Divergences in Particle Processes in Intense External Fields

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The study of fundamental interactions in intense external fields is a broad, interesting and incomplete area of study. Intense external fields are present in the charged bunch collisions at particle colliders, being responsible for the beamstrahlung, pair background processes and potentially affecting all collider physics processes. Pair annihilation in the intense fields near the surfaces of magnetars maybe responsible for intense gamma ray bursts. The intense colomb fields present in heavy ion collisions are also known to affect physics processes. In this paper I review the general QED in intense external fields, discuss the divergences present in the beamstrahlung transition rate and their potential mitigation.

1 QED in the Bound Interaction Picture

For Quantum Electrodynamical physics processes that take place in the presence of an external potential A^e the Lagrangian Density is written

$$\mathcal{L}_{QED} = -\tfrac{1}{4}\,F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ + eA + eA^e - m)\psi \tag{1}$$

In the Interaction Picture (IP), the second term in Equation 1 is expanded and the term trilinear in Dirac and Maxwell field operators makes up the interaction Lagrangian $\mathcal{L}_I = e\bar{\psi}(A+A^e)\psi$. However if the external field is intense enough the coupling term arising from the external potential A^e approaches one, rendering the perturbation theory invalid. It is necessary to leave the external potential in the Dirac part of the Lagrangian so that, $\mathcal{L}_D = e\bar{\psi}(i\partial\!\!\!/ + A^e - m)\psi$.

In order to proceed in this Bound Interaction Picture (BIP) it is necessary to solve the Dirac Equation with the inclusion of the external potential. This Bound Dirac equation can be solved exactly for the case of the external potential being a plane wave electromagnetic field. The Bound Dirac Field Operator is a product of the normal free fermion solution with an extra phase and a magnetic moment term,

$$\Psi^V_p(x) = \left(1 + \tfrac{e\not{k}\not{A}^e}{2(kp)}\right)\, e^{iS(x)}\, u(p)$$
$$\text{where} \quad S(x) = -i\int_0^{(k\cdot x)} \left[\tfrac{e(A^e p)}{(kp)} - \tfrac{e^2 A^{e2}}{2(kp)}\right] d\phi$$

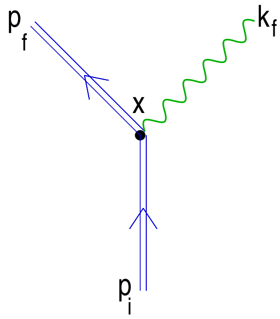

Figure 1: The Beamstrahlung Feynman diagram.

2 Beamstrahlung divergences

Photon radiation from a fermion scattering inelastically from the field of an oncoming charge bunch in a collider - the Beamstrahlung process (Figure 1) is considered in the BIP with Bound Dirac solutions in a constant crossed electromagnetic field. This form for external field leads to Bessel K functions of fractional order (which approach infinity as their argument tends to zero) in the transition probability,

$$W = \frac{\alpha m^2}{\pi\sqrt{3}\epsilon_i}\int_0^{(k\cdot p_i)} \frac{du}{(1+u)^2}\left[\int_{\xi(u)}^{\infty} K_{5/3}(y)dy - \frac{u^2}{1+u}K_{2/3}(\xi(u))\right]$$
$$\text{where} \quad \xi(u) = \frac{2u}{3\nu(k\cdot p_i)}, \quad u = \frac{(k\cdot k_f)}{(k\cdot p_i)(k\cdot p_f)} \tag{2}$$

The Beamstrahlung Transition Rate (Equation 2) reaches infinity at the lower bound of the integration over the variable u which suggests that the beamstrahlung transition rate is divergence whenever the scalar product $(k\cdot k_f)$ is zero. The two apparent conditions for the infinity are when the radiated photon is soft (an IR divergence) or for radiation in the direction of the external field 4-momenta (a collinear divergence). The latter divergence is particularly glaring since it is known that the majority of radiated photons are emiited in a relativistic $1/\gamma$ cone about the initial fermion momentum (Figure 2).[1]

3 Divergence handling in the Bound Interaction Picture

In order to remove the divergence from the Beamstrahlung Transition Rate, it is tempting simply to follow the procedure of Regularization and Cancellation/Renormalization in the IP. However there are subtleties that need to be taken into account.

One such is that the regularization procedure doesnt lead to a cutoff-dependent logarithm, rather to a change in the lower bound of the integration over the parameter u. However this can be achieved using at least one of the usual at least methods - introducing a small photon mass to render $(k\cdot k_f)$ non-zero.

A second difficulty is that the Beamstrahlung process has one vertex with the emitted photon on the mass shell, whereas the usual Bremstrahlung process is second order with a virtual emitted photon. At the level of counting coupling constants, it can be seen that a correction to Bremstahlung can come from

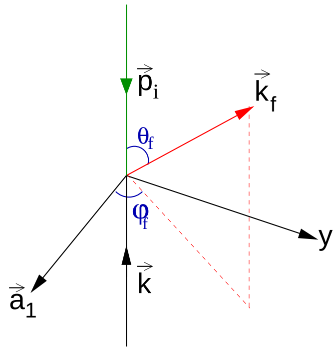

Figure 2: Coordinate system for the beamstrahlung process for a fermion (p_i) approaching an oncoming bunch field (k).

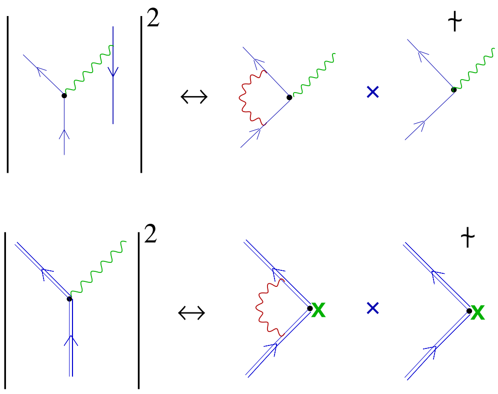

Figure 3: Equivalent Feynman diagrams for divergence cancellation in the IP (upper) and the BIP (lower).

the interference term of the vertex correction and elastic scattering. For the Beamstrahlung process it must be the interference term of the 1st and 3rd order elastic scattering in the BIP (Figure 3).

Work is under way to calculate the 3rd order BIP elastic scattering and to use it to remove the divergence from the Beamstrahlung transition rate. Beyond that it will be necessary to show explicitly that divergences in the BIP cancel to all orders.

4 Bibliography

References

[1] A.F. Hartin J.Phys.Conf.Ser.198 012004 (2009).