Quasi-degenerate neutrinos and maximal mixing in hybrid seesaws

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We address the question of generating maximal neutrino mixing in the context of hybrid seesaw mechanisms with at least two sources (or two seesaws) in the neutrino mass matrix. In the case where both sources predict small mixing angles, we show that the total neutrino mixing can become maximal if the neutrinos have a quasi-degenerate pattern.

1 Introduction

The data from neutrino experiments is consistent with the presence of three light active neutrinos with one maximal, one large and one small mixing angle. The seesaw mechanism has been tremendously successful in giving tiny Majorana masses to neutrinos naturally. It also has the ingredients for generating the baryon asymmetry of the universe via leptogenesis. However, it predicts mixing angles which are small if quark-like angles are replicated in the leptonic sector. In this work we exploit the fact that in most models of grand unified theories (GUT), usually more than one seesaw mechanism is at work. It is then possible to obtain maximal mixing even if the individual seesaw mixings are small. Also, we point out that the enhancement of mixing from small to maximal is linked to quasi-degeneracy in the light neutrino mass spectrum.

2 Type I and Type II enhancement

Before we describe the idea of hybrid enhancement [1], let us briefly review the existing mechanisms of enhancement of mixing in the neutrino sector. It was shown [2] that by appropriate choice of flavor structure and the hierarchies in matrices m_D and M_R (stronger hierarchy in M_R compared to m_D), one could generate large mixing in m_ν in the Type I seesaw formula $m_\nu = -m_D M_R^{-1} m_D$. The mixing in individual matrices m_D and M_R was taken to be small (like the quark mixing). This was called the "seesaw effect (Type I enhancement)". This mechanism fails if the flavor structure of the right handed neutrinos is same as that of other fermions.

To overcome this problem, the authors in Ref. [3] exploited the interplay of the two terms in Type I plus II seesaw formula inorder to enhance the mixing in the neutrino mass matrix (even if the small mixing was of same order in the matrices m_D, M_R, m_L). For the case of normal

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hierarchy (NH) and degenerate contribution of two terms ($m_\nu^I = m_\nu^{II}$) in mass formula under suitable conditions (cancellation of dominant $\mathcal{O}(1)$ entries) it was possible to obtain large mixing. This was referred to as the "Type II enhancement". However, for inverted hierarchy (IH), such a procedure would require unnatural cancellations within several independent elements of m_ν. Without additional symmetries, it was impossible to accomodate maximal or zero mixing angles in their framework [3].

3 Hybrid enhancement of neutrino mixing

Clearly the enhancement mechanisms mentioned above worked in certain parameter regions and for NH but had limitations for example, the maximal mixing could not be explained easily. We analyse a general situation where we have two sources[1] of neutrino masses (both containing small mixings and of comparable magnitude) and phenomenologically find conditions under which enhancement of neutrino mixing can occur. The sources of neutrino mass can be independent seesaws like any two of the Type I, Type II or Type III seesaws or two copies of the same seesaw mechanism itself (infact sources other than seesaws can also contribute). We will refer to our proposed mechanism of generating maximal angles as "hybrid enhancement". The general texture analysis shows that this can only happen if the resulting pattern of neutrino masses is quasi-degenerate and this requires that the dominant elements in the submatrices are not cancelled in the total mass matrix (unlike in [3]), which in turn means that we have submatrices having NH and IH forms. Let us illustrate the idea for two and three generation cases below.

3.1 Two generations

Denoting the two terms contributing to the left-handed light neutrino mass matrix as

$$\mathbb{M}_\nu^{(1)} = \begin{pmatrix} m_{ee}^{(1)} & m_{e\mu}^{(1)} \\ m_{e\mu}^{(1)} & m_{\mu\mu}^{(1)} \end{pmatrix}, \qquad \mathbb{M}_\nu^{(2)} = \begin{pmatrix} m_{ee}^{(2)} & m_{e\mu}^{(2)} \\ m_{e\mu}^{(2)} & m_{\mu\mu}^{(2)} \end{pmatrix}, \tag{1}$$

we have the mixing angle θ of $\mathbb{M}_\nu = \mathbb{M}_\nu^{(1)} + \mathbb{M}_\nu^{(2)}$,

$$\tan 2\theta \quad = \quad \tan 2\theta^{(1)} \frac{1}{(1+d)} + \tan 2\theta^{(2)} \frac{d}{(1+d)} \ , \tag{2}$$

where $d = (m_{\mu\mu}^{(2)} - m_{ee}^{(2)})/(m_{\mu\mu}^{(1)} - m_{ee}^{(1)})$ and $\theta^{(1)}$ and $\theta^{(2)}$ are the mixing angles of $\mathbb{M}_\nu^{(1)}$ and $\mathbb{M}_\nu^{(2)}$ respectively. Given that $\theta^{(1)}$ and $\theta^{(2)}$ are small, θ would be maximal when $d = -1$. Assuming at least one of the diagonal entries in each of the matrix $\mathbb{M}_\nu^{(i)}$ is large, we have the following three solutions for $d = -1$:- (A) $m_{\mu\mu}^{(2)} = -m_{\mu\mu}^{(1)}$ (Both $\mathbb{M}_\nu^{(i)}$ NH), (B) $m_{ee}^{(2)} = -m_{ee}^{(1)}$ (Both $\mathbb{M}_\nu^{(i)}$ IH), and (C) $m_{ee}^{(2)} = m_{\mu\mu}^{(1)}$ or $m_{\mu\mu}^{(2)} = m_{ee}^{(1)}$ ($\mathbb{M}_\nu^{(1)}$ NH and $\mathbb{M}_\nu^{(2)}$ IH). For illustration, let us consider the (sub)-case of (C) with $m_{\mu\mu}^{(2)} = m_{ee}^{(1)}$,

$$\mathbb{M}_\nu = \mathbb{M}_\nu^{(1)} + \mathbb{M}_\nu^{(2)} \quad = \quad m_1 \begin{pmatrix} 0 & x \\ x & 1 \end{pmatrix} + m_2 \begin{pmatrix} 1 & -y \\ -y & 0 \end{pmatrix} \ , \tag{3}$$

where x, y are small entries. In the limit of exact degeneracy between m_1 and m_2, the mixing is maximal as is evident if both the m_1 and m_2 have the same CP parity. Thus to convert *one*

[1]The discussion of Ref. [3] is just a subcase of all possible cases.

small mixing angle in two matrices to *one* maximal mixing in the total matrix, we would require a *pair* of (quasi)-degenerate eigenvalues with the same CP parities, ordered oppositely in the sub-matrices. This count would be useful when we extend this *degeneracy induced large mixing* to three generations. The Left-Right Symmetric (LRS) model naturally has the ingredients for hybrid enhancement to work [1, 4].

3.2 Three generations

Extending the idea to three generations with two sources, we can find out the conditions when we can obtain only one maximal while the other two large and small respectively. From our arguments above, it appears that we can generate only one large mixing angle in the case when there are only two sub-matrices, because of the important constraint that the third mixing angle (θ_{13}) must not be large. Given that we can only generate one large mixing from the small mixing using the degenerate conditions, we will have to assume that at least one of the submatrices has intrinsically one maximal/large mixing angle. However, the presence of this mixing should not disturb the smallness of θ_{13} angle in the total mass matrix. In the following, we will consider one of the sub-matrices to have pseudo-Dirac structure and other one to have one large eigenvalue and all the three mixing angles small. This is because the pseudo-Dirac structure not only gives maximal mixing but also has the eigenvalues with opposite CP parities.

$$\mathbb{M}_\nu = m_1 \begin{pmatrix} x^2 & x & y^2 \\ x & 0 & 1 \\ y^2 & 1 & 0 \end{pmatrix} + m_2 \begin{pmatrix} 1 & z & t^3 \\ z & z^3 & t^3 \\ t^3 & t^3 & z^3 \end{pmatrix}, \tag{4}$$

where x, y, z, t are small entries compared to m_1, m_2. There can be other textures such that the first of the matrices has only one large eigenvalue in a NH with maximal mixing and the second one has two large eigenvalues with one maximal mixing and two small mixings with IH [1].

4 Conclusion

In the present work, we have concentrated on the case with two seesaw mechanisms at work which occurs naturally in many examples like LRS models, SO(10) based GUT models *etc*. We have shown that if both these seesaw mechanisms result in mass matrices which only have small mixing in them, then the only pattern of mass eigenvalues which is *naturally* consistent with maximal/large mixing is the quasi-degenerate pattern for the total mass matrix. All the arguments presented in the present work are independent of the details of the sources of neutrino masses. While the present work is purely a phenomenological study, it is known that quasi-degeneracy in the neutrino sector would generally imply some symmetry in the Lagrangian. The details of such a symmetry are model dependent.

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