Multi-gluon correlations in the color glass condensate

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We discuss recent work on gluon correlations in high energy collisions and argue that they are most naturally understood in the Color Glass Condensate framework. We discuss first the dense-dense regime which is relevant for, e.g., the "ridge"-correlation observed at midrapidity in AA and pp collisions. We then describe recent progress in understanding two-particle correlations in the dilute-dense system, relevant for forward dihadron production in deuteron-gold collisions. This requires computing the energy dependence of higher point Wilson line correlators from the JIMWLK renormalization group equation. We find that the large N_c approximation used so far in the phenomenological literature is not very accurate. On the other hand a Gaussian finite N_c approximation is a surprisingly good approximation of the result from the full JIMWLK equation.

1   Introduction

The physics of high energy hadronic or nuclear collisions is dominated by the gluonic degrees of freedom of the colliding particles. These small x gluons form a dense nonlinear system that is, at high enough $\sqrt{s}$, best described as a classical color field and quantum fluctuations around it. The Color Glass Condensate (CGC, for reviews see [1, 2, 3, 4]) is an effective theory developed around this idea. It gives an universal description that can equally well be applied to small-x DIS as to dilute-dense (pA or forward AA) and dense-dense (AA or very high energy pp) hadronic collisions. The CGC is based on an effective description of large-x partons as a color charge density and small-x ones as a classical field radiated by these charges. The most convenient parametrization of the dominant gauge field is in terms of Wilson lines that describe the eikonal propagation of a projectile through it. The cutoff separating the large-x and small-x degrees of freedom is an arbitrary factorization scale, thus the requirement that physical observables cannot depend on it leads to a renormalization group equation. This nonlinear equation, known by the acronym JIMWLK [5, 6, 7, 11, 12], describes the evolution in rapidity of the probability distribution of the Wilson lines. It reduces, in a large N_c- mean field approximation, to the BK [13, 14] equation and further, in the dilute linear regime, to the BFKL one.

The nonlinear interactions of the small-x gluons generate dynamically a new transverse momentum scale, the saturation scale Q_s. The saturation scale grows with energy, as the increased density of gluons makes their interactions nonlinear at higher transverse momenta. At high enough energy the color glass condensate is thus a one-scale system, characterized by a dominant momentum scale Q_s that is hard enough to justify a weak coupling calculation.
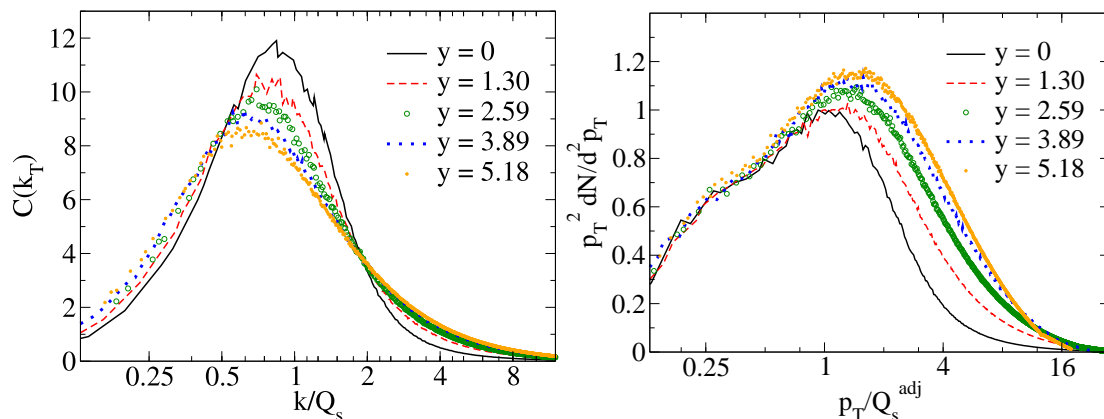
Figure 1: Left: evolution of unintegrated ("dipole") gluon distribution starting from an MV model initial condition. Right: The resulting gluon spectrum produced in a heavy ion collision. Note that this should be thought of as the gluon spectrum at midrapidity with y representing the logarithm of $\sqrt{s}$.

The scale Q_{s} dominates both the gluon spectrum and multigluon correlations. The nature of a unique saturation scale as both the typical gluon transverse momentum and as the correlation length $1/Q_{\mathrm{s}}$ differentiates the CGC qualitatively from the high-x part of the wavefunction.

2 Bulk particle production

One of the most unique aspects of the CGC framework is the prospect of understanding bulk quantities, such as particle multiplicities integrated over the whole p_T spectrum, in weak coupling. At leading order in α_{s}, calculating the spectrum of gluons produced in the initial stages of a heavy ion collision requires solving the time evolution of the non-perturbative strong classical gauge field (known as the glasma field [15]). This field is then, at late enough times, Fourier-decomposed and the modes interpreted as on-shell gluons. This calculation needs the Wilson lines corresponding to the individual colliding nuclei to provide the initial condition for the glasma fields [16], whose equations of motion must then be solved numerically [17, 18, 19]. In most of the numerical studies in the literature, the Wilson lines have been taken from the MV model [20, 21, 22], which is straightforward to implement numerically.

Only very recently [23] an actual numerical solution of the JIMWLK equation has been used to provide the initial condition for the evolution of the glasma fields. The JIMLWK equation is solved, in practice, by reformulating the RGE for the probability distribution of Wilson lines as a Langevin equation [24, 25, 26] for the rapidity dependence of an ensemble of Wilson lines. These configurations can then directly be used in the initial condition for the glasma fields. The results of this calculation are summarized in Fig. 1. On the left is plotted the correlation function of Wilson lines $U(\mathbf{x}_T)$ (i.e. dipole cross section or dipole gluon distribution) in momentum space:

$$C(\mathbf{k}_T) = \mathbf{k}_T^2 \int \mathrm{d}^2\mathbf{x}_T e^{i\mathbf{k}_T \cdot (\mathbf{x}_T - \mathbf{y}_T)} \frac{1}{N_{\mathrm{c}}} \left\langle \mathrm{Tr} U^\dagger(\mathbf{x}_T) U(\mathbf{y}_T) \right\rangle, \tag{1}$$
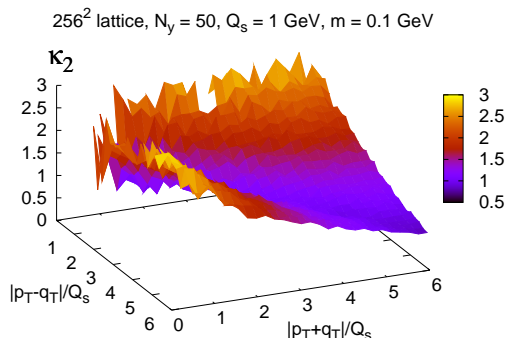
Figure 2: The two gluon correlation function in the MV model, from a full non-perturbative classical field calculation [27]. The near side $\mathbf{p}_T \approx \mathbf{q}_T$ and away side $\mathbf{p}_T \approx -\mathbf{q}_T$ peak structure is clearly seen.

starting from an MV model initial condition at $y = 0$. The main effect of the evolution (as expected from BK evolution) is the hardening of the unintegrated gluon distribution due to the development of a geometric scaling region for $k_T \gtrsim Q_s$. The effect on the gluon spectrum in the glasma is shown in Fig. 1 (right): also the spectrum of gluons in the glasma gets harder with increasing energy. Note that, as advocated above, the spectrum is integrable and the total gluon multiplicity finite without any additional cut-offs. This is a non-trivial consequence of the non-linear interactions of the gluonic field.

3   Correlations in the dense-dense limit

The classical color field is a multi-gluon system, and as such has naturally built in correlations that are long range in rapidity. The formalism for computing the observable correlations in a collision of two high density gluonic systems in the CGC framework was developed in [28, 29, 30]. The essential power-counting argument can be summarized as follows. In the CGC, and the glasma, the gluon fields are non-perturbatively strong; $A_\mu \sim 1/g$. Therefore the correlations arising from quantum fluctuations around the field, which give the typical correlations in a perturbative calculation, are actually subleading compared to the ones arising from the probability distribution of Wilson lines. Physically this means that the dominant correlations are those that are enhanced by large logarithms of x, present already in the wavefunctions of the colliding projectiles and re-summed by the JIMWLK evolution.

The only nonperturbative calculation of the double inclusive gluon spectrum has been performed in Ref. [27]. The main result is shown in Fig. 2 in terms of the quantity

$$\kappa_2(\mathbf{p}_T, \mathbf{q}_T) = S_\perp Q_s^2 \frac{C_2(\mathbf{p}_T, \mathbf{q}_T)}{\left\langle \frac{\mathrm{d}N}{\mathrm{d}y_p\, \mathrm{d}^2\mathbf{p}_T} \right\rangle \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_q\, \mathrm{d}^2\mathbf{q}_T} \right\rangle}, \tag{2}$$

where the correlated double inclusive gluon spectrum is defined as

$$C_2(\mathbf{p}_T, \mathbf{q}_T) = \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_p\, \mathrm{d}^2\mathbf{p}_T} \frac{\mathrm{d}N}{\mathrm{d}y_q\, \mathrm{d}^2\mathbf{q}_T} \right\rangle - \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_p\, \mathrm{d}^2\mathbf{p}_T} \right\rangle \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_q\, \mathrm{d}^2\mathbf{q}_T} \right\rangle. \tag{3}$$
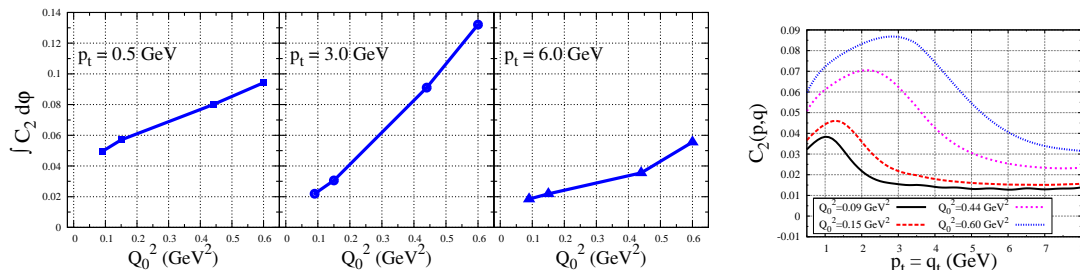
Figure 3: Left: Near side correlated multiplicity (integrated over azimuthal angle) as a function of the saturation scale the CGC calculation. Right transverse momentum spectrum of the correlated secondary particles. Figures from Ref. [31].

Here $S_\perp$ is the transverse area of the collision system. The result displays two characteristic main features. Firstly the double inclusive spectrum, scaled with the number of correlation regions $S_\perp Q_s^2$, is of order one. Secondly one observes a characteristic enhancement in the back-to-back $\mathbf{p}_T \sim -\mathbf{q}_T$ and near side $\mathbf{p}_T \sim \mathbf{q}_T$ regions. While the former is present also in the dilute limit, due to momentum conservation, the latter is a genuine nontrivial high gluon density effect that has no counterpart in a purely perturbative (or string fragmentation, for that matter) picture. This is the basis of the CGC contribution to the "ridge" correlation, a structure at small azimuthal angle and large rapidity separation, that has been observed in AA and pp collisions at high energy.

The calculation of Ref. [27] uses the MV model for the Wilson line distribution. Work on repeating the calculation using the Wilson line configurations from JIMWLK evolution is still ongoing. In the meanwhile the effects of high energy evolution have been analyzed in a k_T-factorized approximation [31, 32], which is valid for particle production at $p_T \gtrsim Q_s$ [33]. The k_T-factorized approximation for the double inclusive gluon spectrum is

$$
\begin{aligned}
C(\mathbf{p}_T, \mathbf{q}_T) = &\frac{\alpha_s^2}{4\pi^{10}} \frac{N_c^2 S_\perp}{(N_c^2 - 1)^3 \, \mathbf{p}_T^2 \mathbf{q}_T^2} \\
&\times \Bigg\{ \int \mathrm{d}^2\mathbf{k}_T \Phi_{A_1}^2(y_p, \mathbf{k}_T) \Phi_{A_2}(y_p, \mathbf{p}_T - \mathbf{k}_T) \Big[\Phi_{A_2}(y_q, \mathbf{q}_T + \mathbf{k}_T) + \Phi_{A_2}(y_q, \mathbf{q}_T - \mathbf{k}_T) \Big] \\
&+ \Phi_{A_2}^2(y_q, \mathbf{k}_T) \Phi_{A_1}(y_p, \mathbf{p}_T - \mathbf{k}_T) \Big[\Phi_{A_1}(y_q, \mathbf{q}_T + \mathbf{k}_T) + \Phi_{A_1}(y_q, \mathbf{q}_T - \mathbf{k}_T) \Big] \Bigg\}. \quad (4)
\end{aligned}
$$

Here Φ_{A_i}, the unintegrated gluon distribution in nucleus i, is related to the correlator $C(\mathbf{k}_T)$ of Eq. (1) simply by $\Phi(\mathbf{k}_T) = C(\mathbf{k}_T)/(4\alpha_s N_c)$. In [31, 32] it is obtained from the mean field BK equation. The qualitative features of the correlation obtained using Eq. (4) are illustrated in Fig. 3. What is shown is the correlation integrated over $\Delta\varphi$, the azimuthal angle separation between the two produced gluons. It rises for increasing Q_s, which corresponds to increasing centrality. Also the dependence of the correlation on the p_T-cutoff, first rising and then decreasing, matches that seen in the CMS data [34]. A more recent detailed analysis [35] confirms the conclusions reached here on a more quantitative level.
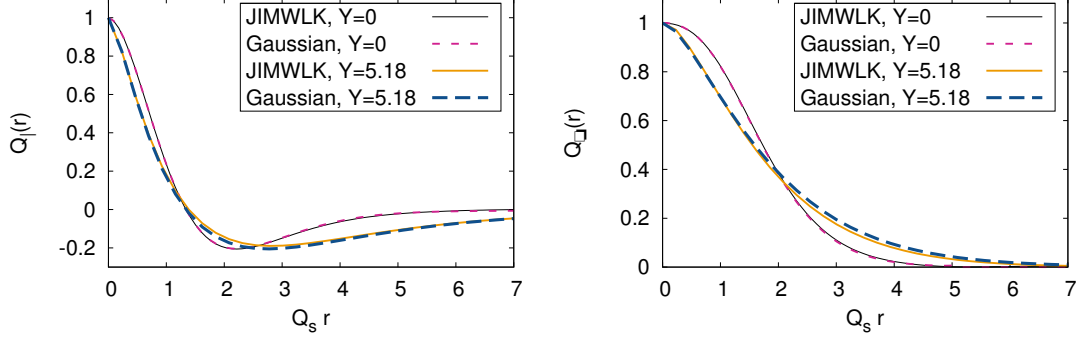
Figure 4: The JIMWLK result for the quadrupole correlator compared to the Gaussian approximation. Shown are the initial condition (MV model) at $y = 0$ and the result after 5.18 units of evolution in rapidity, for the "line" (left) and "square" (right) coordinate configurations. Figures from Ref. [36].

4 Correlations in the dense-dilute limit

One of the more striking signals of saturation physics at RHIC is the observed broadening of the away-side peak in di-hadron correlations in forward deuteron-gold scattering [37, 38]. Our theoretical starting point in analyzing these correlations is to consider the high-x parton from the proton required to produce two relatively large p_T particles at forward rapidity in the final state. We assume the high-x particle to be a quark, since the valence distribution dominates at high x. In order to have a correlated production of two particles this quark must then radiate a gluon, carrying a fraction z of its longitudinal momentum. To leading order we then have a picture of a quark-gluon system propagating (eikonally in our high energy approximation) through the target nucleus. The eikonal matrix element is given by Wilson lines in the appropriate representation for the two particles, leading to a double inclusive cross section

$$\frac{\mathrm{d}\sigma^{qA \to qgX}}{\mathrm{d}^3k_1 \ \mathrm{d}^3k_2} \propto \frac{\alpha_s N_c}{2} \int_{\mathbf{x}_T, \bar{\mathbf{x}}_T, \mathbf{y}_T, \bar{\mathbf{y}}_T} e^{-i\mathbf{k}_{T1}\cdot(\mathbf{x}_T - \bar{\mathbf{x}}_T)} \ e^{-i\mathbf{k}_{T2}\cdot(\mathbf{y}_T - \bar{\mathbf{y}}_T)} \mathcal{F}(\bar{\mathbf{x}}_T - \bar{\mathbf{y}}_T, \mathbf{x}_T - \mathbf{y}_T)$$

$$\left\langle \hat{Q}(\mathbf{y}_T, \bar{\mathbf{y}}_T, \bar{\mathbf{x}}_T, \mathbf{x}_T) \ \hat{D}(\mathbf{x}_T, \bar{\mathbf{x}}_T) - \hat{D}(\mathbf{y}_T, \mathbf{x}_T)\hat{D}(\mathbf{x}_T, \bar{\mathbf{z}}_T) - \hat{D}(\mathbf{z}_T, \bar{\mathbf{x}}_T)\hat{D}(\bar{\mathbf{x}}_T, \bar{\mathbf{y}}_T) \right.$$

$$\left. + \frac{C_F}{N_c}\hat{D}(\mathbf{z}_T, \bar{\mathbf{z}}_T) + \frac{1}{N_c^2}\left(\hat{D}(\mathbf{y}_T, \bar{\mathbf{z}}_T) + \hat{D}(\mathbf{z}_T, \bar{\mathbf{y}}_T) - \hat{D}(\mathbf{y}_T, \bar{\mathbf{y}}_T)\right) \right\rangle, \quad (5)$$

with $\mathbf{z}_T = z\mathbf{x}_T + (1-z)\mathbf{y}_T$ and likewise, $\bar{\mathbf{z}}_T = z\bar{\mathbf{x}}_T + (1-z)\bar{\mathbf{y}}_T$. The kinematical factors denoted by $\mathcal{F}$ can be calculated in light cone perturbation theory [39]. What is then needed to describe the target are the expectation values of the dipole, quadrupole, and sextupole Wilson
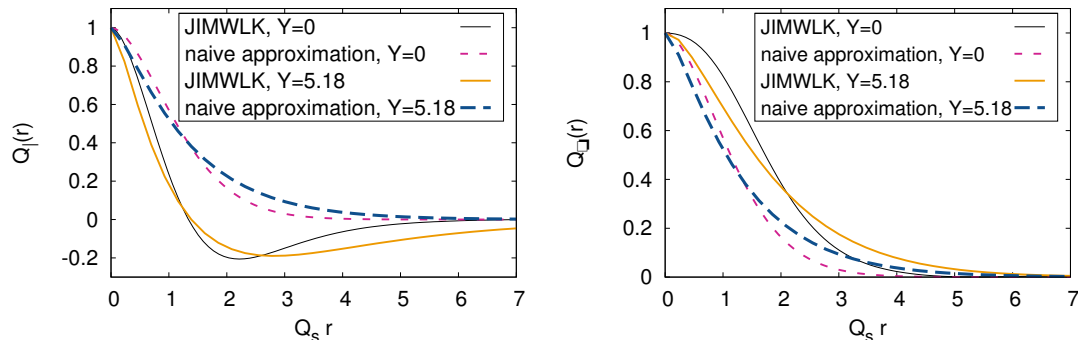
Figure 5: The JIMWLK result for the quadrupole correlator compared to the "naive large N_c" approximation. Shown are the initial condition (MV model) at $y = 0$ and the result after 5.18 units of evolution in rapidity, for the "line" (left) and "square" (right) coordinate configurations. Figures from Ref. [36].

line operators $D = \langle \hat{D} \rangle$, $Q = \langle \hat{Q} \rangle$, and $\langle \hat{D}\hat{Q} \rangle$ defined by

$$\hat{D}(\mathbf{x}_T, \mathbf{y}_T) = \frac{1}{N_\mathrm{c}} \mathrm{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T), \quad \hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) = \frac{1}{N_\mathrm{c}} \mathrm{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) U(\mathbf{u}_T) U^\dagger(\mathbf{v}_T).$$

(6)

For practical phenomenological work it would be extremely convenient to be able to express these higher point correlators in terms of the dipole, which is straightforward to obtain from the BK equation. In the phenomenological literature so far [40, 41] this has been done using a "naive large N_c" approximation as

$$Q(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) \underset{N_\mathrm{c} \to \infty}{\approx} \frac{1}{2} \big(D(\mathbf{x}_T, \mathbf{y}_T) D(\mathbf{u}_T, \mathbf{v}_T) + D(\mathbf{x}_T, \mathbf{v}_T) D(\mathbf{u}_T, \mathbf{y}_T) \rangle \big).$$

(7)

A more elaborate scheme would be a "Gaussian" approximation ("Gaussian truncation" in [42]), where one assumes the relation between the higher point functions and the dipole to be the same as in the (Gaussian) MV model. The expectation value of the quadrupole operator in the MV model has been derived e.g. in Ref. [43]; the one for the 6-point function is unfortunately not known yet.

In Ref. [36] we have studied the validity of these approximations by comparing them numerically to the solution of the JIMWLK equation, following the conjecture of Ref. [44] that the JIMWLK result should significantly deviate from both of them. As studying the full 8-dimensional phase space for the quadrupole operator would be cumbersome, we have concentrated on two special coordinate configurations. The "line" configuration is defined by taking $\mathbf{u}_T = \mathbf{x}_T$ and $\mathbf{v}_T = \mathbf{y}_T$, with $r = |\mathbf{x}_T - \mathbf{y}_T|$ and the "square" by taking $\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T$ as the corners of a square with side r. For these particular configurations the "naive large N_c" approximation reduces to

$$Q_|^{\mathrm{naive}}(r) \approx Q_\square^{\mathrm{naive}}(r) \approx D(r)^2$$

(8)

and the Gaussian approximation to

$$Q_|(r) \approx \frac{N_c + 1}{2} \Big(D(r) \Big)^{2\frac{N_c+2}{N_c+1}} - \frac{N_c - 1}{2} \Big(D(r) \Big)^{2\frac{N_c-2}{N_c-1}} \tag{9}$$

$$Q_\square(r) \approx (D(r))^2 \left[\frac{N_c + 1}{2} \left(\frac{D(r)}{D(\sqrt{2}r)} \right)^{\frac{2}{N_c+1}} - \frac{N_c - 1}{2} \left(\frac{D(\sqrt{2}r)}{D(r)} \right)^{\frac{2}{N_c-1}} \right]. \tag{10}$$

Our results [36] for the quadrupole expectation value are shown in Figs. 4 and 5, with a comparison of the initial and evolved (for 5.18 units in y) results to the approximations. The MV-model initial condition $y = 0$ satisfies the Gaussian approximation by construction. Figure 4 shows that the Gaussian approximation is still surprisingly well conserved by the evolution. A possible explanation for this based on the structure of the JIMWLK equation has recently been proposed in [45, 46]. The naive large N_c approximation used in some phenomenological works, on the other hand, fails already at the initial condition, as shown in Fig. 5. This stresses the importance the various SU(3) group structure constraints violated in this approach. Crucially for the phenomenological consequences, even the characteristic length/momentum scale differs by factor ~ 2 from the actual result.

This result does not yet fully address the effect on the measurable cross section. For that one must perform the integrals in (5) to go from the position space correlator to the momentum space one. Also additional effects such as inelastic contributions [47] and high-x effects in the deuteron (as compared to the proton) must be included, as discussed in [48]. This full calculation is still a work in progress.

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