Searching for tetraquarks on the lattice

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We address the question whether the lightest scalar mesons $\sigma$ and $\kappa$ are tetraquarks. We present a search for possible light tetraquark states with $J^{PC} = 0^{++}$ and $I = 0, 1/2, 3/2, 2$ in the dynamical and the quenched lattice simulations using tetraquark interpolators. In all the channels, we unavoidably find lowest scattering states $\pi(k)\pi(-k)$ or $K(k)\pi(-k)$ with back-to-back momentum $k = 0, 2\pi/L, \cdots$. However, we find an additional light state in the $I = 0$ and $I = 1/2$ channels, which may be related to the observed resonances $\sigma$ and $\kappa$ with a strong tetraquark component. In the exotic repulsive channels $I = 2$ and $I = 3/2$, where no resonance is observed, we find no light state in addition to the scattering states.

It is still not known whether the lightest observed nonet of scalar mesons $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ are conventional $\bar{q}q$ states or exotic tetraquark $qqqq$ states. Tetraquark interpretation was proposed by Jaffe back in 1977 [1] and it is supported by many phenomenological studies, for example [2, 3]. The tetraquarks, composed of a scalar diquark and anti-diquark, form a flavor nonet and are expected to be light. The observed ordering $m_\kappa < m_{a_0(980)}$ favors tetraquark interpretation since the $I = 1$ state $[\bar{s}\bar{d}][us]$ with additional valence pair $\bar{s}s$ is naturally heavier than the $I = 1/2$ state $[\bar{s}\bar{d}][du]$.

It is important to determine whether QCD predicts any scalar tetraquark states below 1 GeV from a first principle lattice QCD calculation. Previous lattice simulations [4, 5] have not given the final answer yet. The strongest claim for $\sigma$ as tetraquark was obtained using the sequential Bayes method to extract the spectrum [4] and needs confirmation using a different method. Our new results, given in this proceeding, are presented with more details in [6, 7].

We calculate the energy spectrum of scalar tetraquark states with $I = 0, 2, 1/2, 3/2$ in dynamical and quenched lattice simulations. Our dynamical simulation ($a \approx 0.15$ fm, $V = 16^3 \times 32$) uses dynamical Chirally Improved $u/d$ quarks [8] and it is the first dynamical simulation intended to study tetraquarks. The quenched simulation ($a \approx 0.20$ fm, $V = 16^3 \times 28$) uses overlap fermions, which have exact chiral symmetry even at finite $a$.

The energies of the lowest three physical states are extracted from the correlation functions $C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle_{x=0} = \sum_n Z_{ij}^n e^{-E_n t}$ with tetraquark interpolators $\mathcal{O} \sim \bar{q}qqq$, where $Z_{ij}^n \equiv \langle 0|\mathcal{O}_i|n\rangle$. In all the channels we use three different interpolators that are products of two color-singlet currents [6]. In addition, we use two types of diquark anti-diquark interpolators in $I = 0, 1/2$ channels [6].

When calculating the $I = 0, 1/2$ correlation matrix, we neglect the so-called single and double disconnected quark contractions [5], as in all previous tetraquark studies. The resulting
states have only a $\bar{q}qqq$ Fock component in this approximation, while they would contain also a $\bar{q}q$ component if single disconnected contractions were taken into account [5]. Since we are searching for “pure” tetraquark states in this pioneering study, our approximation is physically motivated.

All physical states $n$ with given $J^{PC} = 0^{++}$ and $I$ propagate between the source and the sink in the correlation functions. Besides possible tetraquark states, there are unavoidable contributions from scattering states $\pi(k)\pi(-k)$ for $I = 0, 2$ and scattering states $\pi(k)K(-k)$ for $I = 1/2, 3/2$. Scattering states have discrete momenta $\vec{k} = \frac{2\pi}{L} \vec{j}$ on the lattice of size $L$ and energy $(m_\pi^2 + \vec{k}^2)^{1/2} + (m_{K}^2 + \vec{k}^2)^{1/2}$ in the non-interacting approximation. Our main question is whether we find some light state in addition to the scattering states in $I = 0, 1/2$ channels. If such a state is found, it could be related to the resonances $\sigma$ or $\kappa$ with a strong tetraquark component.

The energies $E_n$ are extracted from the correlation functions $C_{ij}(t)$ via the eigenvalues $\lambda^n(t) \propto e^{-E_n(t-t_0)}$ of the generalized eigenvalue problem $C(t)\vec{u}^n(t) = \lambda^n(t, t_0)C(t_0)\vec{u}^n$ at some reference time $t_0$ [9].

**Figure 1:** The resulting spectrum $E_n$ for $I = 0, 2, 1/2, 3/2$ in the dynamical (left) and the quenched (right) simulations. Note that there are two states (black and red) close to each other in $I = 0$ and $I = 1/2$ cases. The lines at $I = 0, 2$ present the energies of non-interacting $\pi(k)\pi(-k)$ with $k = j\frac{2\pi}{L}$ and $j = 0, 1, \sqrt{2}$. Similarly, lines at $I = 1/2, 3/2$ present energies of $\pi(k)K(-k)$.
The resulting spectrum $E_n$ for all four isospins is shown in Fig. 1. The lines present the energies of the scattering states in the non-interacting approximation. Our dynamical and quenched results are in qualitative agreement.

In the repulsive channel $I = 2$, where no resonance is expected, we indeed find only the candidates for the scattering states $\pi(0)\pi(0)$ and $\pi(\pm 2\pi)L\pi(\pm 2\pi)$ with no additional light state. The first excited state is higher than expected due to the smallness of $3 \times 3$ basis. Similar conclusion applies for the repulsive $I = 3/2$ channel with $\pi K$ scattering states.

In the attractive channel $I = 0$ we find two (orthogonal) states close to the threshold $2m_\pi$ and another state consistent with $\pi(\pm 2\pi)L\pi(\pm 2\pi)$, so we do find an additional light state. This leads to a possible interpretation that one of the two light states is the scattering state $\pi(0)\pi(0)$ and the other one corresponds to $\sigma$ resonance with strong tetraquark component (see more general discussion in [10]). In the attractive $I = 1/2$ channel we similarly find the candidates for the lowest two $\pi(k)K(-k)$ scattering states and a candidate for a $\kappa$ resonance with a large tetraquark component. These results have to be confirmed by another independent lattice simulation before making firm conclusions.

We investigate two criterion for distinguishing the one-particle (tetraquark) and two-particle (scattering) states in [7]. The first criteria is related to the time dependence of $C_{ij}(t)$ and $\lambda^a(t)$ at finite temporal extent of the lattice. The second is related to the volume dependence of the couplings $\langle 0|O_i|n \rangle$.

The ultimate method to study $\sigma$ and $\kappa$ on the lattice would involve the study of the spectrum and couplings in presence of $\bar{q}qqq \leftrightarrow \bar{q}q \leftrightarrow \text{vac} \leftrightarrow \text{glue}$ mixing, using interpolators that cover these Fock components. Such a study has to be done as a function of lattice size $L$ in order to extract the resonance mass and width using the Lüscher’s finite volume method [10, 11].

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