\omega t}\psi(\vec{r})$.

As a first model, we consider a magnetic field of the form $B = \Phi\delta^2(x, y)$. This kind of a potential can not be realized in the lab but we will show that the results for this calculation have physical significance in the resonance case, where the scattering becomes isotropic.

Separating the time and space dependence of Ψ and considering the δ function potential gives the following equation of motion

$$[-\vec{\nabla}^2 + g\Phi E\delta^2(x,y)]\psi(\vec{r}) = E^2\psi(\vec{r}) \ , \tag{2}$$

where E is the energy of the particle beam. Solving this equation while regulating the δ function by introducing the cutt-off Λ yields the solution

$$\psi(\vec{r}) = \mathrm{e}^{i\vec{k}\cdot\vec{r}} - G_k(r)\left[\frac{1}{g\Phi E} + \frac{\log(\Lambda/E)}{2\pi} + \frac{i}{2}\right]^{-1} \ , \tag{3}$$

where $G_k(r)$ is Green's function in two dimensions $G_k(r) = \frac{i}{4}H_0^{(1)}(kr) \overset{r\to\infty}{\longrightarrow} \frac{1}{2\sqrt{2\pi kr}}\mathrm{e}^{i(kr+\pi/4)}$.

The scattering amplitude $f(\theta)$ is found from the asymptotic behavior of the scattering wave function $\psi(\vec{r}) \to \mathrm{e}^{i\vec{k}\cdot\vec{r}} + \frac{1}{\sqrt{r}}f(\theta)\mathrm{e}^{i(kr+\pi/4)}$. Since there is no dependence on θ in $f(\theta)$ the scattering here is completely isotropic. Then, to first order in g we find that $\sigma^{\delta}_{tot} = \frac{g^2\Phi^2 E}{4}$. Thus, the probability is given by $P_\delta = \sigma_S/\sigma_G = \frac{g^2\Phi^2 E}{4D} = \frac{\pi^2 g^2 B^2 R^3 E}{8}$, where $\sigma_G = 2RL$ is the geometrical cross-section of the potential.

We wish to obtain eventually measurable quantities which can be incorporated in a laboratory experiment, thus we have to consider a more realistic function to describe the magnetic field generated by the solenoid. First, let us describe the inhomogeneous magnetic field by a Gaussian distribution around the solenoid's axis $\vec{B}(r) = B_0\mathrm{e}^{\frac{-r^2}{R^2}}\hat{z}$. Hence, the scattered wave function is

$$\psi(\vec{r}) = \mathrm{e}^{i\vec{k}\cdot\vec{r}} + \frac{\sqrt{\pi}gB_0R^2\sqrt{E}}{2\sqrt{2r}}\mathrm{e}^{-\frac{1}{4}(Rq)^2}\mathrm{e}^{i(kr+\pi/4)} \ . \tag{4}$$

This gives for the scattering amplitude $f(\theta) = \sqrt{(\pi/8)}gB_0R^2E^{1/2}\mathrm{e}^{-\frac{1}{4}(Rq)^2}$, where the explicit dependence of q (i.e the momentum transfer) on the angle is given by $q^2 = 2k^2(1-\cos\theta) = 4k^2\sin^2(\theta/2)$. Hence, The total 2D cross-section is given by

$$\int_0^{2\pi}|f(\theta)|^2 d\theta = \frac{\pi}{8}(gB_0)^2R^4E\int_0^{2\pi}\mathrm{e}^{-\frac{1}{2}(Rq)^2}d\theta = \frac{\pi^2}{4}(gB_0)^2R^4E\mathrm{e}^{-(Rk)^2}I_0((Rk)^2) \ , \tag{5}$$

where $I_0(x) = J_0(ix)$ is the modified Bessel function. The argument of this function (i.e $(Rk)^2$) is very large (1 eV $\times$ 1 cm $\approx 10^5$) so we can use the asymptotic from of the modified Bessel function which, by keeping only the first order term, gives the result $\sigma^{Gauss}_{tot} = \frac{\pi^{3/2}}{\sqrt{32}}g^2B_0^2R^3$, from which we find the conversion probability to be $P_{Gauss} = \frac{\pi^{3/2}}{8\sqrt{2}}g^2B_0^2R^2$.

Now we turn to consider the magnetic field generated by an ideal solenoidal current which is described by a step function realizing a uniform magnetic field pointing in the $\hat{z}$ direction and constrained to a cylindrical region around the origin: $\vec{B}(\vec{r}) = B_0\hat{z}$ for $r < R$. Using the Fourier transformation of the step function we find that the scattering amplitude is now given by $f(\theta) = \sqrt{\frac{\pi}{2}}\frac{B_0RgE^{1/2}}{q}\mathrm{J}_1(qR)$, where the explicit dependence of q on θ was shown earlier here.

Before evaluating the integral for the total cross-section, let us write the total cross section for the square well case in terms of the delta function cross-section: $\sigma^{well}_{tot.} = \sigma^{\delta}_{tot.}\frac{2}{\pi}I(ER)$, where $I(ER) = \int_0^{2\pi}\left|\frac{J_1(qR)}{qR}\right|^2 d\theta$ is a dimensionless quantity which is a function of the multiplication

$E \cdot R$. Of course, the proportionality constant connects also the conversion probabilities for the δ function and square well cases. Denoting $ER = kR$ by η, the integral can be analytically solved with the solution $I(\eta) = \frac{\pi}{2}\ {}_2F_3(\{\frac{1}{2}, \frac{2}{3}\}; \{1, 2, 3\}\,; -4\eta^2)$, where ${}_2F_3$ is an hypergeometric function.

To analyze this solution we expand the hypergeometric function ${}_2F_3$ to a series. Then, for small η, $I(\eta)$ is converging toward the constant value $\pi/2$, thus giving the equality $\sigma^{well}_{tot.} = \sigma^{\delta}_{tot.}$. This result is expected since considering only small η values is equivalent to considering isotropic scattering because $\eta \ll 1$ means that $ER \ll 1$. Hence, the wavelength of Ψ is very large compared to the length scale of the potential. Therefore, this approximation corresponds to δ function limit of the step function, which, in turn, means that we consider isotropic scattering.

On the other end, we have the expansion for large η which shows that the integral approaches the limit $I \to \frac{8}{3\pi\eta} = \frac{8}{3\pi ER}$ very fast. This limit gives the result $P_{well} = \frac{1}{6}g^2B^2D^2 = \frac{2}{3}g^2B^2R^2$.

So far in this report, we have considered the axion field as a massless field in order to get the $U(1)$ symmetry between axions and photons. In fact, this symmetry holds up whenever the axion mass is equal to the (effective) photon mass inside a medium. Of course, in that case our conclusions will have to be modified. The term that has to be taken under consideration is an $1/\sqrt{(E^2 - m^2)^{1/2}}$ term which comes from the Green's function and will replace the current $1/\sqrt{E}$ in the scattering amplitude. Thus, in the $m_a \sim m_\gamma \neq 0$ case, the total two dimensional cross-section (for the δ function case) would have the following energy dependence: $\sigma_{tot} = \frac{\pi g^2 B^2 R^4 E^2}{4\sqrt{(E^2 - m^2)}}$, and we have a resonance when $E = m$. For an axion rest mass below ~ 1 eV, this can have practical consequences, for example, in laser generated axions when one can control the energy of the axion beam.

The limit of zero momentum means accounting only zero modes of the Fourier Transform, hence the 1D treatment of this process can not be justified since in the limit of zero momentum the scattering amplitude and the differential scattering cross-section become isotropic (i.e independent of the angle) and it is impossible to consider only one direction in the scattering.

To summarize, we have studied here the first examples of scattering which is not one dimensional and we have obtained enhanced probabilities. This effect is further increaed in the case of resonant scattering that appears when $E = m$ and corresponds to isotropic scattering.

In the 1D case the conversion probability is $P_{1D} = g^2B^2l^2/4$, where l is the linear dimension associated with the extent of the magnetic field [4]. Hence, when trying to compare the conversion probability for the cylindrically symmetric geometry found by the method used in this work with the known 1D calculation it is not so obvious what is the correct length scale l that should be taken to calculate P_{1D}. The problem is that the notion of splitting does not make sense in 1D and that the scattering region is not an area but a line. Hence, the best way to discuss the relation between the two calculations will be to average the 1D probability over the scattering region for each case. In other words, we look at the 2D experiment as the weighted average of an infinite number of 1D experiments.

The general case is rather complicated since the scattering region may be infinite and the magnetic field may not be homogenous. However, a 1D analogue to the 2D experiment can be found and the weighted average can be done by choosing the magnetic flux as the averaging measure

$$P^{avg.}_{1D} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{4}g^2 |\int_{-\infty}^{\infty} B(x', y)dx'|^2 B(x, y)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(x, y)dxdy}\ . \tag{6}$$

For the step function case the scattering region is a cylinder with radius R. Hence, the average of P_{1D} in this case is $P_{1D}^{well\ avg.} = \frac{2}{3}g^2 B_0^2 R^2$ a result which coincides with the 2D result.

When considering the Gaussian case the averaged 1D probability is $P_{1D}^{Gauss.\ avg.} = \frac{\pi}{\sqrt{3}}\frac{g^2 B_0^2 R^2}{4}$. The comparison of this result with the 2D result gives $P_{Gauss}/P_{1D}^{Gauss.\ avg.} = 1.085$ and thus the 2D result is bigger by 8.5%.

Despite Eq. (6) not all 2D experiments can have a 1D analogue. We have seen that when considering resonant scattering, the limit of zero momentum implies that the cross section is isotropic and there is no way to describe such a process with an analogous 1D calculation.

When considering scattering from a finite sized potential the enhancement of the conversion probability compared to the 1D case still gives probabilities in the same order of magnitude. This is due to the fact that the wavelength $(1/E)$ of the Ψ wave function is much smaller than the length scale of the potential (R), which essentially results in a quasi-1D behavior of the system. When the wavelength will be smaller, or even comparable to the length scale of the potential we see that we get bigger enhancement since in this case the scattering becomes more and more isotropic and we essentially obtain δ function scattering.

The wavelength is determined by the momentum of the particles. For the massive case, the momentum approaches zero when the energy of the particles is of the order of the particle's mass. This situation, where the wavelength of the particles is much larger than any other length scale in the problem, is realized in the resonant scattering case. There we have shown explicitly that this limit gives an isotropic scattering for a finite potential and thus, conversion probabilities of the order of the δ function case.

Our results may also be applicable for solar scales as well. In the sun, magnetic flux tube can play the role of a solenoidal potential while the energy spectrum of photons is continuous. Thus, we expect to have both isotropic and anisotropic scattering. These magnetic flux tubes are enormous regions of constant magnetic flux with length scale of the order of about 10^2 km in diameter and 10^3 km in length. Hence, the conversion probability will be greatly enhanced.

Acknowledgments

The authors wish to thank J. Jaeckel and J. Redondo for useful correspondence and conversations. E.I. Guendelman thanks the university of Trieste and INFN for the kind hospitality and support during his visit to Trieste. I. Shilon thanks the organizers of the 5th Patras workshop, and in particular J. Jaeckel, for inviting and supporting participation in the workshop.

References

[1] E.I. Guendelman, Mod. Phys. Lett. A **23** 191 (2008), arXiv: 0711.3685 [hep-th]; E.I. Guendelman, Phys. Lett. B **662** 227 (2008), arXiv: 0801.0503 [hep-th]; E.I. Guendelman, Phys. Lett. B **662** 445 (2008), arXiv: 0802.0311 [hep-th].

[2] E. Arik, et al. (The CAST collaboration), JCAP 0902:008 (2009); arXiv: hep-ex/0810.4482.

[3] K. van Bibber, P.M. McIntyre, D.E. Morris and G.G. Raffelt, Phys. Rev. D **39** 2089 (1989). See also: R. Rabadan, A. Ringwald, K. Sigurson, Phys. Rev. Lett. **96** 110407 (2006).

[4] K. van Bibber, N.R. Dagdeviren, S.E. Koonin, A.K. Kerman and H.N. Nelson, Phys. Rev. Lett. **59** 759 (1987).