Theoretical considerations on the double Drell-Yan process as a prototype for multiparton interactions

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DOI: http://dx.doi.org/10.3204/DESY-PROC-2012-03/27

We investigate several ingredients for a theory of multiple hard scattering in hadron-hadron collisions. Issues discussed include the space-time structure of multiple interactions, their power behavior, Sudakov logarithms, and the possibility to constrain multiparton distributions by connecting them with generalized parton distributions.

1 Introduction

The phenomenology of multiparton interactions relies on models that are physically intuitive but involve significant simplifications. So far a systematic description of multiple interactions in QCD remains elusive. Here we report on some steps towards this goal. We will see to which extent the cross section formulae currently used to calculate multiple-scattering processes can be justified in QCD and to which extent they need to be completed.

We consider the case of two hard scatters at parton level. For definiteness we analyze the production of two electroweak gauge bosons with large invariant mass (γ^*, Z, or W). Since the main interest in multiparton interactions is driven by the need to understand details of the final state, we keep the transverse momenta of the produced gauge bosons differential, rather than integrating over them. For the production of a single boson there is a powerful theoretical description based on transverse-momentum dependent parton densities [1], which we aim to extend to the case of multiparton interactions, starting from first-principle QCD. Integrating over transverse momenta gives the more familiar formulation in terms of collinear parton distributions.

The following discussion is a short summary of [2]. Detailed derivations of our results and further discussion are given in [3].

2 Tree-level analysis

We begin with the cross section formula for double parton scattering at tree level. For definiteness we take two colliding protons and consider the case where the two partons coming from

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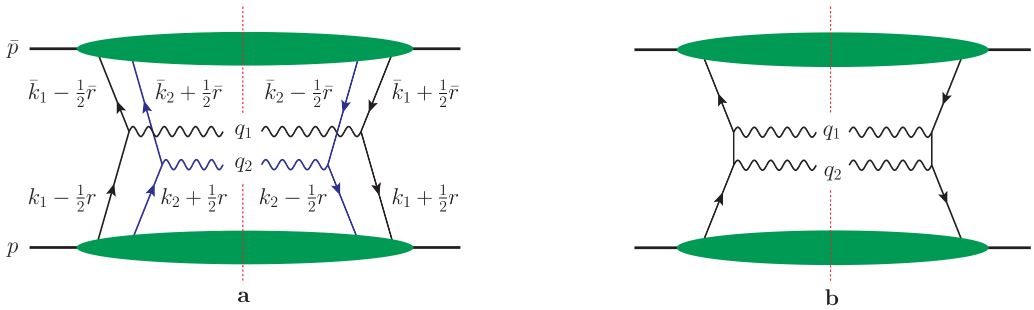

Figure 1: Graphs for the production of two gauge bosons by double (a) or single (b) hard scattering. The dotted line denotes the final-state cut. The decays of the gauge bosons into fermion-antifermion pairs are not shown for simplicity.

one of the protons are quarks. The corresponding graph is shown in Fig. 1a, which also specifies our assignment of momentum variables.

We use light-cone coordinates $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$ and $\boldsymbol{v} = (v^1, v^2)$ for any four-vector v and choose a reference frame where p moves fast to the right and $\bar{p}$ fast to the left, with transverse momenta $\boldsymbol{p} = \bar{\boldsymbol{p}} = \boldsymbol{0}$. We consider kinematics where the invariant masses of the bosons are large and where their transverse momenta are much smaller, i.e. we require $q_T \ll Q$ with $\boldsymbol{q}_1^2 \sim \boldsymbol{q}_2^2 \sim q_T^2$ and $q_1^2 \sim q_2^2 \sim Q^2$.

The two-quark distributions required to describe graph 1a read

$$F_{a_1,a_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) = 2p^+ \int dy^- \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} \, e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \big\langle p \big| \mathcal{O}_{a_2}(0, z_2) \mathcal{O}_{a_1}(y, z_1) \big| p \big\rangle \tag{1}$$

with bilinear operators

$$\mathcal{O}_a(y, z) = \bar{q}(y - \tfrac{1}{2}z)\, \Gamma_a \, q(y + \tfrac{1}{2}z)\Big|_{z^+ = y^+ = 0} . \tag{2}$$

The transverse positions $\boldsymbol{z}_i$ and $\boldsymbol{y}$ are Fourier conjugate to the transverse momenta $\boldsymbol{k}_i$ and $\boldsymbol{r}$ in figure 1a. Furthermore, $a = q, \Delta q, \delta q$ labels the quark polarization and

$$\Gamma_q = \tfrac{1}{2}\gamma^+ , \qquad \Gamma_{\Delta q} = \tfrac{1}{2}\gamma^+ \gamma_5 , \qquad \Gamma_{\delta q}^j = \tfrac{1}{2} i\sigma^{j+}\gamma_5 \tag{3}$$

with $j = 1, 2$. The operators in (2) are well-known from the definitions of single-parton densities for unpolarized, longitudinally polarized and transversely polarized quarks, see e.g. [4]. Analogous definitions hold for antiquarks and for a left-moving hadron. The cross section can finally be written as

$$\frac{d\sigma|_{1\mathrm{a}}}{\prod_{i=1}^2 dx_i \, d\bar{x}_i \, d^2\boldsymbol{q}_i} = \frac{1}{C} \sum_{\substack{a_1, a_2 = q, \Delta q, \delta q \\ \bar{a}_1, \bar{a}_2 = \bar{q}, \Delta\bar{q}, \delta\bar{q}}} \left[\prod_{i=1}^{2} \hat{\sigma}_{i, a_i \bar{a}_i}(q_i^2) \int \frac{d^2\boldsymbol{z}_i}{(2\pi)^2} \, e^{-i\boldsymbol{z}_i \boldsymbol{q}_i} \right] \times \int d^2\boldsymbol{y} \, F_{a_1,a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}) \, F_{\bar{a}_1,\bar{a}_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}) \tag{4}$$

where here and in the following we write $F(x_i, \boldsymbol{z}_i, \boldsymbol{y})$ instead of $F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$ for brevity. The $\hat{\sigma}_{i,a\bar{a}}$ are the partonic cross sections and it is understood that for each $a_i = \delta q$ both F_{a_1,a_2} and $\hat{\sigma}_{i,a\bar{a}}$ carry extra indices j associated with the direction of the transverse quark polarization. Corresponding remarks hold for $\bar{a}_i = \delta\bar{q}$. C is a combinatorial factor related to identical particles in the final state.

Integration of the cross section over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ leads to collinear (i.e. transverse-momentum integrated) two-parton densities

$$F_{a_1,a_2}(x_i, \boldsymbol{y}) = F_{a_1,a_2}(x_i, \boldsymbol{z}_i = \boldsymbol{0}, \boldsymbol{y})\,. \tag{5}$$

The corresponding cross section formula is the basis for the phenomenology of multiple interactions and has been used for a long time. It was derived in [5] for scalar partons and in [6] for quarks.

3 Power behavior

A pair of electroweak gauge bosons can be produced by two hard scatters, but also by a single one. An example graph is shown in figure 1b, and the corresponding cross section formula reads

$$\frac{d\sigma|_{\text{1b}}}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i} = \frac{d\hat{\sigma}}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1} \int \frac{d^2\boldsymbol{z}}{(2\pi)^2} e^{-i\boldsymbol{z}(\boldsymbol{q}_1+\boldsymbol{q}_2)} f_q(x,z) f_{\bar{q}}(\bar{x},z)\,, \tag{6}$$

where $x = x_1 + x_2$, $\bar{x} = \bar{x}_1 + \bar{x}_2$, $\hat{\sigma}$ is the cross section for $q\bar{q}$ annihilation into two gauge bosons and $f_q(x,z)$ is the analog of $F_{q,q}(x_i, z_i, y)$ for a single quark.

Dimensional analysis of (4) and (6) reveals that

$$\frac{d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i} \sim \frac{1}{Q^4\Lambda^2} \tag{7}$$

for both the single and double hard-scattering mechanisms. Here the small scale Λ^2 represents q_T^2 or the scale of non-perturbative interactions, whichever is larger. We thus obtain an important result: multiple hard scattering is *not* power suppressed in cross sections that are sufficiently differential in transverse momenta.

The situation changes when one integrates over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$, because single hard-scattering populates a larger phase space. In single hard-scattering, only the sum $\boldsymbol{q}_1 + \boldsymbol{q}_2$ is restricted to be of order Λ, while the individual momenta can be as large as the hard scale Q. In contrast, both boson transverse momenta are restricted to be of order Λ in double hard-scattering. Thus power counting yields

$$\frac{d\sigma|_{\text{1a}}}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i} \sim \frac{\Lambda^2}{Q^4}\,, \qquad \frac{d\sigma|_{\text{1b}}}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i} \sim \frac{1}{Q^2}\,. \tag{8}$$

In the transverse-momentum integrated cross section multiple hard-scattering is thus power suppressed. This is in fact required for the validity of the usual collinear factorization formulae, which describe only the single hard-scattering contribution.

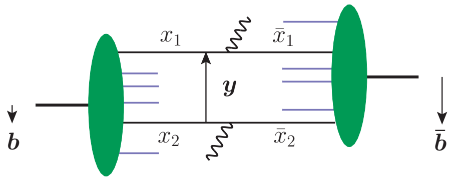

Figure 2: Visualization of the cross section formula (11) when $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are integrated over.

4 Impact parameter

The distributions $F(x_i, \boldsymbol{z}_i, \boldsymbol{y})$ depend on spatial transverse coordinates for the quarks but still refer to a proton with definite (zero) transverse momentum. A representation purely in impact parameter space can be obtained using the methods of [7, 8, 9], where impact parameter densities for a single parton are constructed from generalized parton distributions. To this end we first define non-forward distributions $F(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \boldsymbol{\Delta})$ with proton states $\langle p^+, \frac{1}{2}\boldsymbol{\Delta}|$ and $|p^+, -\frac{1}{2}\boldsymbol{\Delta}\rangle$ with different transverse momenta. Now we introduce the wave packet

$$|p^+, \boldsymbol{b}\rangle = \int \frac{d^2\boldsymbol{p}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle\,, \tag{9}$$

which describes a proton with definite transverse position $\boldsymbol{b}$, and the Fourier transform of $F(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \boldsymbol{\Delta})$,

$$F_{a_1,a_2}\big(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \boldsymbol{b}\big) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\boldsymbol{\Delta}} F_{a_1,a_2}\big(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \boldsymbol{\Delta}\big)\,. \tag{10}$$

Integrating $F(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \boldsymbol{b})$ over $\boldsymbol{b}$ one recovers the distributions $F(x_i, \boldsymbol{z}_i, \boldsymbol{y})$, so that the cross section can be cast into the form

$$\frac{d\sigma|_{1\mathrm{a}}}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i} = \frac{1}{C} \sum_{\substack{a_1,a_2=q,\Delta q,\delta q\\ \bar{a}_1,\bar{a}_2=\bar{q},\Delta\bar{q},\delta\bar{q}}} \Bigg[\prod_{i=1}^{2} \hat{\sigma}_{i,a_i\bar{a}_i}(q_i^2) \int \frac{d^2\boldsymbol{z}_i}{(2\pi)^2}\, e^{-i\boldsymbol{z}_i\boldsymbol{q}_i}\Bigg] \int d^2\boldsymbol{y}\, d^2\boldsymbol{b}\, d^2\bar{\boldsymbol{b}}$$
$$\times F_{a_1,a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \boldsymbol{b})\, F_{\bar{a}_1,\bar{a}_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \bar{\boldsymbol{b}})\,, \tag{11}$$

which has a simple geometric interpretation in impact parameter space. Taking the average of transverse positions in the amplitude and its conjugate, one identifies $\boldsymbol{y}$ as the average distance between the two scattering partons, as can be seen from (2). Likewise, $\boldsymbol{b}$ is the average distance between parton 2 and the right-moving proton, and $\bar{\boldsymbol{b}}$ is the average distance between parton 2 and the left-moving proton. This is illustrated in Fig. 2 for the case where the cross section is integrated over $\boldsymbol{q}_i$, so that $\boldsymbol{z}_i = \boldsymbol{0}$ and the positions in the amplitude and its conjugate coincide.

5 Beyond leading order

Our discussion so far has been concerned with tree graphs as in Fig. 1. At this level, our results can readily be generalized to other hard-scattering processes, in particular to jet production with the well-known subprocesses $qq \to qq$, $qg \to qg$, etc.

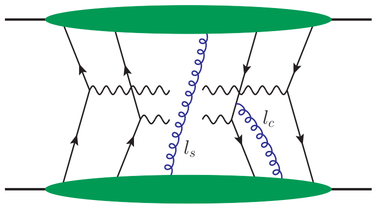

Figure 3: Example graph with a collinear gluon l_c and a soft gluon l_s.

A proper factorization formula in QCD must of course include corrections to the tree-level cross section, and in particular take care of additional gluon exchange. In [3] we argue that the factorization proof for Drell-Yan production can to a large part be extended to double hard-scattering processes producing colorless states such as electroweak gauge bosons. We restrict ourselves to such processes from now on.

There are two types of additional gluon exchange that are not power suppressed by the large scale and hence need to be taken into account systematically. The first type concerns gluons which emerge from the subgraph representing the partons in the right-moving proton and which attach to a hard-scattering subgraph, see Fig. 3. To leading-power accuracy, the effect of these gluons can be represented by Wilson lines that appear in the operators defining parton distributions and make them gauge invariant. For gauge boson pair production, each quark or antiquark field is to be multiplied by a Wilson line according to

$$q_j(z) \to [W(z,v)]_{jk}\, q_k(z)\,, \quad \bar{q}_j(z) \to \bar{q}_k(z)\, [W^\dagger(z,v)]_{kj} \tag{12}$$

with

$$W(z,v) = \mathrm{P}\exp\left[\, ig \int_0^\infty d\lambda\; vA^a(z-\lambda v)\, t^a \right], \tag{13}$$

where j and k are color indices, P denotes path ordering, and the sign convention for the coupling g is such that the covariant derivative reads $D^\mu = \partial^\mu + igA^{\mu,a}t^a$.

In order to avoid rapidity divergences in the parton distributions, we tilt v away from the light-cone [1]. This results in an additional parameter $\zeta^2 = (2pv)^2/|v^2|$ in the parton distributions for proton p. Their ζ dependence is connected with Sudakov logarithms and will be discussed in Section 7. As ζ is a measure of the plus-momentum of the proton [10], an adequate choice of ζ in cross section formulae is the hard scale Q.

The second type of unsuppressed gluon exchange is between the right- and left-moving partons as shown in Fig. 3, provided the gluons are soft and thus do not take partons far off shell. In processes with small observed transverse momenta in the final state, the effects of these gluons do not cancel. Provided that gluons in the Glauber region give no net contribution (which we currently cannot show but have to assume), soft gluon effects can be described by a so-called soft factor, which is defined in terms of vacuum expectation values of Wilson lines. Proper care needs to be taken to prevent double counting, because the Wilson lines $W(z,v)$ in the parton distributions include soft gluon momenta as well [1].

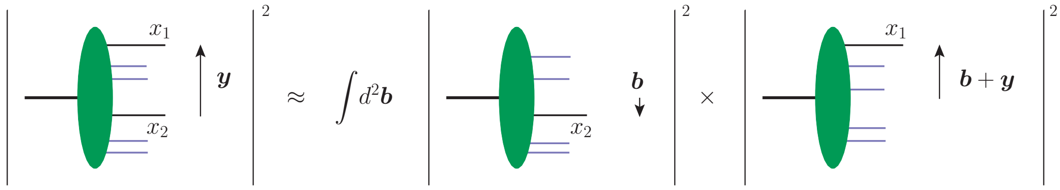

Figure 4: Illustration of the approximation (16) of a two-parton distribution in terms of single-parton distributions. Implicit in the figure is the representation of these distributions as squares of light-cone wave functions in the proton.

6 Connection with generalized parton distributions

In order to obtain a representation of the multiparton distributions in terms of GPDs, we insert a complete set of intermediate states $|X\rangle\langle X|$ between the operators $\mathcal{O}_{a_2}$ and $\mathcal{O}_{a_1}$ in the two-parton distributions in impact parameter space. This gives a product of single-parton operators sandwiched between a proton state and X. If we *assume* that the ground state dominates in the sum over all X and take the intermediate proton states in the impact parameter representation (9), we obtain a representation

$$F_{a_1,a_2}(x_i,\boldsymbol{z}_i,\boldsymbol{y};\boldsymbol{b}) \approx f_{a_2}(x_2,\boldsymbol{z}_2;\boldsymbol{b}+\tfrac{1}{2}x_1\boldsymbol{z}_1)f_{a_1}(x_1,\boldsymbol{z}_1;\boldsymbol{b}+\boldsymbol{y}-\tfrac{1}{2}x_2\boldsymbol{z}_2) \tag{14}$$

in terms of single-quark distributions

$$f_a(x,\boldsymbol{z};\boldsymbol{b}) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\boldsymbol{\Delta}} \int \frac{dz^-}{2\pi}\, e^{ixz^-p^+} \big\langle p^+, \tfrac{1}{2}\boldsymbol{\Delta}\big|\mathcal{O}_a(0,z)\big|p^+, -\tfrac{1}{2}\boldsymbol{\Delta}\big\rangle \tag{15}$$

in impact parameter space.

At $\boldsymbol{z}_i = \boldsymbol{0}$ the relation (14) involves only collinear distributions. Integrating over $\boldsymbol{b}$ we get

$$F_{a_1,a_2}(x_i,\boldsymbol{y}) \approx \int d^2\boldsymbol{b}\; f_{a_2}(x_2;\boldsymbol{b})\, f_{a_1}(x_1;\boldsymbol{b}+\boldsymbol{y})\,, \tag{16}$$

which has a straightforward physical interpretation as sketched in Fig. 4. In different guises, this relation is at the basis of most phenomenological studies and has long been used in the literature, see e.g. [11, 12, 13] and [14, 15].

We emphasize that the relations (14) and (16) are obtained by restricting a sum over all intermediate states to a single proton in a selected helicity state. We do not have a justification or a strong physical motivation for this restriction, other than stating a posteriori that it is tantamount to neglecting any correlation between both partons in the proton. It seems plausible to assume that this is a reasonable first approximation, at least in a certain region of variables, but one should not expect such an approximation to be very precise.

7 Sudakov logarithms

As is well known, transverse momenta $\boldsymbol{q}_i$ which are much smaller than the hard scale Q of a process give rise to Sudakov logarithms in the cross section. These logarithms must be resummed to all orders in perturbation theory, which for single gauge boson production can be done using the Collins-Soper-Sterman formalism [16]. We extended this formalism to gauge boson pair production in [3] and sketch the main results of our analysis in the following.

So far we have glossed over the color structure of double parton distributions. For two quarks one has distributions ${}^{1}F$ and ${}^{8}F$, which describe the cases where the two quark lines with momenta $k_1 - r/2$ and $k_1 + r/2$ in graph 1a are coupled to a color singlet or a color octet, respectively. In the cross section formulae (4) and (11) one should replace FF by ${}^{1}F\,{}^{1}F + {}^{8}F\,{}^{8}F$, with each $\hat{\sigma}_i$ including a color factor 1/3. The relations (14) and (16) hold for ${}^{1}F$.

The dependence of a two-quark distribution on the rapidity parameter ζ defined in Section 5 is governed by the differential equation

$$\frac{d}{d\log\zeta}\begin{pmatrix}{}^{1}F\\ {}^{8}F\end{pmatrix} = \big[G(x_1\zeta,\mu) + G(x_2\zeta,\mu) + K(\boldsymbol{z}_1,\mu) + K(\boldsymbol{z}_2,\mu)\big]\begin{pmatrix}{}^{1}F\\ {}^{8}F\end{pmatrix} + \mathbf{M}(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})\begin{pmatrix}{}^{1}F\\ {}^{8}F\end{pmatrix}, \tag{17}$$

where ${}^{1}F$ and ${}^{8}F$ depend on x_i, $\boldsymbol{z}_i$, $\boldsymbol{y}$ and ζ. They also depend on a renormalization scale μ, but we need not discuss this dependence here. The kernels G and K in (17) already appear in the Collins-Soper equation [10] for single-quark distributions. The matrix $\mathbf{M}$ mixes color singlet and color octet distributions and is μ independent, whereas the μ dependence of G and K is given by a renormalization group equation

$$\gamma_K\big(\alpha_s(\mu)\big) = -\frac{dK(\boldsymbol{z},\mu)}{d\log\mu} = \frac{dG(x\zeta,\mu)}{d\log\mu} \tag{18}$$

and thus cancels in $G + K$. Both K and $\mathbf{M}$ are due to soft gluon exchange and can be defined as vacuum matrix elements of Wilson line operators, similar to those discussed in Section 5. They can only be calculated perturbatively if the transverse distances on which they depend are sufficiently small.

The general solution of (17) can be written as

$$\begin{pmatrix}{}^{1}F(x_i,\boldsymbol{z}_i,\boldsymbol{y};\zeta)\\ {}^{8}F(x_i,\boldsymbol{z}_i,\boldsymbol{y};\zeta)\end{pmatrix} = e^{-S(x_1\zeta,\boldsymbol{z}_1,\boldsymbol{z}_2)-S(x_2\zeta,\boldsymbol{z}_1,\boldsymbol{z}_2)}\, e^{L\mathbf{M}(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})}\begin{pmatrix}{}^{1}F^{\mu_0}(x_i,\boldsymbol{z}_i,\boldsymbol{y})\\ {}^{8}F^{\mu_0}(x_i,\boldsymbol{z}_i,\boldsymbol{y})\end{pmatrix} \tag{19}$$

with

$$S(x\zeta,\boldsymbol{z}_1,\boldsymbol{z}_2) = -\frac{K(\boldsymbol{z}_1,\mu_0) + K(\boldsymbol{z}_2,\mu_0)}{2}\log\frac{x\zeta}{\mu_0} + \int_{\mu_0}^{x\zeta}\frac{d\mu}{\mu}\left[\gamma_K\big(\alpha_s(\mu)\big)\log\frac{x\zeta}{\mu} - G(\mu,\mu)\right] \tag{20}$$

and $L = \log(\sqrt{x_1 x_2}\,\zeta/\mu_0)$. The scale μ_0 specifies the initial condition of the differential equation (17), with a natural choice being $\mu_0 \propto 1\big/\sqrt{|\boldsymbol{z}_1||\boldsymbol{z}_2|}$.

The leading double logarithms of ζ/μ_0 in (19) come from the second term in (20), whereas terms involving K and $\mathbf{M}$ only contain single logarithms. The Sudakov exponent S also appears in the solution of the Collins-Soper equation for single-quark distributions [10],

$$f(x,\boldsymbol{z};\zeta) = e^{-S(x\zeta,\boldsymbol{z},\boldsymbol{z})}\, f^{\mu_0}(x,\boldsymbol{z}). \tag{21}$$

We thus obtain the important result that to double logarithmic accuracy the Sudakov factor for a multiparton distribution is the product of the Sudakov factors for single parton densities, both for color singlet and color octet distributions. A non-trivial cross talk between all partons, and in particular a mixing between color singlet and octet distributions occurs, however, at the level of single logarithms, which are known to be important for phenomenology.

8 Conclusions

We have studied several aspects of multiparton interactions in hadron-hadron collisions. Our theoretical framework is hard-scattering factorization, which requires a large virtuality or momentum transfer in each partonic scattering process but is valid in the full range of parton momentum fractions, i.e. not limited to small x.

The basic cross section formula for multiple interactions can be derived at tree level using standard hard-scattering approximations and has an intuitive geometrical interpretation in impact parameter space. We have shown that it can be formulated at the level of transverse-momentum dependent multiparton distributions, which permits a description of the transverse momenta of the particles produced in the hard scattering. This is particularly important because it is in transverse-momentum dependent cross sections that multiparton interactions are not power suppressed compared with single hard scattering.

To develop a reliable phenomenology, one needs information about the size and kinematic dependences of two-parton distributions. One can relate them to generalized parton distributions for single partons, which are experimentally accessible in exclusive scattering processes, but this requires an approximation whose reliability we cannot quantify.

To go from tree level to genuine factorization formulae, one must be able to sum certain types of collinear and soft gluon exchanges into Wilson lines. We argue in [3] that for double-scattering processes producing color-singlet particles this can be achieved using the methods that have been successfully applied to single Drell-Yan production [1, 10, 17, 18]. This also permits the resummation of Sudakov logarithms, with important results sketched in Section 7. The production of two electroweak gauge bosons thus emerges as a channel where the current perspectives for developing the theory look very good, and where different aspects of multiple interactions can hopefully be explored experimentally at LHC.

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