Double heavy meson production through double parton scattering

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We show that the contribution from double parton scattering to the inclusive production of heavy meson pairs is comparable with the conventional single parton scattering mechanism at the LHC energy. For some species of heavy mesons the double parton scattering is the dominant mode of their production.

1 Introduction

In the last years it has become obvious that multiple parton interactions play an important role in hadron-hadron collisions at high energies and are one of the most common, yet poorly understood [1], phenomenons at the LHC. The presence of such multiple parton interactions in high-energy hadronic collisions has been convincingly demonstrated by the AFS [2], UA2 [3], CDF [4, 5], and D0 [6] collaborations, using events with the four-jets and $\gamma+3$-jets final states, thus providing new and complementary information on the proton structure.

A greater rate of events containing multiple hard interactions is anticipated at the LHC (as compared to the experiments mentioned above) due to much higher luminosity and greater energy of the LHC. Moreover, the reactions with multiple parton interactions will represent important background to signals from the Higgs production and other interesting processes; some certain types of multiple interactions will have distinctive signatures facilitating their detailed experimental investigation.

The main purpose of this talk (based on our previous work [7]) is to bring attention to another important process: the production of heavy meson pairs through double parton scattering, that is definitely not taken into consideration in the current theoretical estimations [8, 9]. Here, however, one should mention a recent paper [10], in which the contribution from the double parton scattering to J/ψ-pair production has been discussed for the first time for the condition of the LHCb experiment.

Let us recall that, with only assuming the factorization of the two hard parton processes A and B (Fig. 1), the inclusive cross section of a double parton scattering process in hadron-hadron

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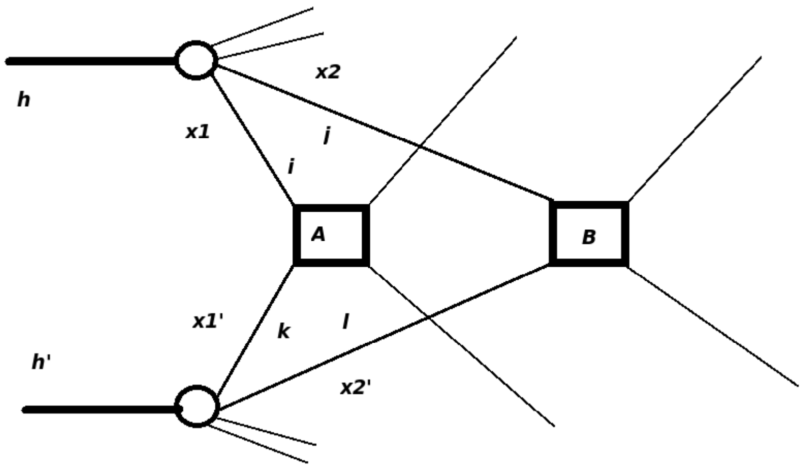

Figure 1: A schematic representation of double parton scattering mechanism.

collisions can be written in the following form [11, 12]

$$\begin{aligned}\sigma_{\rm DPS}^{\rm AB} &= \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_{ij}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)\ \hat\sigma^A_{ik}(x_1,x_1^{'},Q_1^2)\ \hat\sigma^B_{jl}(x_2,x_2^{'},Q_2^2)\\ &\times\ \Gamma_{kl}(x_1^{'},x_2^{'};\mathbf{b_1}-\mathbf{b},\mathbf{b_2}-\mathbf{b};Q_1^2,Q_2^2)\ dx_1dx_2dx_1^{'}dx_2^{'}d^2b_1d^2b_2d^2b, \qquad (1)\end{aligned}$$

where $\mathbf{b}$ is the usual impact parameter, that is, the distance between the centers of colliding hadrons (e.g., the beam and the target) in transverse plane. $\Gamma_{ij}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)$ are the double parton distribution functions, depending on the longitudinal momentum fractions x_1 and x_2 and on the transverse positions $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two partons undergoing the hard processes A and B at the scales Q_1 and Q_2; $\hat\sigma^A_{ik}$ and $\hat\sigma^B_{jl}$ are the parton-level subprocess cross sections. The factor $m/2$ is a consequence of the symmetry with respect to the interacting parton species i and j: $m=1$ if $A=B$, and $m=2$ otherwise.

It is typically taken that the double parton distribution functions may be decomposed in terms of the longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)=D_h^{ij}(x_1,x_2;Q_1^2,Q_2^2)f(\mathbf{b_1})f(\mathbf{b_2}), \qquad (2)$$

where $f(\mathbf{b_1})$ is supposed to be an universal function for all kind of partons with its normalization fixed as

$$\int f(\mathbf{b_1})f(\mathbf{b_1}-\mathbf{b})d^2b_1d^2b=\int T(\mathbf{b})d^2b=1, \qquad (3)$$

and $T(\mathbf{b})=\int f(\mathbf{b_1})f(\mathbf{b_1}-\mathbf{b})d^2b_1$ being the overlap function.

If one makes a further assumption that the longitudinal component $D_h^{ij}(x_1,x_2;Q_1^2,Q_2^2)$ reduces to a product of two independent one parton distributions,

$$D_h^{ij}(x_1,x_2;Q_1^2,Q_2^2)=D_h^i(x_1;Q_1^2)D_h^j(x_2;Q_2^2), \qquad (4)$$

the cross section of the double parton scattering can be expressed in a simple form

$$\sigma_{\rm DPS}^{\rm AB} = \frac{m}{2}\frac{\sigma_{\rm SPS}^{A}\sigma_{\rm SPS}^{B}}{\sigma_{\rm eff}}, \quad \text{with} \quad \sigma_{\rm eff} = [\int d^2b(T(\mathbf{b}))^2]^{-1}. \tag{5}$$

In this representation and at the factorization of longitudinal and transverse components, the inclusive cross section of single hard scattering reads

$$\begin{aligned}\sigma_{\rm SPS}^{A} &= \sum_{i,k}\int D_h^i(x_1;Q_1^2)f(\mathbf{b_1})\hat{\sigma}_{ik}^{A}(x_1,x_1^{'}) \times D_{h'}^k(x_1^{'};Q_1^2)f(\mathbf{b_1}-\mathbf{b})dx_1dx_1^{'}d^2b_1d^2b \\ &= \sum_{i,k}\int D_h^i(x_1;Q_1^2)\hat{\sigma}_{ik}^{A}(x_1,x_1^{'})D_{h'}^k(x_1^{'};Q_1^2)dx_1dx_1^{'}. \end{aligned}\tag{6}$$

These simplifying assumptions, though rather customary in the literature and quite convenient from a computational point of view, are not sufficiently justified and are under revision now. However, the starting cross section formula (1) was derived in many works (see, e.g., Refs. [13, 14, 15, 16]) in the momentum representation using the light-cone variables and the same approximations as those applied to processes with a single hard scattering.

We restrict ourselves to this simple form (5) regarding it as the first estimation of the contribution from the double parton scattering to the inclusive double heavy meson production. The presence of the correlation term in the two-parton distributions results in the decrease [13, 17, 18] of the effective cross section $\sigma_{\rm eff}$ with the growth of the resolution scales Q_1 and Q_2, while the dependence of $\sigma_{\rm eff}$ on the total energy at fixed scales is rather weak [18]. In fact, we obtain the minimal estimate for the contribution of interest. The CDF and D0 measurements give $\sigma_{\rm eff} \simeq 15$ mb, which is roughly 20% of the total (elastic + inelastic) $p\bar{p}$ cross section at the Tevatron energy. We will use this value in our further estimations.

2 Numerical results and discussion

Let us start from the double J/ψ production, since the LHCb Collaboration has recently reported a first measurement [19] of this process

$$\sigma^{J/\psi J/\psi} = 5.6 \pm 1.1 \pm 1.2 \text{ nb} \tag{7}$$

with both J/ψ's in the rapidity region $2 < y^{J/\psi} < 4.5$ and with the transverse momentum $p_T^{J/\psi} < 10$ GeV/c in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 7$ TeV. Earlier this collaboration has already measured [20] the single inclusive J/ψ production cross section with the same kinematic cuts as above

$$\sigma_{\rm SPS}^{J/\psi} = 7.65 \pm 0.19 \pm 1.10_{1.27}^{+0.87} \ \mu\text{b}. \tag{8}$$

Using Eq. (5) we obtain immediately a simple estimation of the contribution from the double parton scattering at the same kinematic conditions

$$\sigma_{\rm DPS}^{J/\psi J/\psi} = \frac{1}{2}\frac{\sigma_{\rm SPS}^{J/\psi}\sigma_{\rm SPS}^{J/\psi}}{\sigma_{\rm eff}} \simeq 2.0 \text{ nb}. \tag{9}$$

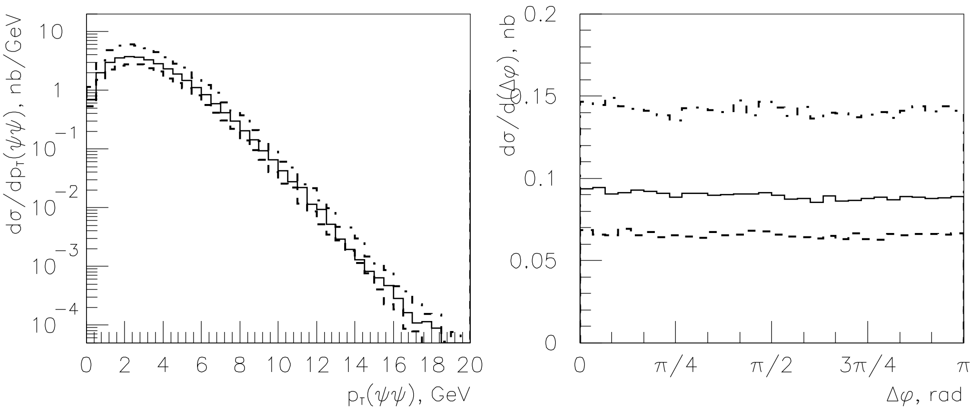

Figure 2: The distributions on the $J/\psi - J/\psi$ pair transverse momentum (left) and the azimuthal angle difference between the two J/ψ mesons (right) in the SPS process. Solid, dashed, and dash-dotted histograms represent calculations with A0, A-, and A+ gluon densities [32], respectively.

This value is quite compatible with the cross section through the "standard" mechanism of the double J/ψ production [9]

$$\sigma_{\rm SPS}^{J/\psi J/\psi} = 4.15 \text{ nb}, \tag{10}$$

and the theoretical prediction for the summary contribution for both scattering modes is then

$$\sigma_{\rm SPS}^{J/\psi J/\psi} + \sigma_{\rm DPS}^{J/\psi J/\psi} = 6.15 \text{ nb}, \tag{11}$$

that is very close to the experimentally observed cross section (7) of double J/ψ production. It is worth mentioning on the other hand that the predictions on the double J/ψ production are very sensitive to the choice of the renormalization scale (because of the $\mathcal{O}(\alpha_s^4)$ dependence of the $\sigma_{\rm SPS}^{J/\psi J/\psi}$ cross section), and so, the LHCb experimental results can also be accommodated by the SPS mechanism alone (see below).

An even better evidence for the double parton scattering process can be found in the production of χ_c pairs. The production of P-wave states is suppressed relative to the production of S-wave states because of the hierarchy of the wave functions $|\mathcal{R}_{J/\psi}(0)|^2 \gg |\mathcal{R}'_{\chi_c}(0)|^2/m_\chi^2$ leading to the inequality $\sigma_{\rm SPS}^{J/\psi J/\psi} \gg \sigma_{\rm SPS}^{\chi_c\chi_c}$. Indeed, as one can learn from Fig. 8 in Ref. [8], the inclusive double χ_c production is suppressed in comparison with the inclusive double J/ψ production by more than two orders of magnitude.

At the same time, the inclusive production of single J/ψ and χ_c states shows nearly the same rates. The latter property is supported by both theoretical [21] and experimental [22, 23, 24] results. The reason for this is, that the χ_c mesons are produced in a direct $2 \to 1$ gluon-gluon fusion $g + g \to \chi_c$, while the J/ψ mesons are produced in a $2 \to 2$ subprocess $g + g \to J/\psi + g$, where an additional final state gluon is required by the color and charge parity conservation. As a consequence, the invariant mass of the produced system is typically much higher in the J case than in the χ_c case. (Besides that, the structure of the matrix element is such that it vanishes when the co-produced gluon becomes soft. This further suppresses the production of low-mass states.)

Taken together, the suppression factors coming from the lower wave function on the χ_c side and from the higher final state mass and extra α_s coupling on the J/ψ side nearly compensate each other making the inclusive production cross sections comparable in size: $\sigma_{\rm SPS}^{\chi_c} \simeq \sigma_{\rm SPS}^{J/\psi}$. As a consequence, we get $\sigma_{\rm DPS}^{\chi_c\chi_c} \simeq \sigma_{\rm DPS}^{J/\psi J/\psi}$ and $\sigma_{\rm DPS}^{\chi_c\chi_c} \gg \sigma_{\rm SPS}^{\chi_c\chi_c}$. Thus, if observed, the production of a $\chi_c\chi_c$ pair would yield a clear and unambiguous indication of the double parton scattering process. The need in detecting the decay photon $\chi_c \to J/\psi + \gamma$ leads to certain difficulties in the experimental procedure, but the task seems still feasible as the production cross section is not small.

Another tempting possibility is to consider the simultaneous production of J/ψ and χ_c. In the SPS mode this process is forbidden at the leading order (LO) by the charge parity conservation but is possible at the next-to-leading order (NLO), $g + g \to J/\psi + \chi_c + g$. The corresponding cross section is then suppressed by one extra power of α_s and by the χ_c wave function. Alternatively, it can proceed via the soft final-state gluon radiation (the so called color octet model). The estimations of the cross section are then model dependent and rather uncertain, but even with the largest acceptable values for the color octet matrix elements one arrives at a suppression factor of about two orders of magnitude [8]. For the DPS mode we still expect no suppression, $\sigma_{\rm DPS}^{J/\psi\chi_c} \simeq \sigma_{\rm DPS}^{J/\psi J/\psi}$.

Yet another interesting process is the production of particles from different flavor families, say, J/ψ and Υ mesons. Once again, this process is not possible at the leading order in the SPS mode and can only occur either at the NNLO (next-to-next-to-leading order) $\mathcal{O}(\alpha_s^6)$, or via the color-octet transitions, or by means of the production and decay of P-wave mesons (i.e., $g + g \to \chi_c + \chi_b$ followed by $\chi_c \to J/\psi + \gamma$ and $\chi_b \to \Upsilon + \gamma$). So, the SPS mode is always suppressed: either by the extra powers of α_s, or by the color-octet matrix elements, or by the P-state wave function, and the DPS mode becomes the absolutely dominant one: $\sigma_{\rm DPS}^{J/\psi\,\Upsilon} \gg \sigma_{\rm SPS}^{J/\psi\,\Upsilon}$.

Now, to be more precise, we will derive some numerical predictions. In doing so, we rely upon perturbative QCD and nonrelativistic bound state formalism [25, 26] with only the color-singlet channels taken into consideration. Also, we accept the k_t-factorization ansatz [27, 28, 29] for the parton model. The computational technique is explained in every detail in Ref. [21], and the parameter setting is as follows. The meson masses are taken from the Particle Data Book [30], and the heavy quark masses are set equal to one half of the respective meson masses; the radial wave functions of J/ψ and Υ mesons are supposed to be known from their leptonic decay widths [30] and are set to $|\mathcal{R}_{J/\psi}(0)|^2 = 0.8$ GeV^3 and $|\mathcal{R}_{\Upsilon}(0)|^2 = 6.48$ GeV^3; the wave functions of the P-states are taken from the potential model [31], $|\mathcal{R}'_{\chi_c}(0)|^2 = 0.075$ GeV^5 and $|\mathcal{R}'_{\chi_b}(0)|^2 = 1.44$ GeV^5; the renormalization scale in the strong coupling $\alpha_s(\mu^2)$ is chosen as the meson transverse mass $\mu^2 = m^2 + p_t^2$; and we use the A0 parametrization from Ref. [32] for the unintegrated gluon density. In the present note we will restrict ourselves to the conditions of the LHCb experiment, since the Collaboration has already recorded the production of J/ψ pairs. Predictions for other experimental conditions can be made in an essentially similar way.

Within the theoretical model described above, we get for the direct inclusive J/ψ production

$$\sigma_{\rm SPS}^{J/\psi}(\text{direct}) = 7.1\ \mu\text{b}, \tag{12}$$

and for the χ_c mesons

$$\sigma_{\rm SPS}^{\chi_1} = 1.5\ \mu\text{b}, \quad \sigma_{\rm SPS}^{\chi_2} = 5.1\ \mu\text{b}. \tag{13}$$

After multiplying these numbers by appropriate branching ratios [30] $Br(\chi_{c1}\to J/\psi+\gamma) = 35\%$ and $Br(\chi_{c2}\to J/\psi+\gamma) = 20\%$ and additions of the direct and indirect contributions, we get for the prompt J/ψ yield:

$$\sigma_{\rm SPS}^{J/\psi} = \sigma_{\rm SPS}^{J/\psi}(\text{direct}) + \sigma_{\rm SPS}^{J/\psi}(\text{from } \chi_c) = 7.1\ \mu\text{b} + 1.6\ \mu\text{b} = 8.7\ \mu\text{b}. \tag{14}$$

This result is in reasonable agreement with the experimental measurement (8), thus giving support to our theoretical model. Quite similarly, we get for the $b\bar{b}$ mesons

$$\sigma_{\rm SPS}^{\Upsilon}(\text{direct}) = 140\ \text{nb}, \quad \text{and} \quad \sigma_{\rm SPS}^{\chi_1} = 18\ \text{nb}, \quad \sigma_{\rm SPS}^{\chi_2} = 91\ \text{nb}. \tag{15}$$

Then one can easily obtain for the DPS mode

$$\sigma_{\rm DPS}^{J/\psi J/\psi} = 1.7\ \text{nb}, \tag{16}$$

$$\sigma_{\rm DPS}^{J/\psi J/\psi}(\text{both from } \chi_c) = 0.9\ \text{nb}, \tag{17}$$

$$\sigma_{\rm DPS}^{J/\psi \Upsilon} = 0.07\ \text{nb}. \tag{18}$$

The reader can continue deriving predictions for other DPS combinations.

To calculate the background contribution $\sigma_{\rm SPS}^{J/\psi J/\psi}$ we use the code developed in [33] and extended now [34] to the k_t-factorization approach:

$$\sigma_{\rm SPS}^{J/\psi J/\psi} = 4\ \text{nb}. \tag{19}$$

Variations in the renormalization scale μ_R^2 within a factor of 2 around the default value $\mu_R^2 = \hat{s}/4$ result in an increase or decrease on the total production rate by a factor of 1.6. Employing some different parametrizations for the unintegrated gluon densities (A+ or A- sets from Ref. [32]) also changes the predicted cross section by a factor of 1.6 up or down. Our central prediction (19) is in reasonable agreement with the data (7).

The proportion between the visible SPS and DPS contributions can, in principle, depend on the experimental cuts on the J/ψ transverse momentum. However, in the particular case which we are considering here, the LHCb Collaboration refers to no cuts on $p_T(J/\psi)$. In fact, there are some soft restrictions on the momenta of the decay muons, $p_T(\mu) > 600$ MeV, but they are taken into account as corrections to the acceptance. The final results reported by the collaboration to compare with are the acceptance-corrected ones.

It is also worth noting that even in the general case the sensitivity of the ratio $\sigma_{\rm DPS}/\sigma_{\rm SPS}$ to the p_T cuts is rather weak, because the DPS and SPS contributions show the same p_T dependence. This is explained in detail in Ref. [34]. Irrespective of the particular properties of the subprocess matrix element, the p_T of the final state is dominated by the transverse momentum of the initial gluons, and the individual J/ψ spectra behave as $1/p_T^4$ in both SPS and DPS modes. Moreover, the momenta of the two J/ψ mesons are not correlated. The latter is evident in the DPS case and was not a priori evident in the SPS case, but turned out to be true (Fig.5 in Ref. [34]). So, the SPS and DPS event topologies are rather similar to each other and can hardly be distinguished from one another.

Our calculations agree with the observation made in Ref. [10] that the effects of initial parton radiation (that are automatically present in the k_t-factorization approach) destroy the original back-to-back $J/\psi J/\psi$ kinematics completely washing out the azimuthal correlations (see Fig. 2). One can potentially distinguish the SPS and DPS modes with rapidity correlations, but we anyway find that looking at some other meson species is more indicative. In particular, the production of $\chi_c\chi_c$, $J/\psi\chi_c$ or $J/\psi\Upsilon$ pairs is totally dominated by the DPS mechanism because the SPS mechanism is suppressed for the reasons given earlier. A similar study has been carried out in Ref.[35].

Summing up, we conclude that the processes with pairs of heavy quarkonia in the final state ($J/\psi J/\psi$, $\chi_c\chi_c$, $J/\psi\chi_c$, $J/\psi\Upsilon$) can serve as precise probes of the double parton scattering at the LHC and can stimulate important steps towards understanding the multiparticle QCD dynamics.

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