

Unintegrated gluon distribution and soft pp collisions at LHC

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We found the parameterization of the unintegrated gluon distribution from the best description of the LHC data on the inclusive spectra of hadrons produced in pp collisions at the mid-rapidity region and small transverse momenta. It is different from the one obtained within perturbative QCD only at low intrinsic transverse momenta k_t . The application of this distribution to analysis of the $e - p$ DIS allows us to get the results which do not contradict the H1 and ZEUS data on the structure functions at low x . So, the connection between the soft processes at LHC and low- x physics at HERA is found.

1 Introduction

As is well known, hard processes involving incoming protons, such as deep-inelastic lepton-proton scattering (DIS), are described using the scale-dependent parton density functions. Usually, these quantities are calculated as a function of the Bjorken variable x and the square of the four-momentum transfer $q^2 = -Q^2$ within the framework of popular collinear QCD factorization based on the DGLAP evolution equations [1]. However, for semi-inclusive processes (such as inclusive jet production in DIS, electroweak boson production [2], etc.) at high energies it is more appropriate to use the parton distributions unintegrated over the transverse momentum k_t in the framework of k_t -factorization QCD approach [3]¹. The k_t -factorization formalism is based on the BFKL [6] or CCFM [7] evolution equations and provides solid theoretical grounds for the effects of initial gluon radiation and intrinsic parton transverse momentum k_t . The theoretical analysis of the unintegrated quark $q(x, k_t)$ distribution (u.q.d.) and gluon $g(x, k_t)$ distribution (u.g.d.) can be found, for example, in [8]- [22]. According to [11], the u.g.d. $g(x, k_t)$ at fixed Q^2 has the very interesting behavior at small $x \leq 0.01$, it increases very fast starting from almost zero values of k_t and decreases smoothly at large k_t . In contrast, the u.q.d. $q(x, k_t)$ is almost constant in the whole region of k_t up to $k_t \sim 100$ GeV/ c and much smaller than $g(x, k_t)$. These distributions were obtained using the so-called KMR formalism within the leading order (LO) and next-to-leading order of QCD (NLO) at large Q^2 from the

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¹See, for example, reviews [4, 5] for more information.

known (DGLAP-evolved [1]) parton densities determined from the global data analysis. The unintegrated parton distributions were successfully applied to analyze the DIS data at low x and the number of processes studied at LHC (see, for example, [13]- [21]). However, at small values of Q^2 the nonperturbative effects should be included to evaluate these distributions. The non-perturbative effects can arise from the complex structure of the QCD vacuum. For example, within the instanton approach [12] the very fast increase of the unintegrated gluon distribution function at $0 \leq k_t \leq 0.5$ GeV/ c and $Q^2 = 1$ (GeV/ c)² is also shown. These results stimulated us to assume, that the unintegrated gluon distribution in the proton can be included by analyzing also the soft hadron production in pp collisions. In this paper we analyze the inclusive spectra of the hadrons produced in pp collisions at LHC energies in the mid-rapidity region including the possible creation of soft gluons in the proton. We estimate the u.g.d. at low intrinsic transverse momenta $k_t \leq 1.5 - 1.6$ GeV/ c and its parameters extracted from the best description of the LHC data at low transverse momenta p_t of the produced hadrons. We also show that our u.g.d. similar to the u.g.d. obtained in [9,10] at large k_t and different from it at low k_t .

2 Inclusive spectra of hadrons in pp collisions

2.1 Unintegrated gluon distributions

As was mentioned above, the unintegrated gluon densities $g(x, k_t^2)$ can be generally described by the BFKL [6] evolution equation, where the leading $\ln(1/x)$ contributions are summed. The conventional integrated gluon distribution $g(x, Q^2)$ can be approximately related to the unintegrated one by [13]

$$g(x, Q^2) \sim g(x, Q_0^2) + \int_{Q_0^2}^{Q^2} dk_t^2 g(x, k_t^2) \quad (1)$$

where Q_0^2 is a starting nonzero value of Q^2 . An appropriate description valid for small and large x is given by the CCFM [7] evolution equation that results in the u.g.d. $g(x, k_t^2, \bar{q}^2)$ as a function of x, k_t^2 and the additional scale \bar{q} (see details in [4, 10] and references therein). The another u.g.d. $g(x, k_t^2)$ satisfies the DGLAP-type evolution equation with respect to k_t^2 [2, 11].

A simple parameterization of the u.g.d. was obtained, for example, within the color-dipole approach in [8, 9] on the assumption of a saturation of the gluon density at low Q^2 which successfully described both inclusive and diffractive $e - p$ scattering. The u.g.d. $xg(x, k_t^2, Q_0^2)$ is given by [9, 10]

$$xg(x, k_t, Q_0) = \frac{3\sigma_0}{4\pi^2\alpha_s(Q_0)} R_0^2 k_t^2 \exp(-R_0^2(x)k_t^2) ; R_0 = \frac{1}{Q_0} \left(\frac{x}{x_0}\right)^{\lambda/2}, \quad (2)$$

where $\sigma_0 = 29.12$ mb, $\alpha_s = 0.2$, $Q_0 = 1$ GeV/ c , $\lambda = 0.277$ and $x_0 = 4.1 \cdot 10^{-5}$. The form for $xg(x, k_t, Q_0)$ given by Eq.(2) was obtained in [9] within the model of the $(q\bar{q})$ dipole [22, 23] on the assumption of the saturation effect for the gluon density, e.g., the cross section σ_{γ^*N} of the interaction of the transverse virtual photons γ^* with a nucleon N is assumed to be a constant at low Q^2 (and low x too). It corresponds to the Gaussian form for the effective dipole cross section $\hat{\sigma}(x, r)$ as a function of x and the relative transverse separation \mathbf{r} of the $q\bar{q}$ pair [9]. In fact, this form could be more complicated. In this paper we study this point and try to find the parameterization for $xg(x, k_t, Q_0)$, which is related to $\hat{\sigma}(x, r)$, from the best description of the

inclusive spectra of charge hadrons produced in pp collisions at LHC energies and mid-rapidity region.

2.2 Quark-gluon string model (QGSM) including gluons

As is well known, the soft hadron production in pp collisions at not large transfer can be analyzed within the soft QCD models, namely, the quark-gluon string model (QGSM) [24]-[26] or the dual parton model (DPM) [27]. The cut n-pomeron graphs calculated within these models result in a reasonable contribution at small but nonzero rapidities. However, it has been shown recently [28] that there are some difficulties in using the QGSM to analyze the inclusive spectra in pp collisions in the mid-rapidity region and at the initial energies above the ISR one. However, it is due to the according to Abramovskiy-Gribov-Kancheli cutting rules (AGK) [29] at mid-rapidity ($y \simeq 0$), when only one-pomeron Mueller-Kancheli diagrams contribute to the inclusive spectrum $\rho_h(y \simeq 0, p_t)$. To overcome these difficulties it was assumed in [28] that there are soft gluons in the proton, which are split into $q\bar{q}$ pairs and should vanish at the zero intrinsic transverse momentum ($k_t \sim 0$). The total spectrum $\rho_h(y \simeq 0, p_t)$ was split into two parts, the quark contribution $\rho_q(y \simeq 0, p_t)$ and the gluon one and their energy dependence was calculated [28]

$$\rho(p_t) = \rho_q(x=0, p_t) + \rho_g(x=0, p_t) = g(s/s_0)^\Delta \bar{\phi}_q(0, p_t) + (g(s/s_0)^\Delta - \sigma_{nd}) \bar{\phi}_g(0, p_t). \quad (3)$$

The following parameterization for $\bar{\phi}_q(0, p_t)$ and $\bar{\phi}_g(0, p_t)$ was found [28]:

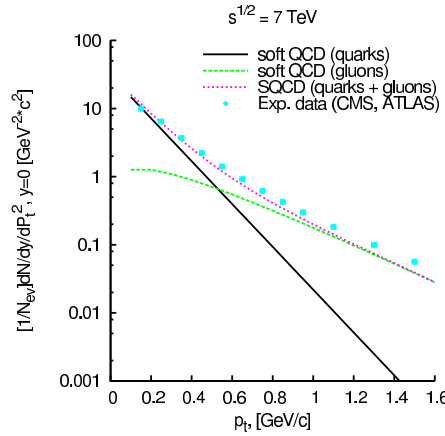


Figure 1: The inclusive spectrum of the charged hadrons as a function of p_t (GeV/c) in the central rapidity region ($y = 0$) at $\sqrt{s} = 7$ TeV at $p_t \leq 1.6$ GeV/c compared with the CMS [30] which are very close to the ATLAS data [31].

$$\begin{aligned} \bar{\phi}_q(0, p_t) &= A_q \exp(-b_q p_t) \\ \bar{\phi}_g(0, p_t) &= A_g \sqrt{p_t} \exp(-b_g p_t), \end{aligned} \quad (4)$$

where $s_0 = 1 \text{ GeV}^2$, $g = 21 \text{ mb}$, $\Delta = 0.12$. The parameters are fixed from the fit to the data on the p_t distribution of charged particles at $y = 0$ [28]: $A_q = 4.78 \pm 0.16 \text{ (GeV/c)}^{-2}$, $b_q = 7.24 \pm 0.11$

$(\text{GeV}/c)^{-1}$ and $A_g = 1.42 \pm 0.05 (\text{GeV}/c)^{-2}$; $b_g = 3.46 \pm 0.02 (\text{GeV}/c)^{-1}$. Figure 1 illustrates the best fit of the inclusive spectrum of charged hadrons produced in pp collisions at $\sqrt{s} = 7$ TeV and the central rapidity region at the hadron transverse momenta $p_t \leq 1.6 \text{ GeV}/c$; the solid line corresponds to the quark contribution ρ_q , the dashed line is the gluon contribution ρ_g , and the dotted curve is the sum of these contributions ρ_h given by Eq.(3).

2.3 Modified unintegrated gluon distributions

Let us assume the existence of the intrinsic gluons in proton, as was suggested in [32], which can be presented as the $q\bar{q}$ pairs similar to the sea $q\bar{q}$ considered in the QGSM [24]. Then, we can calculate the gluon contribution $\tilde{\phi}_g(0, p_t)$ entering into Eq.(4) as the cut graph (Fig.2, right) of the one-pomeron exchange in the gluon-gluon interaction (Fig.2, left) using the splitting of the gluons into the $q\bar{q}$ pair. Actually, the calculation can be made in a way similar to the calculation of the sea quark contribution to the inclusive spectrum within the QGSM, see Eqs.(4,5) in [28] at $n = 2$.

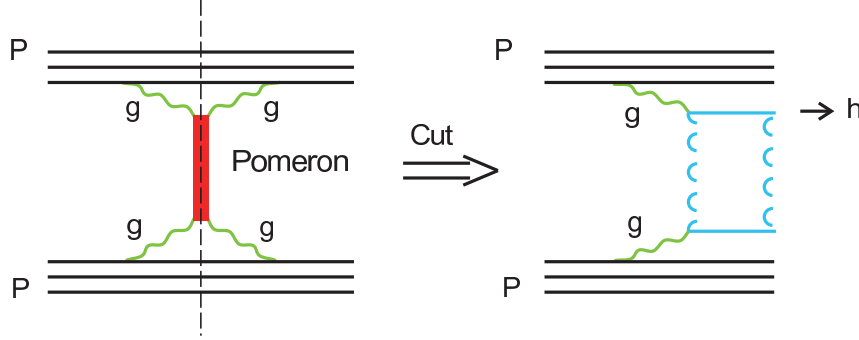


Figure 2: The one-pomeron exchange graph between two gluons in the elastic pp scattering (left) and the cut one-pomeron due to the creation of two colorless strings between quarks/antiquarks (right) [24].

$$\rho_g(x_{\pm}, p_{ht}) = F(x_+, p_{ht})F(x_-, p_{ht}) , \quad (5)$$

where the function $F(x_{\pm}, p_t; p_{ht})$ corresponds to the production of final hadrons from decay of $q\bar{q}$ string. It is calculated as the following convolution:

$$F(x_{\pm}, p_t; p_{ht}) = \int_{x_{\pm}}^1 dx_1 \int d^2 k_{1t} f_{q(\bar{q})}(x_1, k_{1t}) G_{q(\bar{q}) \rightarrow h} \left(\frac{x_{\pm}}{x_1}, p_{ht} - k_t \right) , \quad (6)$$

Here $G_{q(\bar{q}) \rightarrow h}(z, \tilde{k}_t) = z D_{q(\bar{q}) \rightarrow h}(z, \tilde{k}_t)$, $D_{q(\bar{q}) \rightarrow h}(z, \tilde{k}_t)$ is the fragmentation function (FF) of the quark (antiquark) to a hadron h , $z = x_{\pm}/x_1$, $\tilde{k}_t = p_{ht} - k_t$, $x_{\pm} = 0.5(\sqrt{x^2 + x_t^2} \pm x)$, $x_t = 2\sqrt{(m_h^2 + p_t^2)/s}$. The distribution of sea quarks (antiquark) $f_{q(\bar{q})}$ is related to the splitting function $\mathcal{P}_{g \rightarrow q\bar{q}}$ of gluons to $q\bar{q}$ by

$$f_{q(\bar{q})}(z, k_t) = \int_z^1 g(z_1, k_t, Q_0) \mathcal{P}_{g \rightarrow q\bar{q}} \left(\frac{z}{z_1} \right) \frac{dz_1}{z_1} , \quad (7)$$

where $g(z_1, k_{1t}, Q_0)$ is the u.g.d.. The gluon splitting function $\mathcal{P}_{g \rightarrow q\bar{q}}$ was calculated within the Born approximation.

Calculating the diagram of Fig.2 (right) by the use of Eqs.(5-7) for the gluon contribution ρ_g we took the FF to charged hadrons, pions, kaons, and $p\bar{p}$ pairs obtained within the QGSM [33]. From the best description of $\rho_g(x \simeq 0, p_{ht})$, see its parameterization given by Eq.(4), we found the form for the $xg(x, k_t, Q_0)$ which was fitted in the following form:

$$xg(x, k_t, Q_0) = \frac{3\sigma_0}{4\pi^2\alpha_s(Q_0)} C_1 (1-x)^{b_g} \times (R_0^2(x)k_t^2 + C_2(R_0(x)k_t)^a) \exp(-R_0(x)k_t - d(R_0(x)k_t)^3), \quad (8)$$

The coefficient C_1 was found from the following normalization:

$$g(x, Q_0^2) = \int_0^{Q_0^2} dk_t^2 g(x, k_t^2, Q_0^2), \quad (9)$$

and the parameters

$$a = 0.7; C_2 \simeq 2.3; \lambda = 0.22; b_g = 12; d = 0.2; C_1 = 0.3295$$

were found from the best fit of the LHC data on the inclusive spectrum of charged hadrons produced in pp collisions and in the mid-rapidity region at $p_t \leq 1.6$ GeV/c, see the dashed line in Fig.1 and Eq.(4).

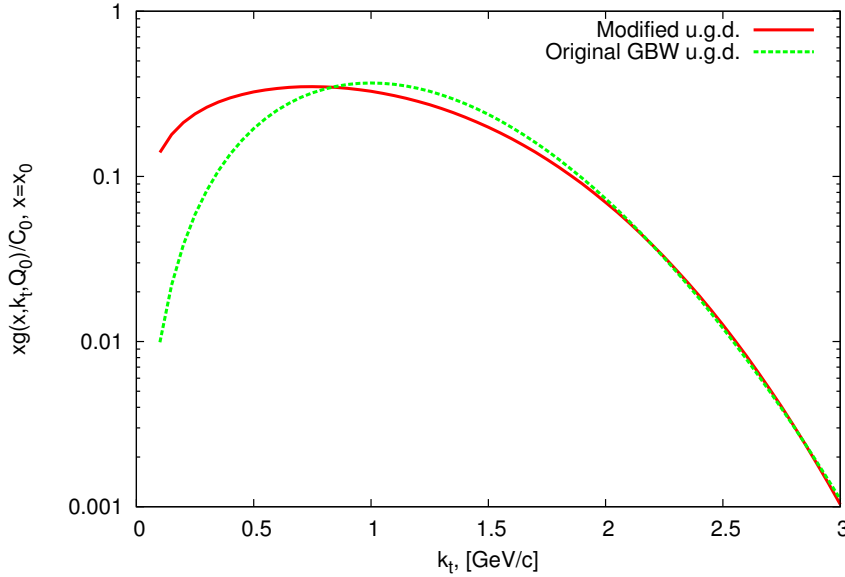


Figure 3: The unintegrated gluon distribution $xg(x, k_t, Q_0)/C_0$ as a function of k_t at $x = x_0$ and $Q_0 = 1$ GeV/c. The dashed curve corresponds to the original GBW [9, 10], Eq.(2), and the solid line is the modified u.g.d. given by Eq.(8).

Figure 3 presents the modified u.g.d. obtained by calculating the cut one-pomeron graph of Fig.2 and the original GBW u.g.d. [9,10] as a function of the transverse gluon momentum k_t . Here $C_0 = 3\sigma_0/(4\pi^2\alpha_s(Q_0))$. One can see that the modified u.g.d. (the solid line in Fig.3) is different from the original GBW u.g.d. [9,10] at $k_t < 1.5$ GeV/c and coincides with it at larger k_t . This is due to the sizable contribution of ρ_g (Eqs.(3,4)) to the inclusive spectrum $\rho(p_t)$ of charged hadrons produced in pp collisions at LHC energies and in the mid-rapidity region, see the dashed line in Fig.1.

3 Proton longitudinal structure function

Within the k_t -factorization approach, the proton longitudinal structure function $F_L(x, Q^2)$ calculated in the leading order of QCD can be presented in the following form [13]:

$$F_L(x, Q^2) = \int_x^1 \frac{dz}{z} \int^{Q^2} dk_t^2 \sum_{flavour(f)} e_f^2 \hat{C}_L^g\left(\frac{x}{z}, Q^2, m_f^2, k_t^2\right) g(x, k_t^2), \quad (10)$$

where e_f^2 is the charge of the quarks of flavor f , the functions $\hat{C}_{2,L}^g(x/z, Q^2, m_f^2, k_t^2)$ are the so called hard structure functions of the off-shell gluons with virtuality k_t^2 which correspond to the quark-box diagram for the photon-gluon interactions [13]. In the present note we calculated $F_L(x, Q^2)$ at the fixed value of the missing mass $W = 276$ GeV using the parameterization for u.g.d. at fixed Q_0^2 given by Eq.(2) and Eq.(8). The results of our calculations are presented in Fig. 4 in comparison with the H1 data [34,35].

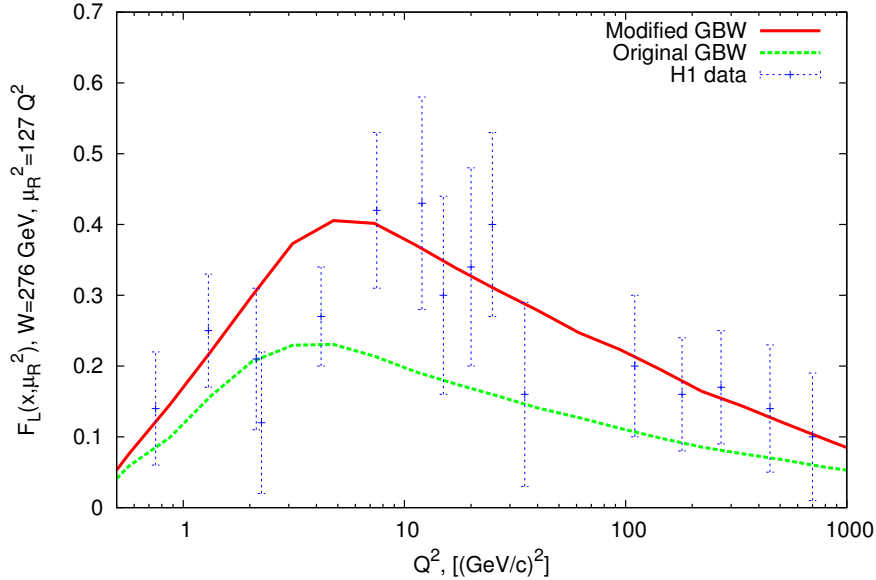


Figure 4: The longitudinal structure function $F_L(\mu_R^2)$ at $W = 276$ GeV and $\mu_R^2 = K \cdot Q^2$, where $K = 127$ [36]. The H1 data are taken from [34,35].

We find that the modified u.g.d. allows us also to describe the proton F_L . Note that in order to take into account the NLO corrections (which are important at low Q^2) in our numerical calculations we apply the method proposed in [14]. Following [14, 36], we use the shifted value of the renormalization scale $\mu_R^2 = K Q^2$, where $K = 127$. As is was shown in [36], this shifted scale in the DGLAP approach at LO approximation leads to the results which are very close to the NLO ones.

4 Conclusion

We fitted the experimental data on the inclusive spectra of charged particles produced in the central pp collisions at energies larger than the ISR starting with the sum of the quark contribution ρ_q and the gluon contribution ρ_g (see Eqs.(3,4)). The parameters of this fit do not depend on the initial energy in that energy interval. Assuming creation of soft gluons in the proton at low transverse momenta k_t and calculating the cut one-pomeron graph between two gluons in colliding protons we found the form for the unintegrated gluon distribution (modified u.g.d) as a function of x and k_t at the fixed value of Q_0^2 . The parameters of this u.g.d. were found from the best description of the LHC data on the inclusive spectra of the charged hadrons produced in the mid-rapidity pp collisions at low p_t . It was shown that the modified u.g.d. is different from the original GBW u.g.d. obtained in [9, 10] at $k_t \leq 1.6$ GeV/c and it coincides with the GBW u.g.d. at $k_t > 1.6$ GeV/c. It was also shown that the modified u.g.d. allows us to describe the longitudinal structure function $F_L(Q^2)$ at the fixed missing mass W better than the original GBW u.g.d. Therefore, some link between soft processes at the LHC and low- x physics at HERA is found.

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