Experimental Status of the CKM Matrix

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We review the present status of experimental results on the magnitudes and phases of the elements of the Cabibbo-Kobayashi-Maskawa matrix. The matrix is found to be consistent with being unitary as predicted by the Standard Model. The matrix is also consistent with being the origin of the observed violations of CP-symmetry in K and B decays.

1 Introduction

In the Standard Model with three generations quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates and the Cabibbo-Kobayashi-Maskawa (CKM) matrix relates these two bases:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{\rm ud} & V_{\rm us} & V_{\rm ub} \\ V_{\rm cd} & V_{\rm cs} & V_{\rm cb} \\ V_{\rm td} & V_{\rm ts} & V_{\rm tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The CKM matrix element $V_{\rm ij}$ describes the strength of the amplitude of the charged-current, flavor-changing quark transition $i \to jW^-$. Since the CP-conjugate decay $\bar{i} \to \bar{j}W^+$ depends on $V^*_{\rm ij}$, the complex nature of the CKM matrix allows for violation of CP-symmetry in quark transitions.

The quark mixing matrix for three generations was first suggested by Makoto Kobayashi and Toshihide Maskawa in 1973 [1] for which they received the 2008 Nobel Prize in Physics (shared with Y. Nambu). In acknowledgement of Nicola Cabibbo's earlier work [2] on quark mixing with two generations we call the quark mixing matrix the CKM matrix $V_{\rm CKM}$.

In the three-generation Standard Model $V_{\rm CKM}$ is a unitary 3×3 matrix. Observables are combinations of matrix elements that are invariant under arbitrary phase transformations:

- doublets $V_{\rm ij}V^*_{\rm ij}$, i.e. the magnitudes of the CKM matrix elements,
- quartets $V_{\rm ij}V_{\rm kl}V^*_{\rm il}V^*_{\rm kj}$, which give access to relative phases between matrix elements,
- sextets $V_{\rm ij}V_{\rm kl}V_{\rm mn}V^*_{\rm il}V^*_{\rm kn}V^*_{\rm mj}$ and higher $2n$-tets constructed in an analogous way.

Due to unitarity constraints the CKM matrix has only four independent parameters. Several parameterizations have been suggested. A common parameterization is the one from Wolfenstein [3], which is an expansion in the small parameter λ that reflects the hierarchy of the magnitudes of the matrix elements.

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

The Standard Model makes no predictions about the values of the CKM matrix elements. However, the unitarity of the CKM matrix provides precise constraints on the relations between matrix elements. A deviation from unitarity would be evidence for physics beyond the Standard Model. The test of the unitarity of the CKM matrix has been a major goal of many flavor physics experiments over the last decade.

2 Magnitudes of the CKM matrix elements

The magnitudes of the CKM matrix elements of the first two rows are all determined from semileptonic decays in order to reduce as much as possible theoretical uncertainties arising from strong interactions between quarks. It is currently not possible to determine the CKM matrix elements which involve the top quark from semileptonic decays. These matrix elements are determined from processes involving virtual top quark pairs or weak production of single top quarks. The measurements of the magnitudes of the CKM matrix elements are limited by the understanding of the influence of the strong interaction in these weak weak processes.

2.1 $|V_{\mathrm{ud}}|$ and $|V_{\mathrm{us}}|$

The most precise determination of the magnitude of V_{ud} comes from super-allowed nuclear β-decays. These decays are pure vector, $0^+ \to 0^+$ transitions within the same isospin multiplet. A recent review [5] gives an average of $\mathcal{F}t = (3071.81 \pm 0.79 \pm 0.27)$ s for thirteen different transitions, where $\mathcal{F}t$ is the product of the Fermi function f, the half-life t and nucleus-dependent corrections for isospin symmetry breaking and internal bremsstrahlung. The magnitude of V_{ud} is obtained from

$$|V_{\mathrm{ud}}|^2 = \frac{m_e^{-5}\pi^3 \ln 2}{G_F^2(1+\Delta_R^V)\mathcal{F}t} ,$$

where m_e is the electron mass, G_F is the Fermi constant taken from muon decay, and Δ_R^V is the electroweak radiative correction. Uncertainties in the calculation of Δ_R^V have recently been reduced by a factor of two [6] and the current value is $\Delta_R^V = (2.361 \pm 0.038)\%$. However, Δ_R^V is still the dominant source of uncertainty for $|V_{\mathrm{ud}}|$. The current world average [5] is given by $|V_{\mathrm{ud}}| = 0.97425 \pm 0.00022$.

The magnitude of V_{us} is determined from semileptonic kaon decays ($Kl3$ decays). Their decay rate is given by

$$\Gamma(Kl3) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{\mathrm{EW}} |V_{\mathrm{us}}|^2 |f_+(0)|^2 I_{Kl} \left(1 + \delta_K^{\mathrm{SU(2)}} + \delta_{Kl}^{\mathrm{EM}}\right),$$

where l refers to either e or μ, M_K is the kaon mass, S_{EW} is the short-distance radiative correction, $\delta_{Kl}^{\mathrm{EM}}$ is the mode-dependent long-distance radiative correction, $f_+(0)$ is the transition form factor calculated at zero momentum transfer for the $l\nu$ system, and I_{Kl} is the phase-space integral, which depends on the measured semileptonic form factors. For charged kaon decay $\delta_K^{\mathrm{SU(2)}}$ is the deviation from one of the ratio of $f_+(0)$ for the charged to neutral kaon decay. C^2 is 1 (1/2) for neutral (charged) kaon decays. Experimentally measured are the $Kl3$ decay widths (from the $Kl3$ branching fractions and K lifetimes) and form factors. The values of S_{EW}, $\delta_{Kl}^{\mathrm{EM}}$, $\delta_K^{\mathrm{SU(2)}}$, and $f_+(0)$ are provided by theory.

An average of $|V_{\mathrm{us}}|f_+(0)$ including new measurements from KLOE, KTeV, ISTRA+ and NA48 has recently been presented at the KAON '09 conference [7]: $\langle |V_{\mathrm{us}}| f_+(0) \rangle = 0.21660(47)$.

The uncertainty of the two most precise measurements in this average from $K_L l3$ decays are dominated by the K_L lifetime uncertainty. The most recent measurements of the K_L [8] and K_S [9] lifetimes from KLOE have not been used in the $|V_{\rm us}|f_+(0)$ average. Using a recent lattice calculation of $f_+(0) = 0.964(5)$ [10] gives $|V_{\rm us}| = 0.2246 \pm 0.0012$.

It is noteworthy that measurements of $|V_{\rm us}|$ from τ decays have reached a comparable precision. From the ratio of branching ratios $BR(\tau \to K\nu)/BR(\tau \to \pi\nu)$ BABAR obtains $|V_{\rm us}| = 0.2246 \pm 0.0023$ [11]. From inclusive τ decays to strange final states $|V_{\rm us}|$ is determined with very small theoretical uncertainties: $|V_{\rm us}| = 0.2165 \pm 0.0026(\text{exp}) \pm 0.005(\text{theo})$ [12]. However, the difference of about 2.6σ with respect to the result from $Kl3$ decays needs to be understood.

The ratio $|V_{\rm us}/V_{\rm ud}|$ is determined from the ratio of decay rates for $K \to \mu\nu$ [13] and $\pi \to \mu\nu$:

$$\left|\frac{V_{\rm us}}{V_{\rm ud}}\right| = 0.2387(4)\sqrt{\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)}} \times \frac{f_\pi}{f_K} \ .$$

Using a recent calculation of the ratio of decay constants $f_\pi/f_K = 1.189(7)$ [14] gives $|V_{\rm us}/V_{\rm ud}| = 0.2321(15)$. From the direct determinations of $|V_{\rm ud}|$ and $|V_{\rm us}|$ and the ratio $|V_{\rm us}/V_{\rm ud}|$ the FlaviaNet collaboration calculates [7]:

$$|V_{\rm ud}| = 0.97424 \pm 0.00022 \quad \text{and} \quad |V_{\rm us}| = 0.2252 \pm 0.0009 \ .$$

2.2 $|V_{\rm cd}|$ and $|V_{\rm cs}|$

Di-muon production measurements by neutrinos on nuclei provide still the best measurement of $|V_{\rm cd}| = 0.230 \pm 0.011$ [15]. However, new measurements of the rate for semileptonic $D \to \pi l\nu$ decays from CLEO-c have a smaller experimental error $|V_{\rm cd}| = 0.234 \pm 0.007(\text{exp}) \pm 0.025(\text{theo})$ [16]. The theoretical error is dominated by the uncertainty of the lattice calculation of the $D \to \pi$ form factor. Our average of these two measurements is

$$|V_{\rm cd}| = 0.231 \pm 0.010 \ .$$

The value $|V_{\rm cs}|$ is determined from measurements of branching ratios of leptonic D_s^+ and semileptonic D decays by CLEO-c [17, 18], BELLE [19] and BABAR [20]. CLEO-c has published recently precise results for $|V_{\rm cs}|f_{D_s}$ from their measurements of the $D_s \to \tau\bar\nu, \mu\bar\nu$ branching ratios [17]. In their paper CLEO-c quote their result in form of the D_s^+ decay constant. We turn this into a number for $|V_{\rm cs}|$ by using $|V_{\rm cs}| = (f_{D_s,\text{meas}}/f_{D_s,\text{LQCD}})|V_{\rm ud}|$ where we use the value of $|V_{\rm ud}|$ from above and an average of $\langle f_{D_s}\rangle = 242 \pm 5$ MeV (calculated from results in [14] and [21] with a scale factor of 1.6 for the error). One obtains $|V_{\rm cs}| = 1.04 \pm 0.04$. CLEO-c measures a consistent result although with a somewhat larger theoretical error (dominated by the $D \to K$ form factor) from semileptonic $D \to Kl\bar\nu$ decays: $|V_{\rm cs}| = 0.985 \pm 0.01(\text{exp}) \pm 0.10(\text{theo})$ [18]. Our average of these numbers is

$$|V_{\rm cs}| = 1.03 \pm 0.04 \ .$$

2.3 Unitarity check of the udsc submatrix

Using the measurements of $|V_{\rm ud}|$, $|V_{\rm us}|$, $|V_{\rm cd}|$, and $|V_{\rm cs}|$ one can check the unitarity of the udsc 2×2 submatrix:

$$\begin{aligned}
|V_{\rm ud}|^2 + |V_{\rm us}|^2 - 1 &= -0.0004 \pm 0.0007 \;\; (-0.6\sigma) \\
|V_{\rm cd}|^2 + |V_{\rm cs}|^2 - 1 &= +0.114 \pm 0.083 \;\; (+1.3\sigma) \\
|V_{\rm ud}|^2 + |V_{\rm cd}|^2 - 1 &= +0.003 \pm 0.005 \;\; (+0.6\sigma) \\
|V_{\rm us}|^2 + |V_{\rm cs}|^2 - 1 &= +0.112 \pm 0.082 \;\; (+1.4\sigma)
\end{aligned}$$

The most precise unitarity test of the udsc matrix comes from the first row elements. The uncertainties of $|V_{\rm ud}|^2$ and $|V_{\rm us}|^2$ contribute roughly the same to the error of this unitarity check. The precision of the udsc submatrix elements is not yet sufficient to predict the existence of the third quark family. Tight constraints on new physics parameters such as the mass of a charged Higgs [22], the coupling to a fourth quark generation [23] and the inclusive branching ratio of exotic muon decays (through G_F) [23] can be obtained from these measurements.

The 2×2 matrix also gives

$$\lambda_{CKM} = 0.2252 \pm 0.0009 \, .$$

2.4 $|V_{\rm cb}|$

The CKM matrix element $|V_{\rm cb}|$ is determined from exclusive and inclusive semileptonic B decays to charmed final states. The differential decay rates for the exclusive $\bar{B} \to D^{(*)} l \bar{\nu}$ decays are given by

$$\begin{aligned}
\frac{d\Gamma}{dw}(\bar{B} \to D^* l \bar{\nu}) &= \frac{G_F^2}{48\pi^3} |V_{\rm cb}|^2 m_{D^*}^3 (w-1)^{1/2} P(w) (F(w))^2, \\
\frac{d\Gamma}{dw}(\bar{B} \to D l \bar{\nu}) &= \frac{G_F^2}{48\pi^3} |V_{\rm cb}|^2 (m_D + m_B) m_D^3 (w-1)^{3/2} (G(w))^2,
\end{aligned}$$

where $m_{D^{(*)}}$ and m_B are the $D^{(*)}$ and B meson masses, w is related to the energy of the $D^{(*)}$ meson in the B rest frame, $P(w)$ is a phase space factor and $F(w)$ and $G(w)$ are the $B \to D^{(*)}$ form factors.

Experiments fit the differential decay rates for $|V_{\rm cb}|F(1)$ and $|V_{\rm cb}|G(1)$ using form factor parameterizations derived from HQET. There have been several new precision results [24] over the last few years from BABAR and BELLE. The latest averages from HFAG [25] calculated for this conference are $|V_{\rm cb}|G(1) = (42.3 \pm 0.7 \pm 1.3) \times 10^{-3}$ and $|V_{\rm cb}|F(1) = (35.75 \pm 0.42) \times 10^{-3}$.

The $|V_{\rm cb}|F(1)$ average does not yet include the recent result $|V_{\rm cb}|F(1) = (35.0 \pm 0.4 \pm 2.2) \times 10^{-3}$ from BELLE's study of $B^- \to D^{*0} l \nu$ decays [26]. Using recent calculations of the $B \to D^{(*)}$ form factors $G(1) = 1.074 \pm 0.018 \pm 0.016$ [27] and $F(1) = 0.921 \pm 0.013 \pm 0.020$ [28] from lattice calculations gives consistent values for $|V_{\rm cb}|$ from $\bar{B} \to D^* l \bar{\nu}$ ($|V_{\rm cb}| = (39.4 \pm 1.4(\text{exp}) \pm 0.9(\text{theo})) \times 10^{-3}$) and $\bar{B} \to D l \bar{\nu}$ ($|V_{\rm cb}| = (38.8 \pm 0.5(\text{exp}) \pm 1.0(\text{theo})) \times 10^{-3}$) decays.

The theoretical uncertainty in $|V_{\rm cb}|$ from $\bar{B} \to D^{(*)} l \bar{\nu}$ decays due to the hadronization process can be avoided if $|V_{\rm cb}|$ is determined from inclusive $b \to c l \nu$ transitions. Using HQET and Operator Product Expansion the inclusive $b \to c l \nu$ decay rate can be expressed by an expansion in powers of $1/m_b$. Non-perturbative corrections up to order $1/m_b^3$ are determined from inclusive distributions in B decays such as the lepton energy spectrum and the hadronic mass spectrum in $b \to cl\nu$ decays and the photon energy spectrum in $b \to s\gamma$ decays. HFAG gives an average of $|V_{\rm cb}|$ from inclusive $b \to cl\nu$ transitions of $|V_{\rm cb}| = (41.67 \pm 0.44 \pm 0.58) \times 10^{-3}$ [25]. This value is about 2.3σ higher than the value from $\bar{B} \to D^{(*)}l\bar{\nu}$ decays. Averaging the two $|V_{\rm cb}|$ values we obtain

$$|V_{\rm cb}| = (40.6 \pm 1.3) \times 10^{-3} \ ,$$

which includes a scale factor for the error of 2.3 .

2.5 $|V_{\rm ub}|$

The B factories determine $|V_{\rm ub}|$ from $B \to \pi l\nu$ decays and inclusive $b \to ul\nu$ decays. There have been several new results in the last few years. The total decay rate for $B \to \pi l\nu$ is now measured by BABAR [29], BELLE [30] and CLEO [31] with a precision of about 5%.

The product $|V_{\rm ub}|f_+(q^2)$ is obtained from the differential decay rate

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 |V_{\rm ub}|^2 |f_+(q^2)|^2 \ .$$

The $B \to \pi$ form factor $f_+(q^2)$ is calculated from lattice QCD [32] and light-cone sum rules [33]. The two methods are complementary since the lattice calculation is limited to large q^2 and light-cone sum rules provide information close to $q^2 = 0$. A recent review [34] quotes an average value for $|V_{\rm ub}|$ from $B \to \pi l\nu$ decays of $|V_{\rm ub}|_{\rm excl} = (3.5^{+0.6}_{-0.5}) \times 10^{-3}$, where the error is dominated by the form factor calculations. This average does not include a new result of $|V_{\rm ub}|_{\rm excl} = (3.38 \pm 0.36) \times 10^{-3}$ obtained with an improved form factor calculation [35].

The magnitude of $V_{\rm ub}$ can also be determined from measurements of the total inclusive $B \to X_u l\nu$ decay rate:

$$\Gamma(B \to X_u l\nu) = \frac{G_F^2}{192\pi^3} |V_{\rm ub}|^2 m_b^5 (1 + \ \Delta_{\rm hadr}),$$

where m_b is the b quark mass and $\Delta_{\rm hadr}$ are hadronic corrections. The two main challenges for the determination of $|V_{\rm ub}|$ from inclusive decays are the strong dependence of the decay rate on the b quark mass and the large background from $B \to X_c l\nu$ decays, which is about fifty times larger than the $B \to X_u l\nu$ signal. In practice the experiments measure a partial $B \to X_u l\nu$ decay rate in regions of phase space where the background is comparatively small and then use so-called shape functions derived from the photon energy spectrum in $B \to X_s\gamma$ decays and theory to extrapolate the rate to the full phase space. Their have been several recent analyses by BABAR and BELLE with varying levels of signal purity and reconstruction efficiency [36]. The average of all measurements of $|V_{\rm ub}|$ from inclusive $B \to X_u l\nu$ decays within one particular theoretical framework [37] gives $|V_{\rm ub}| = (4.20 \pm 0.16^{+0.22}_{-0.23}) \times 10^{-3}$. However, there are several such frameworks [38] and their calculations for $|V_{\rm ub}|$ vary between $(4.05 - 4.87) \times 10^{-3}$.

Taking the average of the $|V_{\rm ub}|$ value from [37] and the $|V_{\rm ub}|$ measurement from $B \to \pi l\nu$ decays gives

$$|V_{\rm ub}| = (4.07 \pm 0.38) \times 10^{-3} \ .$$

This average does not account for the spread between theory frameworks for the value of $|V_{\rm ub}|$ from inclusive $B \to X_u l\nu$ decays. However, the error is scaled by 1.5 to account for the difference between the values for $|V_{\rm ub}|$ from inclusive and exclusive decays.

2.6 $|V_{\rm td}|$ and $|V_{\rm ts}|$

The large value of $|V_{\rm tb}|$ and the relatively small size of currently available top samples do not allow the determination of $|V_{\rm td}|$ and $|V_{\rm ts}|$ from semileptonic top quark decays. Instead these CKM matrix elements are determined from the oscillation frequencies of B_d^0 and B_s^0 mesons, respectively. The $B\bar{B}$ oscillation process is dominated by a 2nd order weak box diagram involving a $t\bar{t}$ pair. The oscillation frequencies are given by

$$\Delta m_{d(s)} = \frac{G_F^2}{6\pi^2} m_{B_{d(s)}} f^2_{B_{d(s)}} \hat{B}_{B_{d(s)}} \eta_{\rm QCD} |V_{\rm td(ts)}|^2 |V_{\rm tb}|^2 S_0(m_t^2/m_W^2),$$

where $f_{B_{d(s)}}$ and $\hat{B}_{B_{d(s)}}$ are the $B_{d(s)}$ weak decay constant and bag parameter, respectively, $\eta_{\rm QCD}$ is a QCD correction factor and S_0 is a function that depends on the square of the ratio of the top quark mass and the W boson mass [39].

The B-factories BABAR and BELLE provide the most precise measurements of Δm_d [41] while CDF and D0 measure Δm_s best [42]. The current world averages [25] are given by $\Delta m_d = (0.507 \pm 0.004){\rm ps}^{-1}$ and $\Delta m_s = (17.78 \pm 0.12){\rm ps}^{-1}$. With new lattice results for $f_{B_{d(s)}}\sqrt{\hat{B}_{B_{d(s)}}}$ [40] one obtains

$$|V_{\rm td}| = (8.1 \pm 0.6) \times 10^{-3} \qquad \text{and} \qquad |V_{\rm ts}| = (38.7 \pm 2.3) \times 10^{-3}\ .$$

Since about half of the theoretical error budget cancels in the calculation of $(f_{B_d}/f_{B_s})\sqrt{\hat{B}_{B_d}/\hat{B}_{B_s}}$, the ratio $|V_{\rm td}/V_{\rm ts}|$ has a correspondingly smaller relative error: $|V_{\rm td}/V_{\rm ts}| = 0.209 \pm 0.001 \pm 0.006$. BABAR [43] and BELLE [44] have measured this ratio also from radiative B decays to $K^*\gamma$ and $(\rho/\omega)\gamma$ final states to be $|V_{\rm td}/V_{\rm ts}| = 0.210 \pm 0.15 \pm 0.018$ [45], which is consistent with the value determined from the oscillation frequencies, but has a larger error.

2.7 $|V_{\rm tb}|$

The value of $|V_{\rm tb}|$ is determined from the production cross-section of single top events in $p\bar{p}$ collisions. CDF [46] and D0 [47] have both reported observations of this process with significances above 5σ. The experiments presented an updated average of $|V_{\rm tb}|$ at this conference [48]:

$$|V_{\rm tb}| = 0.91 \pm 0.08\ .$$

2.8 Unitarity check of the CKM matrix element magnitudes

From the measurements of the magnitudes of all CKM matrix elements one can check the unitarity of the CKM matrix:

$$\begin{aligned}
|V_{\rm ud}|^2 + |V_{\rm us}|^2 + |V_{\rm ub}|^2 - 1 &= -0.0004 \pm 0.0007\ \ (-0.6\sigma) \\
|V_{\rm cd}|^2 + |V_{\rm cs}|^2 + |V_{\rm cb}|^2 - 1 &= +0.11 \pm 0.08\ \ (+1.3\sigma) \\
|V_{\rm td}|^2 + |V_{\rm ts}|^2 + |V_{\rm tb}|^2 - 1 &= +0.00 \pm 0.20\ \ (+0.0\sigma) \\
|V_{\rm ud}|^2 + |V_{\rm cd}|^2 + |V_{\rm td}|^2 - 1 &= +0.003 \pm 0.005\ \ (+0.6\sigma) \\
|V_{\rm us}|^2 + |V_{\rm cs}|^2 + |V_{\rm ts}|^2 - 1 &= +0.11 \pm 0.08\ \ (+1.4\sigma) \\
|V_{\rm ub}|^2 + |V_{\rm cb}|^2 + |V_{\rm tb}|^2 - 1 &= +0.00 \pm 0.20\ \ (+0.0\sigma)
\end{aligned}$$

The magnitudes of the CKM matrix fulfill the unitarity requirements well. From the matrix elements $|V_{\rm cb}|$ and $|V_{\rm ts}|$ one obtains

$$A\lambda^2_{CKM} = (40.1 \pm 1.1) \times 10^{-3} \ .$$

3 Phases of the CKM matrix elements

Six unitarity constraints involving the relative phases between CKM matrix elements can be expressed by the products of one row (or column) of the CKM matrix with the complex-conjugate transpose of another row (or column) and graphically displayed as triangles in the complex plane. The triangle derived from the first and third column of the CKM matrix has become known as the Unitarity Triangle. The inner angles of the Unitarity Triangle α, β, and γ and a fourth phase, β_s related to B_s mixing are defined as

$$\alpha = \arg\left(\frac{-V_{\rm td}V^*_{\rm tb}}{V_{\rm ud}V^*_{\rm ub}}\right), \quad \beta = \arg\left(\frac{-V_{\rm cd}V^*_{\rm cb}}{V_{\rm td}V^*_{\rm tb}}\right), \quad \gamma = \arg\left(\frac{-V_{\rm ud}V^*_{\rm ub}}{V_{\rm cd}V^*_{\rm cb}}\right), \quad \beta_s = \arg\left(\frac{-V_{\rm ts}V^*_{\rm tb}}{V_{\rm cs}V^*_{\rm cb}}\right).$$

By dividing all sides by $V_{\rm cd}V^*_{\rm cb}$, the apex of the Unitarity Triangle is given by $(\bar\rho, \bar\eta)$ where, for example, $\bar\rho = \rho(1 - \lambda^2/2)$. In the Wolfenstein convention γ is the phase of $V_{\rm ub}$ and $\beta_{(s)}$ is the phase of $V_{\rm td(ts)}$. Experimental sensitivity to the phases of the CKM matrix elements comes from the interference of two decay amplitudes with different weak phases and from the comparison of CP asymmetries from B and $\bar B$ decays. Since hadronic uncertainties largely cancel in the ratios of amplitudes between B and $\bar B$ decays, the measured values of the CKM phases have small theoretical uncertainties and turn out to be experimentally limited.

3.1 β

The value of $\sin(2\beta)$ can be determined without large theoretical uncertainties [49] from the time-dependent CP asymmetries in B decays to final states with a charmonium meson and a neutral kaon. These decays proceed either through the direct $b \to (c\bar c)s$ amplitude or through the $B\bar B$ mixing amplitude followed by $\bar b \to (c\bar c)\bar s$. The most precise determination of $\sin(2\beta)$ comes from measurements by BABAR and BELLE of B decays to $J/\psi K^0$, $J/\psi K^{*0}$, $\psi(2S)K^0_S$, $\eta_c K^0_S$, and $\chi_{c1}K^0_S$ [50]. The world average [25] is given by $\sin(2\beta) = 0.672 \pm 0.023$.

Converting $\sin(2\beta)$ into β leaves a two-fold ambiguity for $\beta < 90°$. The solution with negative $\cos(2\beta)$ has been ruled out by measurements of the CP asymmetries in decays with contributions from CP-odd and CP-even amplitudes ($B \to J/\psi K^{*0}$, $B \to D^*D^*K^0_S$ and $B \to Dh^0$ (with $D \to K^0_S\pi^+\pi^-$)). This gives

$$\beta = (21.1 \pm 0.9)^\circ \ .$$

A sensitive test of the predictions of the CKM theory regarding CP asymmetries comes from comparing the above value of $\sin(2\beta)$ with the CP asymmetry obtained from B decays through suppressed penguin loop diagrams. In the Standard Model the weak phase in $b \to (q\bar q)s$ penguin loop decays, where $q\bar q$ is a light quark pair, is the same as in decays to charmonium final states. Therefore the time-dependent CP asymmetry in $b \to (q\bar q)s$ decays, $\sin(2\beta_{\rm eff})$, is expected to be close to $\sin(2\beta)$. Theoretical calculations give $\Delta\sin(2\beta) \equiv \sin(2\beta_{\rm eff}) - \sin(2\beta)$ in the range of $0.01 - 0.1$. However, contributions from new physics processes to these rare decays could cause large $|\Delta\sin(2\beta)|$.

BABAR and BELLE have measured the time-dependent CP asymmetries for 9 rare $b \to (q\bar{q})s$ decays. All measurements are consistent with $\Delta \sin(2\beta) = 0$ and no direct CP violation. A few years ago the (naive) average of the $\sin(2\beta_{\text{eff}})$ for these rare modes differed by more than 3σ from zero [25]. With the latest measurements of $\sin(2\beta_{\text{eff}})$ this discrepancy has been been reduced to approximately 1σ [25].

The theoretical uncertainty in $\sin(2\beta_{\text{eff}})$ is believed to be relatively small for the modes $B \to \phi K^0$, $B \to \eta' K^0$ and $B \to K^0_S K^0_S K^0_S$. Our average for these modes is $\sin(2\beta_{\text{eff,clean}}) = 0.59 \pm 0.06$, which is 1.3σ way from $\sin(2\beta)$.

3.2 α

The time-dependent CP asymmetries in B^0 decays proceeding through a $b \to u$ tree amplitude are sensitive to α. The decay $B^0 \to \pi^+\pi^-$, which is experimentally most accessible, suffers from the contribution of a relatively large $b \to d$ penguin amplitude. The time-dependent CP asymmetries of $B^0 \to \pi^+\pi^-$ are sensitive to $\sin(2\alpha + \delta_{\pi\pi})$ and the phase $\delta_{\pi\pi}$ needs to be measured through an isospin analysis of the branching ratios of neutral and charged B decays to $\pi\pi$ final states [51]. The large size of the $B \to \pi\pi$ penguin amplitude and discrete ambiguities in the determination of $\delta_{\pi\pi}$ currently only allow to exclude the range of $12^\circ < \alpha < 78^\circ$ (at 95% C.L.) [54].

The decays $B \to \rho^+\rho^-$ proceed through the same Feynman diagrams as $B \to \pi^+\pi^-$ decays. The $\rho\rho$ final state consists of two vector mesons and thus separate isospin analyses are in principle required for each of the three polarization amplitudes. However, since the fraction of $B^0 \to \rho^+\rho^-$ decays proceeding through the longitunal polarization amplitude is $f_L(\rho^+\rho^-) = 0.978^{+0.025}_{-0.022}$ [25] in practice only one isospin analysis in needed. In addition, the penguin contribution to $B \to \rho^+\rho^-$ is rather small as is evident from the small ratio of branching ratios for $BR(B \to \rho^0\rho^0)/BR(B^+ \to \rho^+\rho^0)$. As a result the discrete ambiguities for $\delta_{\rho\rho}$ all overlap. A recent update of the branching ratio of $B^+ \to \rho^+\rho^0$ by BABAR constrains $\delta_{\rho\rho}$ further [52]. An average for α from the most recent measurements of time-dependent CP asymmetries in $B \to \pi\pi$, $\rho\rho$ and $\rho\pi$ [53] and corresponding branching fractions has been calculated by the CKMFitter group to be [54]:

$$\alpha = (89.0^{+4.4}_{-4.2})^\circ \; .$$

A first determination of α by BABAR using B decays to the axial-vector final state $a_1\pi$ of $\alpha = (79 \pm 7 \pm 11)^\circ$ [55] is not yet included in this average.

3.3 γ

The angle γ is determined from the interference between $b \to c$ and $b \to u$ transition amplitudes in $B^\pm \to D^{(*)}K^\pm$ decays, where the $D^{(*)}$ meson decays to final states accessible to D^0 and $\bar{D}^0$. Several neutral D final states have been investigated by BABAR, BELLE and CDF including D decays to CP eigenstates [56], D decays to flavor states involving doubly Cabibbo-suppressed transitions [57] and D decays to three-body final states [58].

BABAR recently found evidence at 3.4σ for the decay $B^- \to \bar{D}^0 K^-$ [59], which proceeds through a doubly Cabibbo-suppressed D decay amplitude. The best sensitivity to γ comes from the time-dependent Dalitz analysis of $B^- \to D^{(*)0}K^-$ decays where the D^0 subsequently decays to a three-body $K^0_S\pi^+\pi^-$ or $K^0_S K^+K^-$ final state. BELLE recently updated their result and now includes also $D^{*0} \to D^0\gamma$ decays in their analysis. They obtain $\gamma = (78^{+11}_{-12} \pm 3.6 \pm 9)^\circ$ [60], where the first errors is statistical, the second is systematical and the last one is due to the D^0 decay model. BABAR was able in their recent measurement of γ [61] to reduce the error due to the D^0 decay model to 5°, based on a study of their large sample of D^{*+}-tagged $D^0 \to K^0_S\pi^+\pi^-$, $K^0_S K^+K^-$ decays. This error can ultimately be reduced to about 2° using information from the phase in $\psi(3770) \to D\bar{D}$ decays obtained by CLEO-c [62]. An average of the γ measurements by the UTFit group gives [63]:

$$\gamma = (75.0 \pm 12)^\circ \ .$$

An additional constraint on γ comes from the measurements of time-dependent CP asymmetries in $B^0 \to D^{(*)}\pi$ and $B^0 \to D\rho$ decays giving $\sin(2\beta+\gamma) = (\pm 90 \pm 32)^\circ$ [63].

3.4 β_s

The phase β_s is predicted to be very small in the Standard Model with a value of approximately 0.02 [15, 64], but can be much larger in new physics models. It can be extracted from the time-dependent CP asymmetries in B_s decays to the $J/\psi\phi$ final state. Both CDF and D0 extract simultaneously β_s and $\Delta\Gamma_s$, the width difference between the heavy and light B_s mass eigenstates, from the time and angular-dependent decay time distributions of $B_s \to J/\psi\phi$ [65]. The combined result from CDF and D0 allows a range for β_s between 0.10 and 1.42 at the 95% C.L. and differs from the Standard Model prediction by 2.0σ [65].

4 Global CKM matrix fits

A simple way to test the unitarity of the CKM matrix is to check the sum of the inner angles of the unitarity triangle. It is found to be consistent with 180°:

$$\alpha + \beta + \gamma = (185 \pm 13)^\circ \ .$$

The error of this check is dominated by the experimental uncertainty on γ.

Global tests use all information on the sides and angles of the Unitarity Triangle to determine and overconstrain its apex position. The CKMFitter [54] and UTFit [63] groups use different statistical approaches. While the CKMFitter group uses a frequentist method, the UTFit group employs Bayesian statistics. As a result the uncertainties quoted by the CKMFitter group are often more conservative. The two groups determine the position of the apex of the unitarity triangle to be

$$\begin{array}{llcll} \text{CKMFitter}: & \bar{\rho} = 0.139^{+0.025}_{-0.027}, & & \text{UTFit}: & \bar{\rho} = 0.154 \pm 0.022, \\ & \bar{\eta} = 0.341^{+0.016}_{-0.015}, & & & \bar{\eta} = 0.342 \pm 0.014 \ . \end{array}$$

There is a "tension" at the 2σ level between $\sin(2\beta)$ and ϵ_K (the CP-asymmetry in neutral K decays) and the value of $|V_\mathrm{ub}|$ from inclusive $b \to ul\nu$ decays [67]. It would increase to close to 3σ if an 8% correction is applied to ϵ_K as proposed in [68]. Due to their more conservative treatment of the systematic errors the CKMFitter group obtains a p-value of 45% for their global CKM fit [69]. There is also a 2.4σ tension between the branching fraction $BR(B \to \tau\nu)$ and the result of global Unitarity Triangle fit without information from $B \to \tau\nu$ [66].

As of now there is no significant evidence from global CKM fits that the CKM matrix is not unitary. Comparisons between sets of measurements that might be sensitive to new physics such as quantities derived from tree processes versus quantities derived from loop processes, CP-conserving versus CP-violating processes, etc. do not shown any inconsistency either[54, 63].

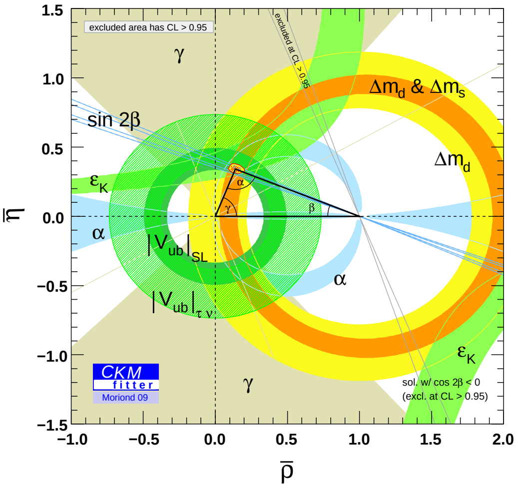

Figure 1: Experimental constraints on the Unitarity Triangle.

5 Conclusions

There have been a wealth of new measurements regarding quark flavor mixing in the last few years that continue to constrain the CKM matrix elements with increasing precision. All current experimental results in quark mixing and CP violation are described by the CKM mechanism, which has proven to be the dominant mechanism for these phenomena. There are some intrinsic discrepancies that need to be resolved (e.g. $V_{\rm us}$ from inclusive versus exclusive tau decays and strange final states and $V_{\rm ub}$ and $V_{\rm cb}$ from inclusive versus exclusive B decays). There are also a few interesting tensions at the 2-3σ level (β versus ϵ_K and $V_{\rm ub}$, β_s and $BR(B \to \tau\nu)$, which should be monitored closely in the future. This will be particulary exciting as with the turn on of new experiments such as LHCb and the Super B factories and improved lattice calculations significant improvements particulary for γ, β_s, $|V_{\rm ub}|$ and $|V_{\rm td}/V_{\rm ts}|$ are expected.

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Discussion

Andrei Golutvin (CERN/ Imperial College, London/ ITEP, Moscow: A discrepancy between Vub determined from exclusive and inclusive measurements limits significantly the accuray of the comparison of beta with the opposite side of the unitarity triangle. Consequently, a sensitivity to a possible contribution from the phases of new particles, if any, to the angle beta is limited.