# Equation of State for Supernova

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In this contribution we discuss several new complete EoS for supernova, whose detailed composition is important for the neutrino dynamics. We focus on one important distinction for various EoS - the density dependence of symmetry energy $E'_{sym}$, and its interesting correlation with the radii of neutron star, as well as properties of neutron distribution in neutron rich nuclei.

## 1 Introduction

The equation of state (EoS) for hot, dense matter relates energy and pressure to temperature, density, and composition. The properties of hot dense matter, for example its pressure at high baryon density - larger than normal nuclear density $3\times10^{14}$ g/cm$^3$, have been the focus of many extradinary terrestrial experiments, including heavy ion collisions with Au [1]. The pressure of nuclear matter at high density determines how large a neutron star our nature could realize.

The properties of nuclear matter depends on its composition, particularly the proton-neutron number asymmetry, or iso-spin dependence, conveniently characterized by the parameter called (a)symmetry energy $E_{sym}$. Most stable nuclei have a small such asymmetry and tell us little about how the EoS changes with the asymmetry. The neutron/proton rich isotopes will be studied with new tools like the Facility for Rare Isotope Beams (FRIB) [2], a heavy ion accerelator to be built in Michigan State University. Studies by these new tools will help us understand when the nuclei would become unstable upon too many neutrons (or protons) added, and ultimately tell us the composition of nuclear matter in supernova given temperature, density, and proton-neutron asymmetry.

Density dependence of symmetry energy $E'_{sym}$ is one key unknown in nuclear physics and nuclear astrophysics, where neutron rich matter is particularly relevant. There are many interesting correlations with $E'_{sym}$ that have been studied in recent years. The pressure of nuclear matter is proportional to $E'_{sym}$. With a higher pressure if $E'_{sym}$ is large , neutron rich nucleus, such as $^{208}$Pb, is found to have a larger neutron radius [3]. This has motivated the Lead Radius Experiment (PREX) [4] to accurately measure the neutron radius in $^{208}$Pb with parity violating electron scattering [5]. On the other hand, the radius of a canonical 1.4 solar mass neutron star is determined by the nuclear matter at similar density inside $^{208}$Pb, therefore a larger pressure at such density tends to give a bigger radius for 1.4 solar mass neutron star [6]. Current large uncertainties in the EoS lead us to generate several big tables of EoS based on dinstinct properties of nuclear matter at high densities, which could be used in astrophysical simulations such as proto-neutron star evolution to identify astrophysical observables with related nuclear matter properties.

Table 1: Range of temperature $T$, density $n_B$, and proton fraction $Y_p$ in the finely spaced interpolated EoS table.

| Parameter | minimum | maximum | number of points |
|---|---|---|---|
| T [MeV] | 0, $10^{-0.8}$ | $10^{1.875}$ | 109 |
| $\log_{10}(n_B)$ [fm$^{-3}$] | -8.0 | 0.175 | 328 |
| $Y_P$ | 0, 0.05 | 0.56 | 1($Y_P$=0)+52 |

## 2 Complete EoS for Supernova

We used a relativistic mean field (RMF) model to self-consistently calculate non-uniform matter at intermediate density and uniform matter at high density, and used a virial expansion for nonideal gas of nucleons and thousands of different nuclei to obtain the EoS at low densities. Altogether these two EoS models cover the large range of temperatures, densities, and proton fractions. Discussion of matching the two results can be found in Ref. [7]. There are 73,840 data points from the virial calculation at low densities, 17,021 data points from the nonuniform Hartree calculation, and 90,478 data points from uniform matter calculations. The overall calculations took 7,000 CPU days in Indiana University's supercomputer clusters.

We use a hybrid interpolation scheme to generate a full EoS table on a fine grid that is thermodynamically consistent. The range of parameter spaces is shown in Table 1. This insures that the first law of thermodynamics is satisfied and that entropy is conserved during adiabatic compression. Our EoS is an improvement over the existing Lattimer-Swesty [8] and H. Shen *et al.* [9, 10] equations of state, because our EoS includes thousands of heavy nuclei and is exact in the low density limit.

We also generated a second EoS based on the RMF effective interaction FSUGold [11, 12], whereas our earlier EoS was based on the RMF effective interaction NL3. The FSUGold interaction has a lower pressure at high densities compared to the NL3 interaction. The original FSUGold interaction produces an EoS, that we call FSU1.7, that has a maximum neutron star mass of 1.7 solar masses. A modification in the high density EoS is introduced to increase the maximum neutron star mass to 2.1 solar masses and results in a slightly different EoS that we call FSU2.1. Finally, the EoS tables for NL3, FSU1.7 and FSU2.1 are available for download.

## 3 Symmetry Energy in Various EoS

The bulk properties of infinite nuclear matter have been collected in Table 2, for NL3, FSUGold, as well as a new effective RMF interaction IUFSU [13]. One important distinction among various EoS is the slope of the symmetry energy at saturation density, $L/3 = \rho_0 E'_{sym}(\rho_0)$. The pressure around saturation density is proportional to $L$, which plays a crucial role both in the terrestial context where it affects the neutron density distribution in neutron rich nuclei and in astrophysics where it affects the structure and thermal evolution of neutron stars.

Brown first realized the correlation between $L$ and the neutron skin thickness of $^{208}$Pb [3], which has 126 neutrons and 82 protons. A larger pressure - due to larger $L$ inside the nucleus will push neutrons to the surface, therefore leads to a bigger neutron skin thickness. This is clearly demonstrated in left panel of Fig. 1, where the proton and neutron densities inside $^{208}$Pb are shown from several model predictions. The proton density is well constrained to 1%. In contrast the neutron density has sizable variations among different model predictions. The

| Model | $\rho_0$ (fm$^{-3}$) | $\varepsilon_0$ (MeV) | $K_0$ (MeV) | $E_{sym}$ (MeV) | $L$ (MeV) |
|---|---|---|---|---|---|
| NL3 | 0.148 | −16.24 | 271.5 | 37.29 | 118.2 |
| FSU | 0.148 | −16.30 | 230.0 | 32.59 | 60.5 |
| IU-FSU | 0.155 | −16.40 | 231.2 | 31.30 | 47.2 |

Table 2: Bulk parameters characterizing the behavior of infinite nuclear matter at saturation density $\rho_0$. The quantities $\varepsilon_0$ and $K_0$ represent the binding energy per nucleon and incompressibility coefficient of symmetric nuclear matter, whereas $E_{sym}$ and $L$ represent the energy and slope of the symmetry energy at saturation density.

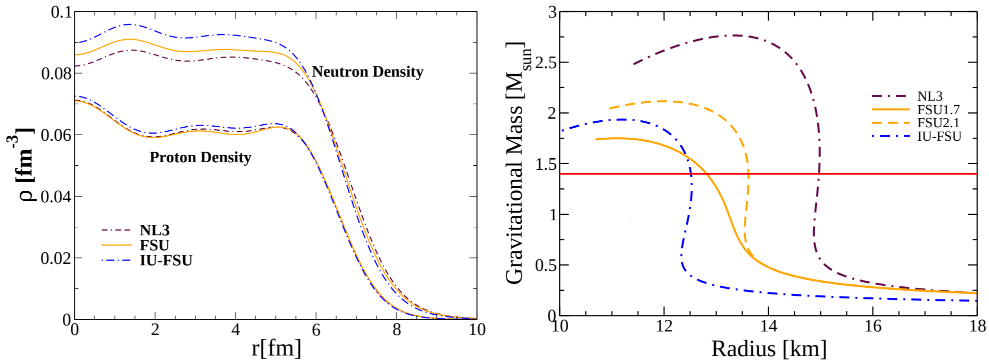


Figure 1: (color online) Left: model predictions for the proton and neutron densities of $^{208}$Pb. Right: Neutron Star *Mass-vs-Radius* relation predicted by the relativistic mean-field models discussed in the text.

values of $L$ are 118.2, 60.5, and 47.2 MeV for NL3, FSU, and IUFSU, respectively. The resulting neutron skin thickness is 0.28, 0.21, and 0.16 fm for NL3, FSU, and IUFSU, respectively. The pressure of neutron matter around saturation density also influences the radius of cold neutron star [6]. In the right panel of Fig. 1, we show the neutron star mass-radius relation for various RMF models. For 1.4 solar mass neutron star, the corresponding radii are 15, 12.8, and 12.5 km for NL3, FSU, and IUFSU, respectively. FUS2.1 has a larger pressure than FSU1.7 at density above 0.2 fm$^{-3}$, and gives rise to larger radius for 1.4 solar mass neutron star, 13.6 km. Due to their common relation to the derivative of symmetry energy $L$, there exists a correlation between the neutron skin thickness and neutron star radius [6].

It's our goal to calculate EoS tables with different pressures. This will allow one to correlate features of astrophysical simulations with EoS properties. In this work we discussed two new EoS based on different RMF effective interactions NL3 and FSUGold. In future we will present a third EoS based on IU-FSU like effective interaction which has a softer symmetry energy $L$.

# 4 Conclusions

Equation of state of nuclear matter at finite temperature and its dynamics is the key to understand the evolution of supernova. The pressure at high densities determines how big and large a neutron star nature could make. The composition of matter in supernova and its dynamical response with neutrinos is important for the evolution of supernova. The emergent neutrino spectra from neutrino-sphere is crucial for the neutron fraction in the neutrino-driven wind and possible r-process that could make heavy nuclei beyond iron. The EoS of nuclear matter has been the focus of heavy ion collision experiments and a future heavy ion accelerator FRIB.

We construct several new EoS of nuclear matter for a wide range of temperatures, densities, and proton fractions. We employ fully microscopic relativistic mean field calculation for matter at intermediate density and high density, and the virial expansion of a nonideal gas (with nucleons and 8981 kinds of nuclei) for matter at low density. The EoS was obtained at over 180,000 grid points in 3-dimensional parameter spaces (temperature, density, and proton fraction). We used hybrid interpolation scheme to generate the final table, as shown in Table 1. The thermodynamic consistency in our table is checked via usual adiabatic compression test, where the oscillation in entropy is limited to 1% using the finer table Ref. [7, 12].

Due to the rather large uncertainties in the EoS at high density, particularly the density dependence of symmetry energy, we generated two EoS tables based on a stiff EoS at high density, NL3, and a softer EoS at low density, FSU1.7, and the modified FSU2.1 which could support a 2.1 solar mass neutron star. In future we will generate a third EoS based on IU-FSU like interaction, which is soft around saturation density but stiff at higher densities. Altogether these EoS tables could cover the uncertainties in the properties of nuclear matter at high density, and astrophysical simulations with them could identify observational phenomena with dinstinct nuclear matter property.

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