Strong Scaling Ansatz of Flavor Neutrino Mass Matrix and Normal Mass Hierarchy

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Considering Strong Scaling Ansatz (SSA) which predicts $U_{e3} = 0$ and the inverted mass hierarchy, we discover the possibility to realize the normal mass hierarchy by introducing tiny breakings of SSA. In this case, we can automatically reproduce the small mass squared difference ratio ($\Delta m^2_{\odot}/\Delta m^2_{atm} \ll 1$) instead of the suppressed U_{e3} ($|U_{e3}| \ll 1$).

1 Introduction

We start with the definition of the PMNS matrix U_{PMNS} [1], which gives the transformation of flavor eigenstate of neutrinos $\nu_f (f = e\mathrm{A}\mu\mathrm{A}\tau)$ into mass eigenstate of neutrinos $\nu_i (i = 1\mathrm{A}2\mathrm{A}3)$ as $\nu_f = \sum_{i=1}^{3} \left(U_{PMNS}\right)_{fi} \nu_i$. We employ U_{PMNS} determined to be [2]:

$$U_{PMNS} = \Omega \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12}e^{i\rho} & s_{13}e^{-i\delta} \\ -s_{12}e^{-i\rho}c_{23} - s_{13}e^{i\delta}c_{12}s_{23} & c_{12}c_{23} - s_{13}e^{i\delta}s_{12}e^{i\rho}s_{23} & c_{13}s_{23} \\ s_{12}e^{-i\rho}s_{23} - s_{13}e^{i\delta}c_{12}c_{23} & -c_{12}s_{23} - s_{13}e^{i\delta}s_{12}e^{i\rho}c_{23} & c_{23}c_{13} \end{pmatrix} K, \tag{1}$$

where $K = diag\begin{pmatrix} e^{i\beta_1} & e^{i\beta_2} & e^{i\beta_3} \end{pmatrix}$, $\Omega = diag\begin{pmatrix} 1 & e^{i\gamma} & e^{-i\gamma} \end{pmatrix}$ and θ_{ij} is the $\nu_i - \nu_j$ mixing angle. Assuming the seesaw model [3], we obtain the flavor neutrino mass matrix as

$$M_\nu = -\left\langle v \right\rangle^2 Y_\nu^T M_R^{-1} Y_\nu \tag{2}$$

through the higgs mechanism, where Y_ν is a coupling constant of the higgs interaction and $\langle v \rangle$ is a vacuum expectation value of the higgs boson. U_{PMNS} can be transformed from Eq.(1) into the Particle Data Group (PDG) version [4] by removing additional phases of ρ and γ. It should be noted that observable Dirac CP phase is given by $\delta_{CP} = \delta + \rho$ and Majorana CP phase β_1 is changed into $\beta_1' = \beta_1 - \rho$ in the case of U_{PMNS} of the PDG version. The experimental data [5] shows us atmospheric neutrino mass squared differences $\Delta m^2_{atm} \equiv m_3^2 - \left(m_2^2 + m_1^2\right)/2$, solar neutrino mass squared differences $\Delta m^2_{\odot} \equiv m_2^2 - m_1^2$ and mixing angles as follows:

$$\begin{array}{l} \sin^2\theta_{13} < 0.016^{+0.01}_{-0.01},\ \sin^2\theta_{23} \approx 0.466^{+0.073}_{-0.058},\ \sin^2\theta_{12} \approx 0.312^{+0.018}_{-0.019}, \\ \Delta m^2_{atm} \approx 7.67^{+0.16}_{-0.19} \times 10^{-5},\ \Delta m^2_{\odot} \approx 2.39^{+0.08}_{-0.11} \times 10^{-3}. \end{array} \tag{3}$$

Strong Scaling Ansatz (SSA) [6] requires that all ratios of $M_{f\mu}/M_{f\tau}$ are equal as

$$\mathbf{c} \equiv -\sigma\frac{M_{e\mu}}{M_{e\tau}} = -\sigma\frac{M_{\mu\mu}}{M_{\mu\tau}} = -\sigma\frac{M_{\tau\mu}}{M_{\tau\tau}} \tag{4}$$

where $\sigma \equiv \pm 1$. There is an advantage of SSA that the condition of Eq.(4) is invariant under the renormalization running because its effects can be canceled between numerator and denominator [6]. Under the condition of Eq.(4), M_ν and also $\mathbf{M} \equiv M_\nu^\dagger M_\nu$ are determined as follows:

$$M_\nu^{(+)} = \begin{pmatrix} a & be^{i\chi} & \frac{-\sigma b}{\mathbf{c}}e^{i\chi} \\ be^{i\chi} & d & -\sigma\frac{d}{\mathbf{c}} \\ \frac{-\sigma b}{\mathbf{c}}e^{i\chi} & -\sigma\frac{d}{\mathbf{c}} & \frac{d}{\mathbf{c}^2} \end{pmatrix}, \quad \mathbf{M}^{(+)} = \begin{pmatrix} A & |B|\, e^{i\eta} & \frac{-\sigma|B|}{\mathbf{c}}e^{i\eta} \\ |B|\, e^{-i\eta} & D & \frac{-\sigma D}{\mathbf{c}} \\ \frac{-\sigma|B|}{\mathbf{c}}e^{-i\eta} & \frac{-\sigma D}{\mathbf{c}} & \frac{D}{\mathbf{c}^2} \end{pmatrix}. \quad (5)$$

We, respectively, use χ and η in Eq.(5) to denote phases of $e\mu$ and $e\tau$ elements of $M_\nu^{(+)}$ and $\mathbf{M}^{(+)}$, while $a, b, d, A, D, \mathbf{c}$ are real. We calculate the eigenvectors of $\mathbf{M}^{(+)}$, which does not contain the Majorana CP phases, and obtain

$$|\lambda_+\rangle = \begin{pmatrix} \cos\theta \\ -\sin\theta\frac{\mathbf{c}}{\sqrt{1+\mathbf{c}^2}}e^{-i\eta} \\ \sin\theta\frac{\sigma}{\sqrt{1+\mathbf{c}^2}}e^{-i\eta} \end{pmatrix}, |\lambda_-\rangle = \begin{pmatrix} \sin\theta e^{i\eta} \\ \frac{\mathbf{c}}{\sqrt{\mathbf{c}^2+1}}\cos\theta \\ -\frac{\sigma}{\sqrt{\mathbf{c}^2+1}}\cos\theta \end{pmatrix}, |\lambda\rangle = \frac{1}{\sqrt{1+\mathbf{c}^2}}\begin{pmatrix} 0 \\ \sigma \\ \mathbf{c} \end{pmatrix}, \quad (6)$$

where $\tan\theta = \frac{|B|\sqrt{1+\mathbf{c}^2}}{\mathbf{c}(\lambda_- - A)}$. Corresponding eigenvalues are given by $\lambda_\pm = \frac{1}{2}\left\{\left(D\left(1+1/\mathbf{c}^2\right)+A\right) \pm \omega\right\}$ and $\lambda = 0$, where $\omega = \sqrt{\left\{D\left(1+1/\mathbf{c}^2\right)-A\right\}^2 + 4\,|B|^2\left(1+1/\mathbf{c}^2\right)}$. We obtain $U_{e3} = 0$ if $|\lambda\rangle$ is assigned to the state of ν_3. In this case, we also obtain $\theta = \theta_{12}$, $t_{23} = \sigma/\mathbf{c}$A $\rho = \eta$. This properties of SSA allow us to reproduce experimental data of Eq.(3) whose best fit values indicate the smallness of θ_{13} and the small deviation of θ_{23} from $\sigma\pi/4$. However, we couldn't have realized any hierarchy except for the inverted mass hierarchy [6].

We have discovered the possibility to realize the normal mass hierarchy in the paradigm of SSA when $|\lambda\rangle$ is assigned to the state of ν_2. In this proceedings, we call the former case (C1) and the latter case (C2). In (C2), we obtain $\theta_{12} = 0$ instead of $\theta_{13} = 0$ though it should be improved by introducing tiny breakings of SSA as well as $\theta = \theta_{13}$, $t_{23} = -\sigma\mathbf{c}$ and $\delta = -\eta$. In the next section, we consider a tiny breaking term to be added to M_ν in the case of (C2).

2 Effects of breakings of SSA

Breakings of SSA can be defined by adding mass matrix $M_\nu^{(-)}$ to $M_\nu^{(+)}$ as $M_\nu = M_\nu^{(+)} + M_\nu^{(-)}$ where

$$M_\nu^{(-)} = \varepsilon \begin{pmatrix} 0 & b' & \sigma b'/\mathbf{c} \\ b' & d'+d'' & \sigma d'/\mathbf{c} \\ \sigma b'/\mathbf{c} & \sigma d'/\mathbf{c} & (d'-d'')/\mathbf{c}^2 \end{pmatrix}. \quad (7)$$

The parameter ε denotes the tiny SSA breaking and $\tan\theta_{23}$ abbreviated as t_{23} is written as $t_{23} = -\sigma\mathbf{c}(1-\Delta)/(1+\Delta)$ in (C1) and $t_{23} = \sigma\,(1-\Delta)/\mathbf{c}\,(1+\Delta)$ in (C2), where Δ is also small because it induces to break SSA. We neglect the second order of breaking terms of SSA. Using formulas of [2], we easily compute the masses of neutrinos up to O(ε) as

$$m_1 e^{-2i\beta_1'} \approx \frac{1}{2}e^{2i\rho}a + d_0 - z_1,\; m_2 e^{-2i\beta_2} \approx \frac{1}{2}e^{2i\rho}a + d_0 + z_1,\; m_3 e^{-2i\beta_3} \approx 2\varepsilon d', \quad (8)$$

where $z_1 = \frac{2b_0 e^{i(\rho+\chi)}}{\mathbf{c}\sqrt{2}\sin 2\theta_{12}}$ in (C1),

$$m_1 e^{-2i\beta_1'} \approx \frac{1}{2}e^{2i\rho}a + \varepsilon d' - z_2,\; m_2 e^{-2i\beta_2} \approx \frac{1}{2}e^{2i\rho}a + \varepsilon d' + z_2,\; m_3 e^{-2i\beta_3} \approx 2d_0, \quad (9)$$

where $z_2 = \sqrt{2}e^{i\rho}\frac{\varepsilon b' + (i\gamma + \Delta)e^{i\chi}b_0}{\sin 2\theta_{12}}$ in (C2). As a result, we observe the new possibility of the normal mass hierarchy in (C2) though we need the condition of $|a| \ll 1$, whereas we are only allowed to have the inverted mass hierarchy in (C1) as have been already known.

The formulas enable us to readily understand the dependdence of mixing angles on flavor neutrino masses as

$$\begin{aligned} \tan 2\theta_{12}e^{i\rho} &\approx \frac{2\sqrt{2}b_0\left(a_0e^{i\chi}+2d_0e^{-i\chi}\right)}{\mathbf{c}\left\{4\left(b_0^2+d_0^2\right)-a_0^2\right\}}, \\ \tan 2\theta_{13}e^{-i\delta} &\approx \sigma 2\sqrt{2}\frac{b_0\left(a_0e^{i\chi}+2d_0e^{-i\chi}\right)(\Delta - i\gamma) - \varepsilon\left(a_0b' + b_0d''e^{-i\chi}\right)}{a_0^2+2b_0^2}, \end{aligned} \tag{10}$$

for (C1),

$$\begin{aligned} \tan 2\theta_{12}e^{i\rho} &\approx -2\sqrt{2}\frac{\varepsilon\left(a_0b'+b_0d''e^{-i\chi}\right)+b_0(\Delta+i\gamma)\left(a_0e^{i\chi}+2d_0e^{-i\chi}\right)}{a_0^2+2b_0^2}, \\ \tan 2\theta_{13}e^{-i\delta} &\approx -\sigma 2\sqrt{2}b_0\frac{a_0e^{i\chi}+2d_0e^{-i\chi}}{\mathbf{c}\left(4d_0^2-a_0^2\right)}, \end{aligned} \tag{11}$$

for (C2), where γ is also a small parameter because it obviously breaks SSA. We naturally obtain the smallness of θ_{13} in (C1), while we need $\left|b_0\left(a_0e^{i\chi}+2d_0e^{-i\chi}\right)\right| \ll \left|4d_0^2 - a_0^2\right|$ for (C2). Moreover, Dirac CP phase $\delta_{CP} = \delta + \rho$ and Majorana CP phases β_1', β_2 and β_3 are found to large in both cases. Mass squared differences in (C2) are calculated from Eq.(9) as

$$\begin{aligned} \Delta m_\odot^2 &\approx \frac{2\sqrt{2}}{\sin 2\theta_{12}}\mathrm{Re}\left[\left(e^{-i\rho}a + 2\varepsilon d'e^{i\rho}\right)\left\{\varepsilon b' + (i\gamma+\Delta)\,e^{i\chi}b_0\right\}\right], \\ \Delta m_{atm}^2 &\approx 4d_0^2 - \frac{1}{4}a\left(a + 4\varepsilon d'\cos 2\rho\right). \end{aligned} \tag{12}$$

Therefore, there is another virtue of SSA in (C2) that smallness of the ratio of the mass squared difference $\Delta m_{atm}^2/\Delta m_\odot^2 \ll 1$ is automatically satisfied as suggested by experiment because of the smallness of ε, γ and Δ, which serve as the SSA breaking parameters.

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