Baryonium in Confining Gauge Theory

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We show a new class of embedding solutions of D5 brane, which wraps on S^5 in the $AdS \times S^5$ space-time and contains fundamental strings as U(1) flux to form a baryon vertex. This configuration is regarded as a $D5 - \bar{D}5$ bound state, and we propose this as a baryonium state. We could also show their stability.

1 String Model

Quark confinement is well pictured by colored string confining quarks. The string has "orientation" because, when the string is cut by pair creation of quarks, the sequence of q and $\bar{q}$ is unique. When we define the orientation by the direction toward a confined quark, there should exist in the baryon a "singular point" from which the three strings emerge and where the three colors are neutralized. This point is called " junction" . In 1977 we proposed string junction model[1] (abbreviated as SJM), and investigated the nature of baryon and baryonium shown in Fig.1. The reason why they are so difficult to be observed was attributed to their complex structure, in particular, to the nature of junction.

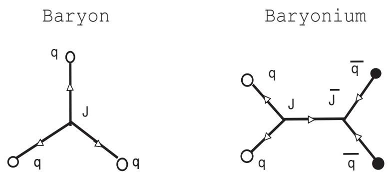

Figure 1: Baryon and baryonium in the string-junction model.

We try to explain the baryon and baryonium states in the framework of AdS/QCD. @They are expressed by D5($\bar{D}$5) brane in a 10D supergravity bavkground which is dual to a confining gauge theory.

2 D5 brane in $AdS_5 \times S^5$ space

We derive baryon and baryonium states from the equations of motion by the action of D5-brane which is embedded in a supersymmetric 10d background of type IIB theory. The background solution should be dual to confining gauge theory and we consider the following background,

$$ds_{10}^2 = e^{\Phi/2}\left(\frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right) \ . \tag{1}$$

The dilaton Φ and the axion χ are given as

$$e^\Phi = 1 + \frac{q}{r^4} \ , \quad \chi = -e^{-\Phi} + \chi_0 \ , \tag{2}$$

and with self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = 4R^4\left(\mathrm{vol}(S^5)d\theta_1 \wedge \ldots \wedge d\theta_5 - \frac{r^3}{R^8}dt \wedge \ldots \wedge dx_3 \wedge dr\right). \tag{3}$$

The D5-brane action is thus written as by the Dirac-Born-Infeld (DBI) plus WZW term

$$\begin{aligned} S_{D5} &= -T_5\int d^6\xi e^{-\Phi}\sqrt{-\det\left(g_{ab} + \tilde{F}_{ab}\right)} + T_5\int d^6\xi \tilde{A}_{(1)} \wedge \mathcal{G}_{(5)} \ , \\ g_{ab} &\equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \ , \qquad \mathcal{G}_{a_1\ldots a_5} \equiv \partial_{a_1}X^{\mu_1}\ldots\partial_{a_5}X^{\mu_5}G_{\mu_1\ldots\mu_5} \ . \end{aligned} \tag{4}$$

where $\tilde{F}_{ab} = 2\pi\alpha' F_{ab}$ and $T_5 = 1/(g_s(2\pi)^5 {l_s}^6)$ is the brane tension. And $\mathcal{G}_{(5)}$ denotes the induced five form field strength.

2.1 Baryon(D5 brane)

The D5 brane is embedded in the world volume $\xi^a = (t, \theta, \theta = 2, \ldots\theta = 5)$. Under the some assumptions, we obtain an energy functional from the above action,

$$U = \frac{N}{3\pi^2\alpha'}\int ds \ e^{\Phi/2}\sqrt{r^2\dot{\theta}^2 + \dot{r}^2 + (r/R)^4\dot{x}^2} \ \sqrt{V_\nu(\theta)}, \tag{5}$$

$$V_\nu(\theta) = D(\nu,\theta)^2 + \sin^8\theta. \tag{6}$$

Then, we obtain the following canonical equations of motion,

$$\dot{r} = p_r \ , \ \ \dot{p}_r = \frac{2}{r^5}p_x^2 R^4 + \frac{p_\theta^2}{r^3} + \frac{1}{2}\left(V_\nu(\theta)\right) \ e^\Phi \partial_r \Phi, \tag{7}$$

$$\dot{\theta} = \frac{p_\theta}{r^2} \ , \ \ \dot{p}_\theta = -6\sin^4\theta\left(\pi\nu - \theta + \sin\theta\cos\theta\right) \ e^\Phi, \tag{8}$$

$$\dot{x} = \frac{R^4}{r^4}p_x, \ \ \dot{p}_x = 0 \tag{9}$$

Baryon is given by the classical solution with the boundary conditions[2],
$r(\theta_c) = r_c, \quad r(\pi) = r_{max}, \quad r(0) = r_0, \quad x(\theta_c) = 0.$

3 Baryonium(D5-anti D5 brane)

Baryonium is obtained by the classical solution by the same equations as baryon with the different boundary conditions,
$\theta'(0) = 0, \quad \theta(\pm x_c) = \pm\pi$. We obtain the baryonium solutions numerically and calculate their energies[3].

3.1 Stability

In ordr to consider the fluctuations, we back to the action of D5 brane (4), and expand it with respect to the fluctuations, $\delta r(t,\theta) = r - \bar{r}, \delta x(t,\theta) = x - \bar{x}$ and $\delta A_t(\theta,t) = A_t - \bar{A}_t$, up to their quadratic terms. Here $\bar{r}$, $\bar{A}_t(\theta)$ and $\bar{x}$ are the solutions of the equations of motion. The modified quadratic term is obtained as

$$\begin{aligned}\tilde{L}_{(2)} &= \tilde{A}_{(2)}\delta r^2 + \frac{B_{(0)}}{2}\Bigg[\left(\frac{R^4}{r^2}+x'^2\right)\left(-\delta\dot{r}^2+\left(\frac{r}{R}\right)^4\frac{1}{Q_{(0)}}\delta r'^2\right)+2x'r'\delta\dot{x}\delta\dot{r} \\ &\quad +\left(r^2+r'^2\right)\left(-\delta\dot{x}^2+\left(\frac{r}{R}\right)^4\frac{1}{Q_{(0)}}\delta x'^2\right)-\left(\frac{r}{R}\right)^4\frac{1}{Q_{(0)}}2x'r'\delta x'\delta r'\Bigg] \\ &\quad +Q_{(1)}r'\delta r'\delta r+Q_{(2)}x'\delta x'\delta r \end{aligned} \tag{10}$$

By assuming the following form for the fluctuations,

$$\delta r(t,\theta) = e^{i\omega t}\phi_r(\theta), \quad \delta x(t,\theta) = e^{i\omega t}\phi_x(\theta)\,, \tag{11}$$

we estimate the value of frequency ω by solving the equations and obtain the stable regions($\omega^2 > 0$) shown in Fig.2[4].

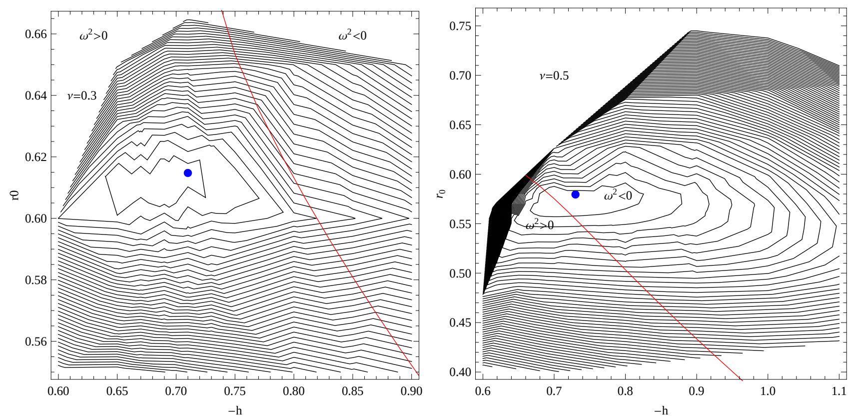

Figure 2: The equi-ω_2^2 curves in $(-h,r_0)$ plane. The equi-U curves in parameter plane. The lines express the border where the sign of ω^2 changes. And a blob denotes $U = U_{\min}$ point. The results for $\nu = 0.3$(left) and $\nu = 0.5$(right) are shown.

References

[1] M. Imachi,S. Otsuki and F. Toyoda, Prog.Theor.Phys.**57**,517 (1977).

[2] K. Ghoroku and M. Ishihara,Phys.Rev.D**77**,086003(2008) arXiv:hep-th/08060195.

[3] K. Ghoroku,M. Ishihara,A. Nakamura and F. Toyoda, JHEP**04**,041(2009) arXiv:hep-th/08091137.

[4] K. Ghoroku,M. Ishihara,A. Nakamura and F. Toyoda, arXiv:hep-th/09071170.