Status of Neutrino Theory

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A summary of neutrino oscillation results is given along with a discussion of neutrino mass generation mechanisms, including high and low-scale seesaw, with and without supersymmetry, as well as recent attempts to understand flavor. I argue that if the origin of neutrino masses is intrinsically supersymmetric, it may lead to clear tests at the LHC. Finally, I briefly discuss thermal leptogenesis and dark matter.

1 Status of neutrino oscillation experiments

The discovery of neutrino oscillations provides the first evidence of physics beyond the Standard Model (SM), marking the beginning of a new era in particle physics. Thanks to their brilliant confirmation by reactor and accelerator experiments, oscillations constitute the only viable explanation for the observed flavor conversion of "celestial" neutrinos [1–3], requiring both neutrino mass and mixing, as expected in theories without conserved lepton number [4, 5].

Even in its simplest 3×3 unitary form, the lepton mixing matrix $K = \omega_{23}\omega_{13}\omega_{12}$ [5] differs from the quark mixing matrix in that each ω factor carries a physical phase: one is the KM-analogue and appears in oscillations, while the other two are Majorana phases and appear in lepton number (L)-violating processes. Current experiments are insensitive to CP violation, so that oscillations depend only on the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and on the two squared-mass splittings $\Delta m^2_{21} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{31} \equiv m^2_3 - m^2_1$ characterizing solar and atmospheric transitions. Setting $\Delta m^2_{21} = 0$ in the analysis of atmospheric and accelerator data, and Δm^2_{31} to infinity in the solar and reactor data analysis one obtains the neutrino oscillation parameters, as summarized in Figs. 1 and 2. Fig. 1 gives the allowed values of "atmospheric" and "solar" oscillation parameters, θ_{23} & Δm^2_{31}, and θ_{12} & Δm^2_{21}, respectively. The dot, star and diamond in the left panel of Fig. 1 indicate the best fit points of atmospheric, MINOS and global data, respectively. Similarly the "solar" oscillation parameters are obtained by combining solar and reactor neutrino data, as shown in the right panel. The dot, star and diamond indicate the best fit points of solar, KamLAND and global data, respectively. In both cases minimization is carried out with respect to the undisplayed parameters. One sees that data from artificial and natural neutrino sources are clearly complementary: reactor and accelerators give the best determination of squared-mass-splittings, while solar and atmospheric data mainly determine mixings. The right panel in Fig. 2 shows how data slightly prefer a nonzero θ_{13} value, though currently not significant, leading to an upper bound at 90%C.L. (3σ):

$$\sin^2\theta_{13} \leq \begin{cases} 0.060\ (0.089) & \text{(solar+KamLAND)} \\ 0.027\ (0.058) & \text{(CHOOZ+atm+K2K+MINOS)} \\ 0.035\ (0.056) & \text{(global data)} \end{cases} \tag{1}$$

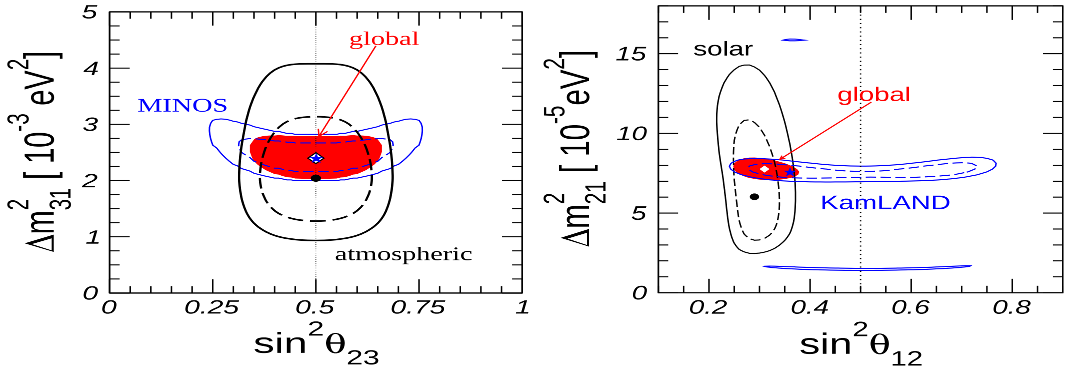

Figure 1: Current neutrino oscillation parameters, from a global analysis of the world's data [3].

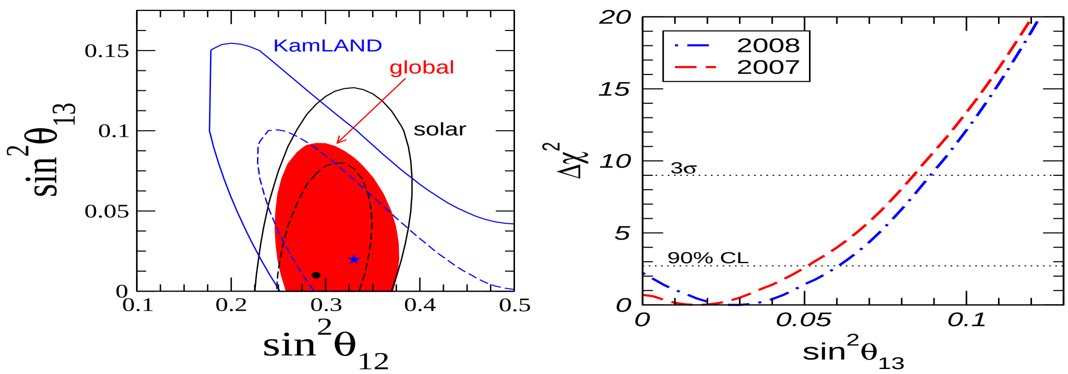

Figure 2: Constraints on $\sin^2\theta_{13}$ from different neutrino oscillation data sets [3].

given for 1 dof, while the regions in Fig. 2 (left) correspond to 90% CL for 2 dof. The confirmation of a non-zero θ_{13} would strongly encourage the search for CP violation in upcoming neutrino oscillation experiments [6, 7]. Note that the small parameter $\alpha \equiv \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|}$ is currently well-determined experimentally as $\alpha = 0.032\,, \quad 0.027 \leq \alpha \leq 0.038 \quad (3\sigma)\,.$

Before closing let us note that many effects may distort the "celestial" neutrino fluxes reaching our detectors, such as regular [8–10] and random [11, 12] solar magnetic fields. These would induce spin-flavor precession in the convective zone [13–15] as well as density fluctuations deep inside the Sun's radiative zone [16–18]. Although these can modify the solar neutrino survival probabilities [19, 20], they can not have an important impact on the determination of oscillation parameters, thanks to the KamLAND reactor neutrino spectrum data. The result is that oscillations remain robust against astrophysical uncertainties, and of all oscillation solutions allowed by solar data [21], only the large mixing angle solution survives KamLAND's measurements [22].

Often the generation of neutrino mass in gauge theories (left panel in Fig. 4) is accompanied by effective sub-weak strength ($\sim \varepsilon G_F$) flavour-changing (FC) or non-universal (NU) dimension-6 operators, as seen in the right panel. Such non-standard neutrino interactions (NSI) are expected in low-scale seesaw schemes, such as the inverse [23–25] and the linear [26] seesaw. In such schemes NSI would arise from the effectively non-unitary form of the corresponding lepton mixing matrix [5] [65]. Relatively sizeable NSI strengths may also be induced in models with radiatively induced neutrino masses [66,67]. Current determination of solar neutrino parameters is not yet fully robust against the presence of large NSI, allowing for a new "dark side" solution that survives the inclusion of reactor data [27].

In contrast, thanks to the large statistics of atmospheric data over a wide energy range, the determination of atmospheric parameters Δm^2_{31} and $\sin^2\theta_{23}$ is fairly robust even in the presence of NSI, at least within the 2–neutrino approximation [28], a situation likely to improve with future neutrino factories [29]. However, NSI operators may have dramatic consequences for the sensitivity to θ_{13} at a neutrino factory [30] and may affect the interpretation of future supernova neutrino data in an important way [31–33]. Improved NSI tests will also shed light on the origin of neutrino mass, helping discriminate between high and low-scale schemes.

2 Neutrino mass and neutrinoless double beta decay

Neutrino oscillations can not probe absolute neutrino masses, for this we need cosmic microwave background and large scale structure observations [34], high sensitivity beta decay and $0\nu\beta\beta$ (neutrinoless double beta decay) studies [35]. The observation of neutrino oscillations suggests that light Majorana neutrino exchange will induce $0\nu\beta\beta$ as illustrated in the left panel of Fig. 3. This nuclear process would hold the key to probe the nature (Dirac versus Majorana) of neutrinos [36] since, in a gauge theory, it would imply a Majorana mass for at least one neutrino [36], as illustrated in the middle panel of Fig. 3.

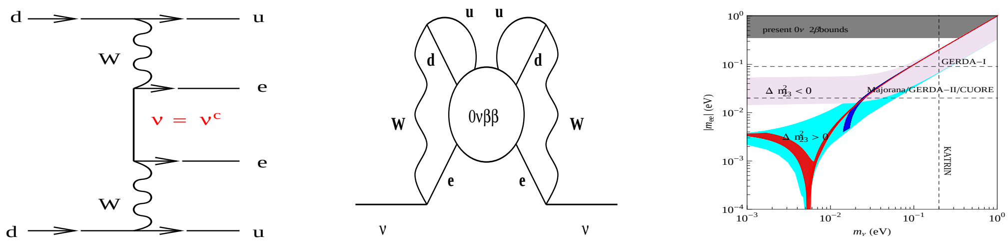

Figure 3: Neutrino mass mechanism (left), black box theorem (center) [36] and $0\nu\beta\beta$ decay parameter versus lightest neutrino mass for inverse and linear seesaw models (right), from [37].

Such "black-box" theorem [36] holds in any "natural" gauge theory, though its implications are rather difficult to quantify in general [38]. The $0\nu\beta\beta$ detection prospects were discussed in [39] and are summarized in the right panel in Fig. 3. The broad bands are allowed by current neutrino oscillation data [2, 3] for normal and inverse neutrino mass hierarchy, while the narrow bands correspond to the case of lepton mixing angles fixed to the tri-bimaximal values [37]. Note that for normal hierarchy there is in general no lower bound on $0\nu\beta\beta$ as there can be a destructive interference among the three neutrinos [1]. In contrast, the inverted neutrino mass hierarchy always gives a generic "lower" bound for the $0\nu\beta\beta$ amplitude. On the other hand, quasi-degenerate neutrino models [43, 44] give the largest possible $0\nu\beta\beta$ signal. Taking into account state-of-the-art nuclear matrix elements [45] one can determine the best current limit, which comes from the Heidelberg-Moscow experiment, as well as future experimental sensitivities [39], summarized in the right panel in Fig. 3 for GERDA, Majorana and CUORE.

[1] Specific flavor models may, however, lead to a lower bound on the $0\nu\beta\beta$ decay rate even for normal hierarchy neutrino spectra, as discussed in Refs. [37] [40–42].

3 Generating neutrino masses

Underpinning the origin of neutrino mass remains a challenge despite the tremendous progress we have achieved. Neutrino masses are markedly different from those of charged fermions in the SM. The latter arise by coupling the two chiral species to the Higgs scalar, hence linear in the electroweak symmetry breaking vacuum expectation value (vev) $\langle\Phi\rangle$ of the Higgs scalar doublet $\Phi \equiv H$. By contrast, being electrically neutral, neutrinos may get mass with just one chiral species: in other words, on general grounds they are of Majorana type [5]. The lowest-dimensional lepton number violating (LNV) operator has $\Delta L = 2$, namely $\lambda L\Phi L\Phi$, where L denotes a lepton doublet [4], see left panel in Fig. 4 [2].

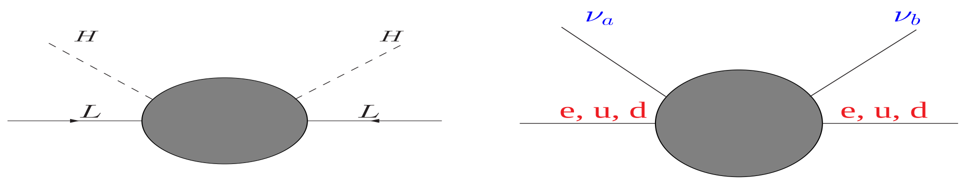

Figure 4: Neutrino mass [4] and non-standard neutrino interactions (NSI) operators [5].

Irrespective of their specific origin, the smallness of neutrino masses would come from the fact that they violate lepton number. The big issue is to identify which *mechanism* gives rise to the L-violating operator, its associated mass *scale* and its *flavor structure*. As for its magnitude, it may naturally be suppressed either by a high-scale M_X in its denominator, or may involve a low-mass-scale in its numerator.

It is often argued that gravity breaks global symmetries [48, 49]. This would induce the L-violating operators suppressed by the Planck scale. The resulting Majorana neutrino masses are too small, hence the need for physics beyond the SM [50]. It is usual to assume that this physics lies at a large sub-Planck scale, say, associated to unification. However, λ may be suppressed by *small* scales, Yukawas and/or loop-factors [51]. There are three classes of mechanisms: (i) tree level, (ii) radiative, and (iii) hybrid, all of which may have high- or low-scale realizations, suggesting a fair chance that the origin of neutrino mass may be probed at accelerator experiments like the LHC. Depending on the nature of spontaneous lepton-number breaking there may be an extra neutral gauge boson [26, 52] or a Nambu-Goldstone boson coupled to neutrinos [53].

3.1 Minimal high-scale seesaw

Weinberg's dimension-5 operator [4] may arise from the exchange of heavy fermion states with masses close to the "unification" scale. Depending on whether these are $SU(2)$ singlets or triplets the mechanism is called *type-I* [54–57], or *type-III* seesaw [58], respectively. Neutrino masses may also arise from the exchange of heavy triplet scalars, now called *type-II* seesaw [5] [53, 59], as seen in the right panel in Fig. 5. The hierarchy of vevs required to account for the small neutrino masses $v_3 \ll v_2 \ll v_1$ is consistent with the minimization of the scalar potential. The resulting perturbative diagonalization of the seesaw mass matrix was given in Ref. [53] in the most general form that may be used in any model. From a phenomenological viewpoint, however, the most basic effective description of *any* seesaw is in terms of the SM $SU(3) \otimes SU(2) \otimes U(1)$ gauge structure with explicit L-violation [5].

[2]Note that neutrino masses may also arise from higher dimension effective operators [46, 47].

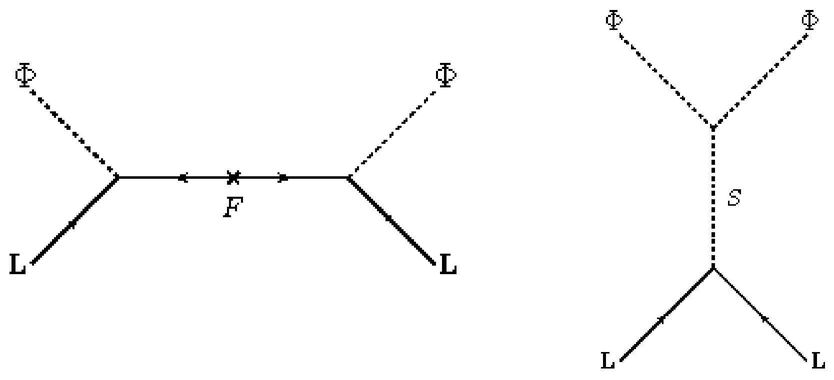

Figure 5: Type-I and III (left) and Type-II (right) realizations of the seesaw mechanism.

3.2 "Non-minimal" seesaw

The seesaw mechanism is not any particular *theory* but rather represents a broad *language* in terms of which to phrase neutrino mass model-building and many are its pathways. Indeed, it can be implemented non-minimally with lepton number broken explicitly or spontaneously, over a wide range of energy scales, in a variety of models with different gauge groups and multiplet contents, with or without supersymmetry. There is no point to attempt giving here a full taxonomy of seesaw schemes. However one should stress that any model must ultimately reduce to the SM. Hence what is phenomenologically most relevant, for example to describe neutrino oscillation data, is the effective structure of the SM seesaw mixing matrix, given in [5]. In addition to the mixing angles characterizing oscillations, the latter includes non-unitarity effects which will become an important topic in the agenda of future studies probing neutrino propagation beyond oscillations.

An attractive class of non-minimal seesaw schemes employs, in addition to the left-handed SM neutrinos ν_L, two $SU(3) \otimes SU(2) \otimes U(1)$ singlets ν^c, S [23] (see also e.g. [61–64]). The basic parameter characterizing the violation of lepton-number can be small [25,37] and may be calculable from supersymmetric renormalization group evolution effects [24].

3.3 Radiative neutrino masses

Neutrino masses may be radiatively calculable [66,67], with no need for a large scale. In this case the suppression comes from small loop-factors and Yukawa couplings. The same way as low-scale seesaw schemes, the states responsible for generating neutrino masses in radiative models may lie at the weak-scale, opening the door to phenomenology at the LHC [60].

4 Understanding and probing flavor

There is no reasonable doubt that flavor is violated in neutrino propagation [2,3]. Current oscillation experiments indicate solar and atmospheric mixing angles which are unexpectedly large when compared to quark mixing angles. To a good approximation they are given by [68],

$$\tan^2\theta_{23} = \tan^2\theta^0_{23} = 1 \qquad \sin^2\theta_{\text{Chooz}} \quad = \sin^2\theta^0_{13} = 0 \qquad \tan^2\theta_{\text{SOL}} = \tan^2\theta^0_{12} = 0.5. \tag{2}$$

Understanding the pattern of lepton mixing angles from first principles constitutes a big challenge to unified theories of flavor where quarks and leptons are related. Many less ambitious schemes have been suggested in order to reproduce the tri-bi-maximal pattern, at least partially, using various discrete flavor symmetry groups containing mu-tau symmetry, e. g. [43, 44, 69–76]. In general one expects the flavor symmetry to be valid at high energy scales. Deviations from tri-bi-maximal ansatz [77] may be calculable by renormalization group evolution [78–80]. A simple possibility is that, as a result of a given flavor symmetry such as A4 [43, 44], neutrino masses unify at high energies [81], the same way as gauge couplings do, due to supersymmetry. Such quasi-degenerate neutrino scheme predicts maximal atmospheric angle and vanishing θ_{13}, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$, leaving the solar angle θ_{12} unpredicted, but Cabibbo-unsuppressed, $\theta_{12} = \mathcal{O}(1)$. If CP is violated θ_{13} becomes arbitrary and the Dirac phase is maximal [70]. The lower bound on the absolute Majorana neutrino mass scale $m_0 \gtrsim 0.3$ eV ensures that the model will be probed by future cosmological data and $0\nu\beta\beta$ searches.

It is natural to expect that, at some level, lepton flavor violation will also show up as transitions involving the charged leptons, since these sit in the same electroweak doublets as neutrinos. Rates for lepton flavour violating processes $l_j \to l_i + \gamma$ often lie in the range of sensitivity of coming experiments, providing an independent test. There are two basic mechanisms: (i) neutral heavy lepton exchange [82–84] and (ii) supersymmetry [85–87]. Both exist in supersymmetric seesaw-type schemes of neutrino mass, the interplay of both depends on the seesaw scale, and has been considered in [88]. Barring fine-tunings, high-scale seesaw models require supersymmetry in order to have sizeable LFV rates. The most interesting feature of these models is that they bring in the possibility of direct lepton flavour violation in the production of supersymmetric particles. As seen in Fig. 6 this provides the most direct way to probe LFV at the LHC in high-scale seesaw models.

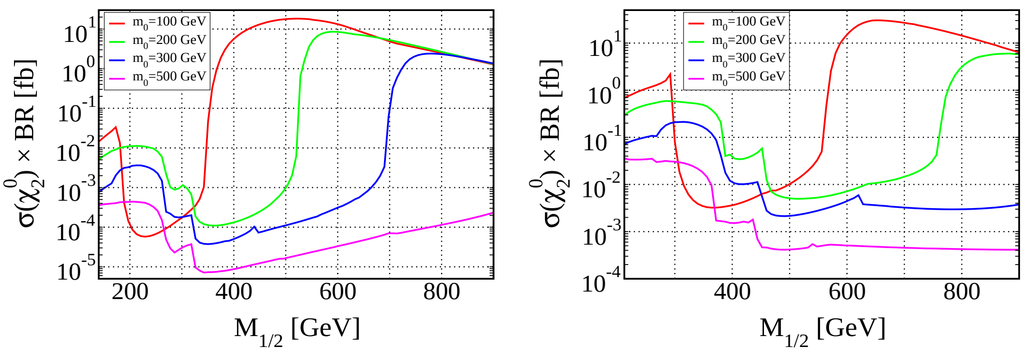

Figure 6: LFV rate for μ-τ lepton pair production from χ^0_2 decays versus $M_{1/2}$ for the indicated m_0 values, assuming minimal supergravity parameters: $\mu > 0$, $\tan\beta = 10$ and $A_0 = 0$ GeV, for type-I (left) and for type-II seesaw (right). Here $\lambda_1 = 0.02$ and $\lambda_2 = 0.5$ are Type-II seesaw parameters, and we imposed the contraint $\text{Br}(\mu \to e + \gamma) \leq 1.2 \cdot 10^{-11}$, from Ref. [89].

In low-scale seesaw schemes, by contrast, the sizeable admixture of "right-handed" (RH) neutrinos in the charged current (rectangular nature of the lepton mixing matrix [5]) induces

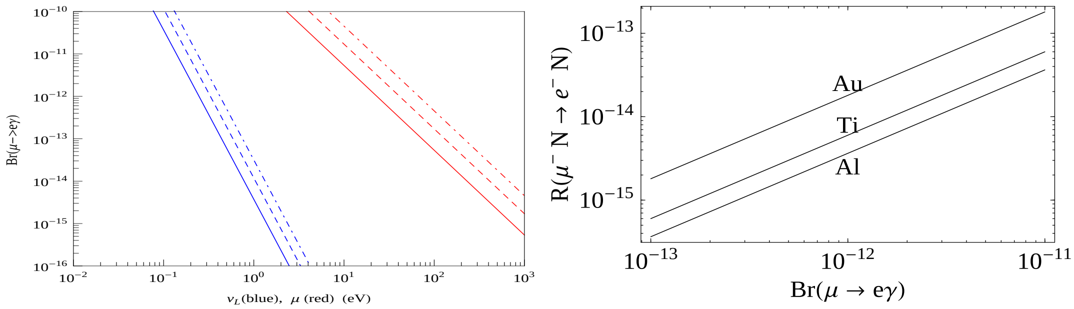

Figure 7: Left: $Br(\mu \to e\gamma)$ versus the LNV scale for inverse seesaw (top: red color) and linear seesaw (bottom, blue color). In both low-scale seesaw models we fix $M = 100\,GeV$ (continuous line), $M = 200\,GeV$ (dashed line) and $M = 1000\,GeV$ (dot-dashed line), from [37]. The right panel shows typical correlation between mu-e conversion in nuclei and $Br(\mu \to e\gamma)$, from [90].

potentially large LFV rates even in the absence of supersymmetry [82]. Indeed, an important point to stress is that LFV [82,83] and CP violation [91,92] can occur in the massless neutrino limit, hence their attainable magnitude is unrestricted by the smallness of neutrino masses. In Fig. 7 we display $Br(\mu \to e\gamma)$ versus the small LNV parameters μ and v_L for two different low-scale seesaw models, the inverse and the linear seesaw, respectively. Clearly the LFV rates are sizeable in both cases, the different slopes with respect to μ and v_L follow from the fact that $\Delta L = 2$ in the first case and $\Delta L = 1$, in the second.

Similarly [90] in low-scale seesaw models the nuclear $\mu^- - e^-$ conversion rates lie within planned sensitivities of future experiments such as PRISM [93]. Note that models with specific flavor symmetries, such as those in [25, 37] relate different LFV rates. To conclude we mention that some seesaw schemes, like type-III with tiny Yukawas [58] or inverse type-III [25], may be directly probed at the LHC by directly producing the TeV states with gauge strength [60].

5 Probing neutrino properties at the LHC

In supersymmetric models lepton number can be broken together with the so-called R parity, leading to an intrinsically supersymmetric origin for neutrino masses [94–96]. This may happen spontaneously, driven by a nonzero vev of an $SU(3) \otimes SU(2) \otimes U(1)$ singlet sneutrino [97–99], leading to an effective model with bilinear violation of R parity [100, 101]. The latter provides the minimal way to break R parity and add neutrino masses to the MSSM [101]. One finds that, typically, the atmospheric scale is generated at tree level by neutralino-exchange *weak-scale seesaw*, while the solar scale is radiatively induced [102]. Unprotected by any symmetry, the lightest supersymmetric particle (LSP) decays. Given the masses indicated by neutrino experiments these decays will happen inside typical detectors [102–104] but with a decay path that can be experimentally resolved, leading to a so-called displaced vertex [105], see left panel in Fig. 8. More strikingly, LSP decay properties correlate with the neutrino mixing angles. Indeed, as seen in the right panel in Fig. 8 the LSP decay pattern is predicted by the low-energy measurement of the atmospheric angle [103, 106, 107], allowing for a clear test at the LHC [60], namely a high-energy redetermination of θ_{23}. Similar correlations hold in variant models based on alternative supersymmetry breaking mechanisms, where other states appear as LSP [108].

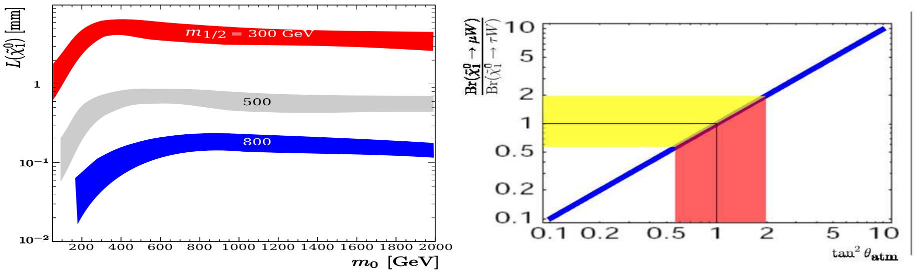

Figure 8: $\tilde\chi^0_1$ decay length versus m_0 for $A_0 = -100$ GeV, $\tan\beta = 10$, $\mu > 0$, and various $m_{1/2}$ values. The three shaded bands around $m_{1/2} = 300,\ 500,\ 800$ GeV correspond to the variation of the BRpV parameters in such a way that the neutrino masses and mixing angles fit the required values within 3σ. The right panel gives the ratio of branching ratios, $\mathrm{Br}(\chi^0_1 \to \mu q'\bar q)$ over $\mathrm{Br}(\chi^0_1 \to \tau q'\bar q)$ in terms of the atmospheric angle in bilinear R parity violation [103].

6 Neutrinos as cosmological probes

Neutrinos can probe very early epochs in the evolution of the Universe, previous to the electroweak phase transition. For example, the high-scale generation of neutrino masses through the seesaw mechanism may induce the observed baryon asymmetry of the Universe, as well as the dark matter, as I now discuss.

6.1 Thermal leptogenesis

The observed cosmological matter-antimatter asymmetry in the Universe may arise from the C/CP-violating out-of-equilibrium decays of the heavy RH neutrinos present in the seesaw. These take place before the electroweak phase transition [109] through the diagrams in the left panel in Fig. 9. The lepton asymmetry thus produced gets converted, through sphaleron processes, into a baryon asymmetry. This so-called leptogenesis scenario [110, 111] is a frame-

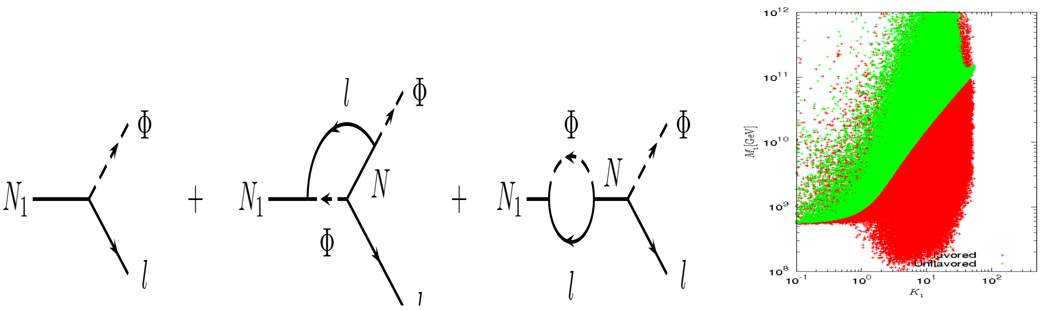

Figure 9: Diagrams (left), flavor effects on minimum required leptogenesis scale (right) [113].

work to explain just one number, namely the baryon asymmetry, currently well-determined by

WMAP [112]. It turns out that, as displayed in the right panel in Fig. 9, from Ref. [113], after taking into account carefully washout and flavor effects one finds that the required asymmetry can be achieved for a given range of model parameters which fits neutrino masses indicated by neutrino oscillation experiments. It did not have to be so, *a priori*, hence this may be taken as a success. The figure also shows how the inclusion of flavor effects lowers the minimum value of the lightest RH neutrino mass required for successful leptogenesis. Nevertheless, in order to avoid gravitino overproduction [114], which would destroy the standard Big Bang Nucleosynthesis predictions, one also requires an upper bound on the reheat temperature T_R after inflation, incompatible with Fig. 9 [115]. One way to prevent such *gravitino crisis* is to assume enhancement coming from resonant leptogenesis [116]. Alternatively, there are many ways to go beyond the minimal type-I supersymmetric seesaw [117–119]. For example one may add a small R-parity violating superpotential term $\lambda_i \hat{\nu^c}_i \hat{H}_u \hat{H}_d$, where $\hat{\nu^c}_i$ are RH neutrino supermultiplets [118]. In the presence of this term the produced asymmetry can be enhanced. In extended SO(10) supersymmetric seesaw schemes leptogenesis can occur at relatively low scales, through the decay of a new singlet, as illustrated in the left panel in Fig. 10. This not only avoids the gravitino crisis but also opens the possibility of detecting the new neutral gauge boson at the LHC [119, 120]. The right panel illustrates how a sizeable asymmetry may be achieved just with the leptonic CP violation parameter δ that characterizes neutrino oscillations.

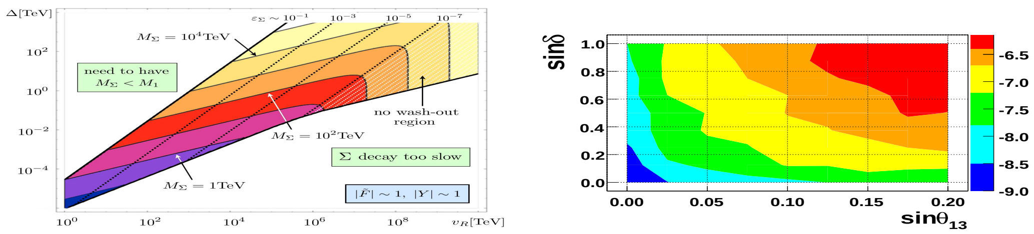

Figure 10: Low-scale leptogenesis in supersymmetric SO(10) models, from [119, 120].

6.2 Neutrino masses and dark matter

Neutrinos may get mass through the spontaneous breaking of ungauged lepton number. Due to quantum gravity effects the associated Goldstone boson - the majoron - is likely to pick up a mass, and play the role of late-decaying Dark Matter, decaying mainly to neutrinos [121, 122]. Cosmic microwave background observations place constraints on the majoron lifetime and mass, illustrated in left and middle panels of Fig. 11. This decaying dark matter scenario arises in type-II seesaw models, where the majoron couples to photons through the Higgs triplet and may be *tested* through the mono-energetic emission line from its sub-dominant decay to two photons, as illustrated in the right panel in Fig. 11.

Neutrino masses may also open new possibilities for "conventional" supersymmetric dark matter. For example, within the inverse seesaw mechanism minimal supergravity is more likely to have a *sneutrino* as the lightest superparticle than the conventional neutralino. Such schemes naturally reconcile the small neutrino masses with the correct relic sneutrino dark matter abundance and accessible direct detection rates in nuclear recoil experiments [125].

Acknowledgments

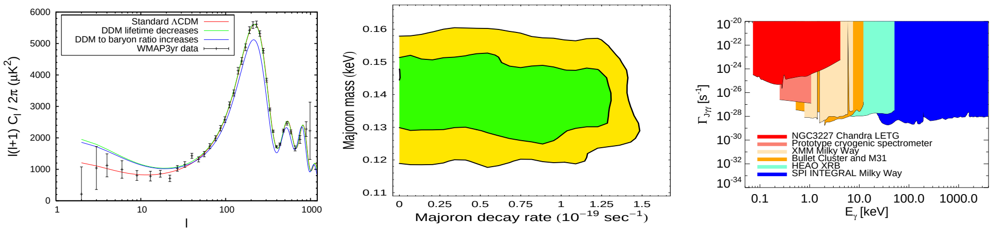

Figure 11: Late decaying majoron dark matter: decay parameters allowd by the CMB [123] (left-middle); probing sub-leading decay to two photons $J \to \gamma\gamma$ (right), from [124].

Work supported by Spanish grants FPA2008-00319/FPA, MULTIDARK Consolider CSD2009-00064 and PROMETEO/2009/091, and by European network UNILHC, PITN-GA-2009-237920.

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Discussion

Bennie Ward (Baylor University):

Could you comment on the degree of fine tuning in the prediction of θ_{ij} and Δm_{ij}^2 from high scale SUSY scenarios?

Answer: While supersymmetric seesaw schemes in unified gauge theories provide a potentially ideal framework to describe flavor, consistent predictions for neutrino properties as well as quark mass and mixing parameters are very hard to obtain in this context. We are still far from a unified description of flavor, hence we often play with flavor symmetries at lower scales.

Toru Iijima (Nagoya University):

You discussed about correlation to LFV. Can you comment on the relation to LFV in τ decays?

Answer: In many flavor-symmetry-based models one finds that the limits on muon-number violation are so stringent that they do not leave much room for sizable lepton-flavor-violation in tau decays.

Vali Huseynov (Nakhchivan State University):

At the beginning of your presentation you have mentioned about the see-saw mechanism. If I understood you correctly, you discuss the Majorana neutrinos. Does there exist the anormolous moment of a neutrino according to the theory presented by you?

Answer: When we studied the phenomenology of all types of seesaw mechanisms in early in 1980, Schecheter and I noted that neutrinos were expected to be Majorana-type in any natural gauge theory, irrespective of the mechanism that provides the neutrino mass. For this reason we never considered the neutrino anormalous moment.