Yukawaon model and unified description of quark and lepton mass matrices

Yoshio Koide

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

In the so-called yukawaon model, where effective Yukawa coupling constants Y_f^{eff} $(f = e, \nu, u, d)$ are given by vacuum expectation values of gauge singlet scalars (yukawaons) Y_f with 3×3 flavor components, it is tried to give a unified description of quark and lepton mass matrices. Especially, without assuming any discrete symmetry in the lepton sector, nearly tribimaximal mixing is derived by assumed a simple up-quark mass matrix form.

1 What is a yukawaon model?

First, let us give a short review of the so-called *yukawaon* model: We regard Yukawa coupling constants Y_f as effective coupling constants Y_f^{eff} in an effective theory, and we consider that Y_f^{eff} originate in vacuum expectation values (VEVs) of new gauge singlet scalars Y_f, i.e.

$$Y_f^{eff} = \frac{y_f}{\Lambda}\langle Y_f \rangle, \tag{1}$$

where Λ is a scale of an effective theory which is valid at $\mu \leq \Lambda$, and we assume $\langle Y_f \rangle \sim \Lambda$. We refer the fields Y_f as *yukawaons* [1] hereafter. Note that the effective coupling constants Y_f^{eff} evolve as those in the standard SUSY model below the scale Λ, since a flavor symmetry is completely broken at a high energy scale $\mu \sim \Lambda$.

In the present work, we assume an O(3) flavor symmetry. In order to distinguish each Y_f from others, we assume a $\mathrm{U}(1)_X$ symmetry (i.e. *sector charge*). (The $\mathrm{SU}(2)_L$ doublet fields q, ℓ, H_u and H_d are assigned to sector charges $Q_X = 0$.) Then, we obtain VEV relations as follows: (i) We give an O(3) and $\mathrm{U}(1)_X$ invariant superpotential for yukawaons Y_f. (ii) We solve SUSY vacuum conditions $\partial W / \partial Y_f = 0$. (iii) Then, we obtain VEV relations among Y_f.

For example, in the seesaw-type neutrino mass matrix, $M_\nu \propto \langle Y_\nu \rangle \langle Y_R \rangle^{-1} \langle Y_\nu \rangle^T$, we obtain [2]

$$\langle Y_R \rangle \propto \langle Y_e \rangle \langle \Phi_u \rangle + \langle \Phi_u \rangle \langle Y_e \rangle \tag{2}$$

together with $\langle Y_\nu \rangle \propto \langle Y_e \rangle$ and $\langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle$, i.e. a neutrino mass matrix is given by

$$\langle M_\nu \rangle_e \propto \langle Y_e \rangle_e \left\{ \langle Y_e \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle Y_e \rangle_e \right\}^{-1} \langle Y_e \rangle_e, \tag{3}$$

where $\langle \Phi_u \rangle_u \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$, and $\langle A \rangle_f$ denotes a form of a VEV matrix $\langle A \rangle$ in the diagonal basis of $\langle Y_f \rangle$ (we refer it as f basis). We can obtain a form $\langle \Phi_u \rangle_d = V(\delta)^T \langle \Phi_u \rangle_u V(\delta)$ from the definition of the CKM matrix $V(\delta)$, but we do not know an explicit form of $\langle \Phi_u \rangle_e$. Therefore, in a previous work [2], the author put an ansatz, $\langle \Phi_u \rangle_e = V(\pi)^T \langle \Phi_u \rangle_u V(\pi)$ by supposing $\langle\Phi_u\rangle_e \simeq \langle\Phi_u\rangle_d$, and he obtained excellent predictions of the neutrino oscillation parameters without assuming any discrete symmetry. However, there is no theoretical ground for the ansatz for the form $\langle\Phi_u\rangle_e$.

The purpose of the present work is to investigate a quark mass matrix model in order to predict neutrino mixing parameters on the basis of a yukawaon model (2), without such the ad hoc ansatz, because if we give a quark mass matrix model where mass matrices (M_u, M_d) are given on the e basis, then, we can obtain the form $\langle\Phi_u\rangle_e$ by using a transformation $\langle\Phi_u\rangle_e = U_u\langle\Phi_u\rangle_u U_u^T$, where U_u is defined by $U_u^T M_u U_u = M_u^{diag}$.

2 Yukawaons in the quark sector

We assume a superpotential in the quark sector [3]:

$$W_q = \mu_u[Y_u\Theta_u] + \lambda_u[\Phi_u\Phi_u\Theta_u] + \mu_u^X[\Phi_u\Theta_u^X] + \mu_d^X[Y_d\Theta_d^X] + \sum_{q=u,d}\frac{\xi_q}{\Lambda}[\Phi_e(\Phi_X + a_q E)\Phi_e\Theta_q^X]. \quad (4)$$

Here and hereafter, for convenience, we denotes Tr[...] as [....] simply. From SUSY vacuum conditions $\partial W/\partial\Theta_u = 0$, $\partial W/\partial\Theta_u^X = 0$ and $\partial W/\partial\Theta_d^X = 0$, we obtain $\langle Y_u\rangle \propto \langle\Phi_u\rangle\langle\Phi_u\rangle$,

$$M_u^{1/2} \propto \langle\Phi_u\rangle_e \propto \langle\Phi_e\rangle_e\left(\langle\Phi_X\rangle_e + a_u\langle E\rangle_e\right)\langle\Phi_e\rangle_e, \quad (5)$$

$$M_d \propto \langle Y_d\rangle_e \propto \langle\Phi_e\rangle_e\left(\langle\Phi_X\rangle_e + a_d\langle E\rangle_e\right)\langle\Phi_e\rangle_e, \quad (6)$$

respectively. Here, $\langle\Phi_X\rangle_e$ and $\langle E\rangle_e$ are given by

$$\langle\Phi_X\rangle_e \propto X \equiv \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad \langle E\rangle_e \propto \mathbf{1} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

(Note that the VEV form $\langle\Phi_X\rangle_e$ breaks the O(3) flavor symmetry into S_3.) Therefore, we obtain quark mass matrices

$$M_u^{1/2} \propto M_e^{1/2}\left(X + a_u\mathbf{1}\right)M_e^{1/2}, \quad M_d \propto M_e^{1/2}\left(X + a_d e^{i\alpha_d}\mathbf{1}\right)M_e^{1/2}, \quad (8)$$

on the e basis. Note that we have assumed that the O(3) relations are valid only on the e and u bases, so that $\langle Y_e\rangle$ and $\langle Y_u\rangle$ must be real.

A case $a_u \simeq -0.56$ can give a reasonable up-quark mass ratios $\sqrt{m_{u1}/m_{u2}} = 0.043$ and $\sqrt{m_{u2}/m_{u3}} = 0.057$, which are in favor of the observed values [4] $\sqrt{m_u/m_c} = 0.045^{+0.013}_{-0.010}$, and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$ at $\mu = M_Z$.

3 Yukawaons in the neutrino

However, the up-quark mass matrix (5) failed to give reasonable neutrino oscillation parameter values although it can give reasonable up-quark mass ratios. Therefore, we will slightly modify the model (2) in the neutrino sector.

Note that the sign of the eigenvalues of $M_u^{1/2}$ given by Eq.(8) is $(+,-,+)$ for the case $a_u \simeq -0.56$. If we assume that the eigenvalues of $\langle\Phi_u\rangle_u$ must be positive, so that $\langle\Phi_u\rangle_u$ in

Eq.(2) is replaced as $\langle\Phi_u\rangle_u \to \langle\Phi_u\rangle_u \cdot \mathrm{diag}(+1,-1,+1)$, then, we can obtain successful results except for $\tan^2\theta_{solar}$, i.e. predictions $\sin^2 2\theta_{atm} = 0.984$ and $|U_{13}| = 0.0128$ and an unfavorable prediction $\tan^2\theta_{solar} = 0.7033$.

When we introduce a new field P_u with a VEV $\langle P_u\rangle_u \propto \mathrm{diag}(+1,-1,+1)$, we must consider an existence of $P_u Y_e \Phi_u + \Phi_u Y_e P_u$ in addition to $Y_e P_u \Phi_u + \Phi_u P_u Y_e$, because they have the same $\mathrm{U}(1)_X$ charges. Therefore, we modify Eq.(2) into

$$W_R = \mu_R[Y_R\Theta_R] + \frac{\lambda_R}{\Lambda}\{[(Y_e P_u \Phi_u + \Phi_u P_u Y_e)\Theta_R] + \xi[(P_u Y_e \Phi_u + \Phi_u Y_e P_u)\Theta_R]\}, \quad (9)$$

which leads to VEV relation $Y_R \propto Y_e P_u \Phi_u + \Phi_u P_u Y_e + \xi(P_u Y_e \Phi_u + \Phi_u Y_e P_u)$. The results at $a_u \simeq -0.56$ are excellently in favor of the observed neutrino oscillation parameters by taking a small value of $|\xi|$ (see Table 1):

Also, we can calculate the down-quark sector. We have two parameters (a_d, α_d) in the down-quark sector given in Eq.(8). (See Table 2 in Ref.[3]). The results are roughly reasonable, although $|V_{i3}|$ and $|V_{3i}|$ are somewhat larger than the observed values. Those discrepancies will be improved in future version of the model.

4 Summary

In conclusion, for the purpose of deriving the observed nearly tribimaximal neutrino mixing, a possible yukawaon model in the quark sector is investigated. Five observable quantities (2 up-quark mass ratios and 3 neutrino mixing parameters) are excellently fitted by two parameters. Also, the CKM mixing parameters and down-quark mass ratios are given under additional 2 parameters. The results are summarized in Table 1.

Sector	Parameters	Predictions		
		$\sin^2\theta_{atm}$	$\tan^2\theta_{solar}$	$\|U_{13}\|$
M_ν	$\xi = +0.0005$	0.982	0.449	0.012
	$\xi = -0.0012$	0.990	0.441	0.014
$M_u^{1/2}$	$a_u = -0.56$	$\sqrt{\frac{m_u}{m_c}} = 0.0425$	$\sqrt{\frac{m_c}{m_t}} = 0.0570$	
	two parameters	5 observables: fitted excellently		
M_d	$a_d e^{i\alpha_d}$	$\sqrt{\frac{m_d}{m_s}}$, $\sqrt{\frac{m_s}{m_b}}$, $\|V_{us}\|$, $\|V_{cb}\|$, $\|V_{ub}\|$, $\|V_{td}\|$		
	two parameters	6 observables: not always excellent		

Table 1: Summary of the present model.

It is worthwhile to notice that the observed tribimaximal mixing in the neutrino sector is substantially obtained from the up-quark mass matrix structure (8). Although the model for down-quark sector still need an improvement, the present approach will provide a new view to a unified description of the masses and mixings.

References

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