Turbulence and Supernova Neutrinos

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As neutrinos propagate from the neutrinosphere through the mantle of a core-collapse supernova they will pass through regions of turbulence. The turbulence leads to stochastic neutrino flavor mixing thus leaving fingerprints in the Galactic supernova neutrino burst signal. In this talk I explore the effect of turbulence upon the neutrinos focusing upon the case of large amplitudes and demonstrate that the ensemble of S matrices that describe the neutrino evolution in this limit is Dyson's Circular Ensemble.

1 The signal from the next Galactic supernova

The progress in the field of supernova neutrino over the past decade has been impressive with a constant procession of important discoveries. For a review we refere the reader to Duan & Kneller [1]. We have discovered that the neutrino burst from the next supernova in our Galaxy is dynamic with information about both the neutrino and the supernova embedded within it. Decoding that signal will be a formidable challenge because of the many different processes which alter the neutrino spectra during their voyage to Earth: neutrino self interactions over the first 1000 km or so from the neutrinosphere, the effect of matter - the Mikheev-Smirnov-Wolfenstein (MSW) effect - with the added complication of turbulence, de-coherence as the neutrino propagates to Earth, and then Earth matter effects if the SN is 'shadowed' at the detector. The last two effects are well understood and are simple to account for; the first item on this list is a fascinating subject with a rich and evolving phenomenology and we refer the reader to the contributions by Baha Balantekin, Amol Dighe, Alessadro Mirizzi and Raymond Sawyer. The MSW effects too have been well studied and the expected signals of supernova features such as the shockwave have been described. That leaves the effect of turbulence (density fluctuations) which are not yet satisfactorily included in simulations of the expected neutrino signal because, a) we have little idea of what the turbulence in the supernova may look like, and b) we have no prescription for including turbulence in the signal. Having said this, the general effect of turbulence is well-known: turbulence tends to equilibrate the spectra of the different flavors. In the limit of total equilibration the spectra at Earth are a linear combination of the spectra at the neutrinosphere and thus the features in the spectra which are supposed to indicate unknown neutrino properties, neutrino phenomena such as collective effects, and supernova diagnostics such as the shock wave, are removed. A better understanding of turbulence effects upon supernova neutrinos and the implications for observables would be very desirable. This talk summarizes the work from Kneller & Volpe [3] and Kneller [4] and some more recent work which focused upon these problems.

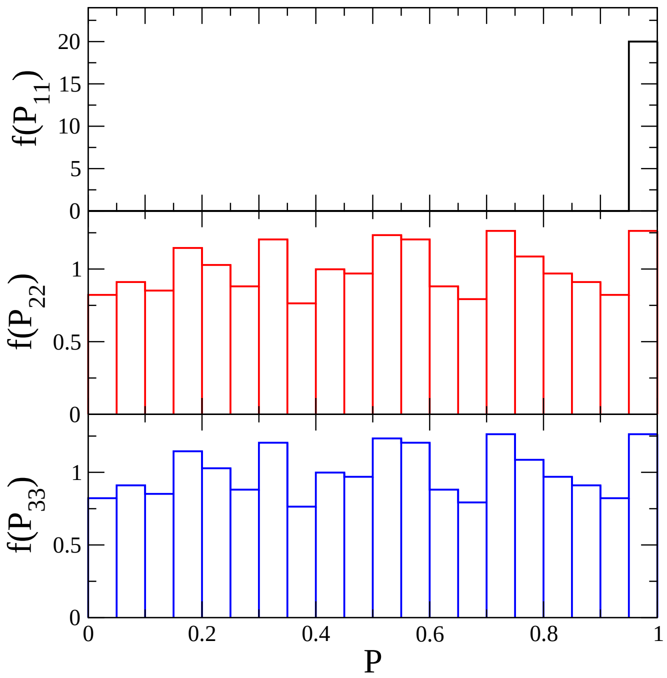

Figure 1: (color online) A normalized frequency histogram of 1012 calculations of P_{11} (top panel) P_{22} (middle panel) and P_{33} (bottom panel) for $E = 25$ MeV neutrinos. The hierarchy is normal, $\sin^2(2\theta_{13}) = 4 \times 10^{-4}$ and $C_\star^2 = 0.01$.

2 Turbulence in supernova

In multi-dimensional hydrodynamical simulations we see turbulence generated by aspherical flows through distorted shocks, convection, etc. For a description of the very interesting results of these simulations we refer the reader to the contributions by Bernhard Mueller, Stephan Bruenn, Thomas Janka and Christian Ott. In order to study the effect of this turbulence upon the neutrinos we obviously first need to have at hand density profiles with turbulence in them. Ideally we would gather such profiles from multi-dimensional hydrodynamical simulations but at present they are all focused upon the inner regions of the star and early times in order to discover the mechanism (or mechanisms) that leads to the explosion. But it is the outer regions, from ~ 1000 km to $\sim 10^7$ km, where the MSW effect occurs and where, several seconds into the signal, the shock wave will generate the turbulence that will most effect the neutrinos. Thus we are forced to take density profiles from one-dimensional simulations which do extend to the outer parts of the star and late times and insert turbulence into them. For this work we choose to model the turbulence as a Gaussian random field $F(r)$ with a rms amplitude $C_\star$ and we shall adopt a Kolmogorov power spectrum. To mimic the turbulence seen in multi-d hydro simulations we restrict the turbulence to the region between the forward and reverse shocks.

$$V_e(r) = (1 + F(r))\langle V_e(r)\rangle \quad (1)$$

The one-dimensional density profile $\langle V_e(r)\rangle$ we use here is the $t = 4.5$ s snapshot of the $Q = 3.36 \times 10^{51}$ erg model taken from Kneller, McLaughlin & Brockman [5].

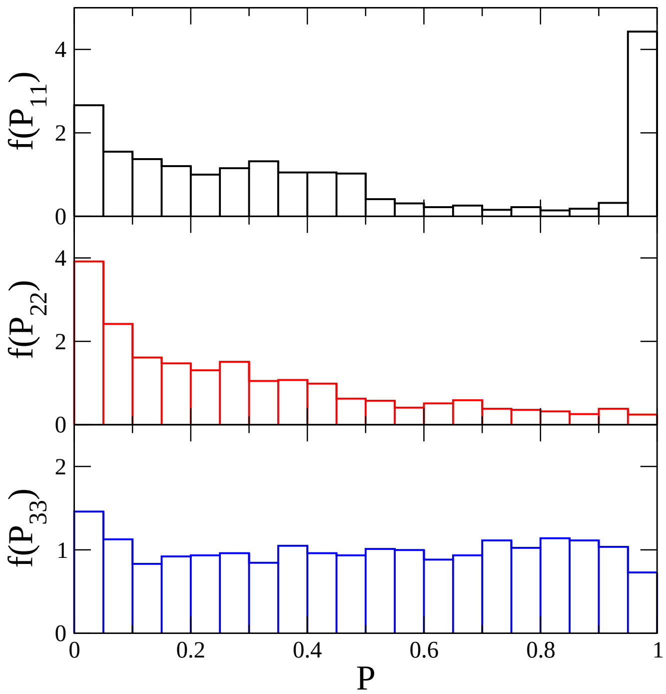

Figure 2: (color online) Normalized frequency distributions of the probabilities P_{11}, P_{22} and P_{33} of 1563 calculations. The hierarchy is normal, $C_\star^2 = 0.3$ and $E = 60$ MeV.

Now that we have our density profiles we have to run the neutrinos through them. The ν state at r is related to the initial state through an operator S and the probability that an initial state j is later detected as state i is given by the square amplitude of the appropriate element of S i.e. $P(\nu_j \rightarrow \nu_i) \equiv P_{ij} = |S_{ij}|^2$. The ideas and methods used to find S are described in Kneller & McLaughlin [2, 6]. For all results in this paper we set the oscillation frequencies and angles to $\delta m_{12}^2 = 8\times10^{-5}\mathrm{eV}^2$, $|\delta m_{23}^2| = 3\times10^{-3}\mathrm{eV}^2$ and by $\sin^2 2\theta_{12} = 0.83$ and $\sin^2 2\theta_{23} = 1$ [7]. The value of the unknown angle θ_{13} will be given when a specific value is used in a calculation. These transition probabilities are not unique: each realization of the random field will give a different set of P_{ij}'s so in order to study the overall effect one needs to generate many many realizations and construct ensemble of S and the P_{ij}'s. From these ensembles one then extracts the transition probabilities as the neutrino exits the turbulence and construct histograms of the results. One such histogram is shown in Figure (1) for the case of relatively small turbulence amplitudes of 10%. The figure indicates that for small amplitudes the final state distributions are quasi two flavor: the transition probability P_{11} is always close to unity while the two probabilities P_{22} and P_{33} are consistent with uniform. But as we increase the amplitude of the turbulence the effect begins to change. For very large amplitudes we begin to see three-flavor effects that occur because we break HL factorization. Breaking HL factorization leads to a change in the distributions of the transition probabilities: the distributions begin to transit to a triangular shape albeit with relic quasi-two flavor features in the example shown in Figure (2).

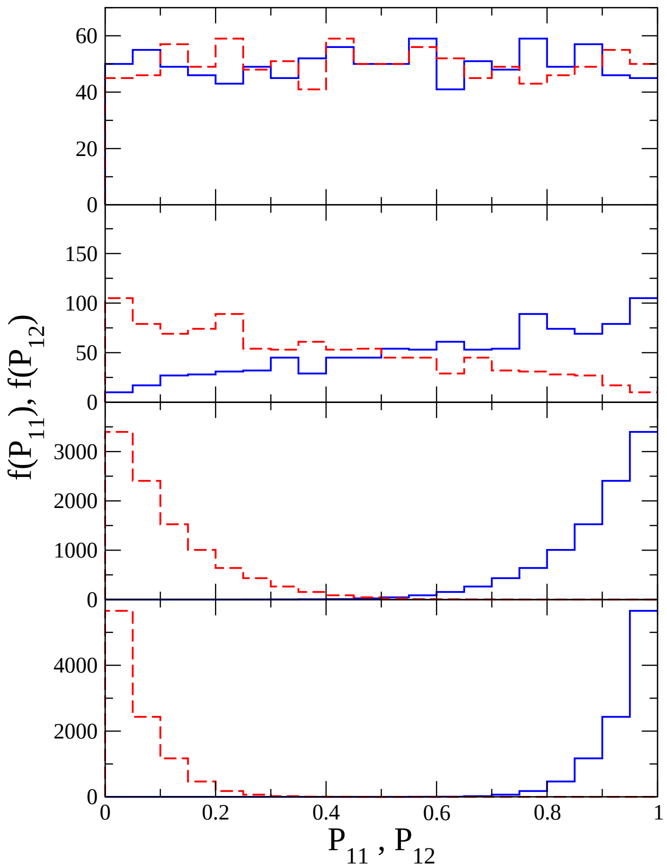

Figure 3: (color online) The distributions of the transition probabilities P_{11} (solid) and P_{12} (dashed) as a function of the number N of products of random 2×2 matrices. From bottom to top the panels are for $N = 1$, $N = 2$, $N = 10$ and $N = 100$.

3 Products of random unitary matrices

The results shown in Figures (1) and (2) correspond to two cases of flavor depolarization: one with 2 flavors and the other with 3. Depolarization means that there is no connection between the initial and final states: all final states are equally likely and when this occurs the ensemble of S matrices one has constructed is a realization of Dyson's Circular Unitary Ensemble CUE(N_f) [8] where N_f is the number of flavors. From this ansatz one can also show analytically [4] that the distribution of the set of probabilities P_{1j}, P_{2j}, ... for observing final states ν_1, ν_2 etc. is uniform over a standard N_f-1 simplex and after integrating over N_f-1 elements of the set one finds that the distribution f of a particular probability P_{ij} is $f(P_{ij}) = (N_f - 1)\,(1 - P_{ij})^{N_f - 2}$. The Circular Ensemble is also relevant to the distribution of the product of N random non-circular matrices in the limit where $N \to \infty$ which has a natural association with S matrices because S-matrices can be factored. So we can think about breaking the integration domain into N subdomains each with one MSW resonance. Each

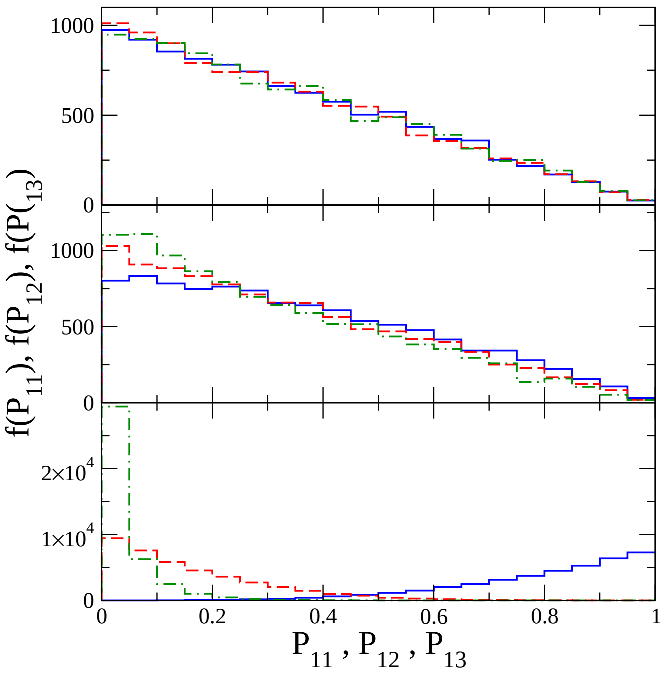

Figure 4: (color online) The distributions of the transition probabilities P_{11} (solid), P_{12} (dashed) and P_{13} (dashdot) as a function of the number N of products of random 3×3 matrices. From bottom to top the panels are for $N = 1$, $N = 10$ and $N = 100$.

domain is described by a S-matrix that we could regard as a random matrix. Thus the S matrix which describes the evolution through the entire turbulence region can be considered as the product of N random $N_f \times N_f$ matrices which individually are not necessarily from the Circular Ensemble. An ensemble of the matrix product of N random, unitary matrix factors is $CUE(N_f)$ as $N \to \infty$ for all distributions of the factors i.e. it is like the central limit theorem for random variates. An example calculation of an ensemble formed as the the product of N random 2×2 matrices is shown in Figure (3). The lowest panel shows the distribution of each matrix factor is diagonally dominant but as the number of products increases we end up with uniform distributions. Figure (4) shows the case of the product of N random 3×3 matrices and we see that as N becomes large we obtain triangular distributions.

4 Summary

The turbulence features very much depend upon the amplitude and the mixing parameters. For small amplitudes: turbulence is quasi two flavor, appears only in the H resonant channel. But for larger amplitudes turbulence breaks HL i.e. it is 3 flavor, and appears in non-resonant channel. Supernova turbulence amplitudes of order $\sim 1 - 10\%$ lead to two-flavor depolarization with uniform distributions. If the amplitudes are of order $\gtrsim 10\%$ or the turbulence extends over a much greater distance than expected then we transit to three-flavor depolarization with triangular distributions.

5 Acknowledgments

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