# Multiparton interactions in Herwig++

*Stefan Gieseke*[1,*], *Christian Röhr*[1], *Andrzej Síodmok*[1,2]

[1]Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany
[2]University of Manchester, Manchester, UK

**DOI:** http://dx.doi.org/10.3204/DESY-PROC-2012-03/45

We summarize the implementation of a model for multiple partonic interactions in HERWIG++. Some studies of colour reconnection models are presented and conclusions are drawn regarding the underlying physics.

## 1 Introduction

Tevatron and early LHC data have shown the importance of Multiple Partonic Interaction (MPI) models in order to give an accurate Monte Carlo simulation of minimum bias events and the underlying event in hard partonic collisions [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The major Monte Carlo event generators HERWIG [14], PYTHIA [15, 16] and SHERPA [17] by now all have an MPI model in order to simulate the underlying event. In this contribution, we summarize the MPI model in the event generator HERWIG++.

## 2 MPI model in Herwig++

The starting point of our model is the observation that the hard inclusive cross section for dijet production,

$$\sigma^{\rm inc}(s;p_t^{\rm min}) = \sum_{i,j} \int_{p_t^{\rm min\,2}} {\rm d}p_t^2 f_{i/h_1}(x_1,\mu^2) \otimes \frac{{\rm d}\hat{\sigma}_{i,j}}{{\rm d}p_t^2} \otimes f_{j/h_2}(x_2,\mu^2)\,,$$

eventually exceeds the total cross section at hadron colliders. This leads us to the interpretation that in fact the inclusive cross section counts not only single hard events but all hard events that occur in parallel during the very same hadron–hadron collision. With the key assumption of *independent* multiple partonic interactions we may interpret this as

$$\sigma^{\rm inc} = \bar{n}\,\sigma_{\rm inel}\ ,$$

with the average number of hard scatters $\bar{n}$ and $\sigma_{\rm inel}$ the 'unitarized' inelastic cross section. With statistically independent scatters (eikonal approximation) we are lead to a Poisson distribution of the number $m$ of additional scatters,

$$P_m(\vec{b},s) = \frac{\bar{n}(\vec{b},s)^m}{m!} {\rm e}^{-\bar{n}(\vec{b},s)}\ .$$

*Speaker

Hence, we get $\sigma_{\rm inel}$:

$$\sigma_{\rm inel} = \int {\rm d}^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b},s) = \int {\rm d}^2\vec{b} \left(1 - {\rm e}^{-\bar{n}(\vec{b},s)}\right) \ .$$

Comparing with a unitarized scattering amplitude in scattering theory in the eikonal approximation $a(\vec{b},s) = \frac{1}{2i}({\rm e}^{-\chi(\vec{b},s)} - 1)$ we can identify the eikonal function $\chi(\vec{b},s)$,

$$\sigma_{\rm inel} = \int {\rm d}^2\vec{b} \left(1 - {\rm e}^{-2\chi(\vec{b},s)}\right) \qquad \Rightarrow \quad \chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) \ .$$

The eikonal function or the average number of scatters is calculated in the parton model as

$$\begin{aligned}\bar{n}(\vec{b},s) &= L_{\rm partons}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int {\rm d}p_t^2 \frac{{\rm d}\hat{\sigma}_{ij}}{{\rm d}p_t^2} \\ &= \sum_{ij} \frac{1}{1+\delta_{ij}} \int {\rm d}x_1 {\rm d}x_2 \int {\rm d}^2\vec{b}' \int {\rm d}p_t^2 \frac{{\rm d}\hat{\sigma}_{ij}}{{\rm d}p_t^2} D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|) \ .\end{aligned}$$

We assume the momentum⊗transverse space distributions to factorize,

$$D_{i/A}(x,p_t^2,|\vec{b}|) = f_{i/A}(x,p_t^2)\, G_A(|\vec{b}|) \ .$$

Here, $f_{i/A}(x,p_t^2)$ are the ordinary parton distribution functions and the spatial distribution of partons $G_A(|\vec{b}|)$, which is obtained from elastic $e^-p$ scattering, so we get

$$\bar{n}(\vec{b},s) = \sum_{ij} \frac{1}{1+\delta_{ij}} \int {\rm d}x_1 {\rm d}x_2 \int {\rm d}^2\vec{b}' \int {\rm d}p_t^2 \frac{{\rm d}\hat{\sigma}_{ij}}{{\rm d}p_t^2} f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|) \ .$$

Now we can carry out the $\vec{b}'$ integration, factor off the overlap function $A(\vec{b})$ and identify the inclusive cross section,

$$\bar{n}(\vec{b},s) = A(\vec{b})\sigma^{\rm inc}(s;p_t^{\rm min}) \ , \qquad A(\vec{b}) = \int {\rm d}^2\vec{b}'\, G_A(|\vec{b}'|) G_B(|\vec{b}-\vec{b}'|) \ .$$

So, finally we arrive at the following expression for the eikonal,

$$\chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) = \tfrac{1}{2}A(\vec{b})\sigma^{\rm inc}(s;p_t^{\rm min}) \ .$$

This model of independent partonic interactions was first implemented in Pythia [18] and similarly in the JIMMY add–on to the old HERWIG program [19]. In all models, first a number of additional hard scatters is computed according to the probability distribution resulting from the Poissonian with $\bar{n}$ as calculated above. The additional hard scatters are simulated as a primary hard scatter. The differences of the available implementations are hidden in the details of the application of the parton shower to the various hard processes and the treatment of the parton distribution functions for the additional scatters and the overlap function. These, as inclusive quantities, are not anymore well–defined when several partons are extracted from the proton and hence the remnant extraction has to be modelled. The underlying event model in SHERPA [17] is quite similar but will be replaced by a new approach soon. The current model in PYTHIA follows the idea of interleaved partonic interactions and showering and differs quite significantly from the model discussed here [20, 21].

In HERWIG++ this model has been implemented and released as well [22]. The only two parameters of the model are the minimum transverse momentum $p_t^{\min}$ of the additional hard scatters and the parameter $\mu^2$ that characterizes the inverse proton radius in the overlap function $A(\vec{b};\mu^2)$. At that stage good agreement with available Tevatron data on the underlying event was found.

As a second step, also soft interactions were implemented [23]. The model is based on adding a soft term to the eikonal function. The simplest possibility, using the same overlap function as for hard interactions, was studied but discarded [24]. The current ansatz we use is the same functional form of the overlap function with a second independent parameter $\mu^2_{\rm soft}$ and the soft cross section $\sigma_{\rm soft}$. The differential distribution of soft scatters in transverse momentum is based on a Gaussian below $p_t^{\min}$, which has been introduced above. The parameters of the Gaussian are fixed by demanding a smooth continuation from hard to soft transverse momenta and the soft cross section. The additional parameter $\mu^2_{\rm soft}$ is fixed with the elastic slope parameter that can also be calculated within our model. With this extension of the model a good description of Tevatron underlying event data has been found.

## 3 Colour reconnection model

First observations of minimum bias events at the LHC [3] have shown that an important model detail is missing: a colour reconnection model. The pseudorapidity distribution of charged particles comes out very much peaked towards the forward regions, opposed to the rapidity plateau found in the data. Hints towards the colour structure were found, as only one model parameter, namely the probability that a soft interaction is colour disrupted from the rest of the hard event or not, has shown some sensitivity to this distribution.

The need for a colour reconnection model is quite clear from the point of view of the colour preconfinement property of QCD which is the basis of the hadronization model in HERWIG. Preconfinement tells us that partons that are close in phase space, particularly in momentum space, will most often also be neighboured in colour space. This property is given in parton shower models as the colour structure evolution during parton shower evolution retains the history of the colour charge. In the multiple interaction model, however, this is not the case. The partons from the additional hard interactions are all extracted from the proton remnants without respecting a possible colour flow of the event as a whole. Hence, it is possible that jets from different hard events in the multiple interaction chain can end up in similar directions in momentum space and therefore should have been created closely in colour space as well. As the multiple interaction model sets up the colour connection between different hard events *ad hoc* we should be able to improve the description of the hadronic final state with colour connections that resemble our QCD picture of preconfinement, which now have to be modelled.

The goal of any colour reconnection model is to ensure that all (or most) colour–anticolour charge pairs end up closely in phase space by some criterion. In our case we define closeness of pairs as having a small invariant mass, or in the HERWIG case, a small cluster mass. We may say that a *colour length*

$$\lambda = \sum_{\text{pairs } ij} m_{ij}^2$$

should be minimized. Here, $ij$ are all $3 \otimes \bar{3}$ pairs that may form a colour neutral cluster in the hadronization model. In practice it is computationally too expensive to find the true minimum for a full partonic final state at the LHC with $\mathcal{O}(100)$ partons. Furthermore, we may not really want to find the true minimum as the colour line picture we use is only true in the limit of infinite colours and we may very well have fluctuations about the minimum.

Based on this physical picture there are two colour reconnection models implemented in HERWIG++. In the *plain* model, which is similar to the model in Fortran HERWIG, all clusters pairs are iterated in a random order and whenever a swap of colours is preferable, i.e. $\lambda$ becomes smaller, this is done with a given probability which is the only model parameter. This model has shown to give the desired results and is implemented in HERWIG++.

One of the shortcomings of this model is that it is not so easy to assess how close we come to the true minimum and which clusters are taken into account as only a single random stream of clusters is presented to the model. In order to study its physical significance we have implemented a second model that we would like to discuss here a bit more in detail. Here, we try to minimize the colour length $\lambda$ with a Metropolis algorithm. This has the advantage that the minimization procedure is quite physical and can be controlled by the parameters of the model which have some kind of thermodynamical counterpart. A similar model was discussed also for PYTHIA [25].

The algorithm is, again, based on random colour rearrangements between randomly chosen clusters, but now in a more controlled way.

- First an initial 'temperature' $T$ is chosen which is related to a typical value of $\Delta\lambda = \lambda_{\text{new}} - \lambda_{\text{old}}$.
- Based on this temperature, we try a certain number of random colour rearrangements, proportional to the total number of available clusters.
- For each rearrangement $\Delta\lambda$ is computed. If $\Delta\lambda < 0$ the new configuration is accepted. If $\Delta\lambda > 0$, we only accept the new configuration with probability $\exp(-\Delta\lambda/T)$.
- After a number of attempts the temperature is decreased by a given factor.
- The algorithm terminates if no more rearrangements were made or a maximum number of loops has been passed.

It is known well, that if run with a suitable set of parameters this algorithm will come very close to the true minimum. The key point is that the algorithm also allows for fluctuations in the wrong direction, controlled by the temperature parameter, in order to also look for minima in previously unexplored paths. We could confirm this for a few examples where we also determined the true minimum by brute force. For our application, however, we left the parameter choice open and had them determined by tuning to minimum bias and underlying event data. The good results were also shown at this workshop [26].

The result is quite interesting, as actually quite small initial temperatures and a quick reduction of temperature is preferred by the model. So, effectively the model is making random colour rearrangements in more or less a *single* stream of clusters, chosen randomly. This is exactly what happens in our plain reconnection model. So, physically, the model does not really want to find the true minimum but rather wants to keep some non–optimal colour correlations due to deviations from the large $N_c$ limit or other fluctuations of colour.

In addition we have studied a few properties of the model performance that are relevant for the formation of the hadronic final state. Fig. 1 shows the relative change in colour length for

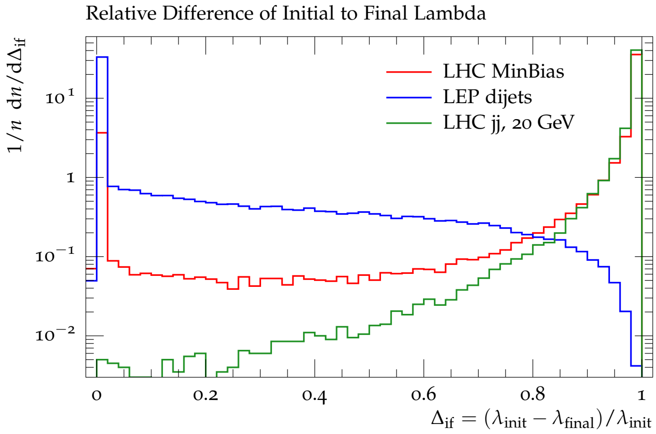


Figure 1: Relative change of colour length in various situations

two types of events at the LHC, namely minimum bias events and dijets ($p_T > 20\,\mathrm{GeV}$), and LEP events. Clearly the colour length presented to the model at LEP is already close to the minimum as there the simulation of whole hadronic final state is controlled by QCD. At the LHC the situation is the opposite as there the underlying events play a role, and possibly also non–perturbative effects from the hadron remnants.

# 4 Conclusions

We have summarized the multiple partonic interaction models in HERWIG++ and described the ongoing work on colour reconnections in some detail. The colour reconnection models have found to be vital for the description of LHC data and we have studied the physical significance of our model with a second model for a controlled minimization of the colour length. This study has confirmed our physical picture of an initial lack of colour preconfinement in modelling hadron collider events with MPI.

**Acknowledgements** We thank the organizers for a very pleasant and fruitful workshop. This work has been supported by the Helmholtz Alliance “Physics at the Terascale”.

# References

[1] T. Affolder et al., "Charged jet evolution and the underlying event in $p\bar{p}$ collisions at 1.8 TeV", Phys.Rev. D **65** (2002) 092002.

[2] T. Aaltonen et al., "Studying the Underlying Event in Drell-Yan and High Transverse Momentum Jet Production at the Tevatron", Phys.Rev. D **82** (2010) 034001. (arXiv:1003.3146).

[3] G. Aad et al., "Charged-particle multiplicities in pp interactions at $\sqrt{s} = 900$ GeV measured with the ATLAS detector at the LHC", Phys.Lett. B **688** (2010) 21–42. (arXiv:1003.3124).

[4] G. Aad et al., "Measurement of underlying event characteristics using charged particles in pp collisions at $\sqrt{s} = 900 GeV$ and 7 TeV with the ATLAS detector", Phys. Rev. D **83** (2011) 112001. (arXiv:1012.0791).

[5] G. Aad et al., "Charged-particle multiplicities in pp interactions measured with the ATLAS detector at the LHC", New J.Phys. **13** (2011) 053033. (arXiv:1012.5104).

[6] G. Aad et al., "Measurements of underlying-event properties using neutral and charged particles in *pp* collisions at 900 GeV and 7 TeV with the ATLAS detector at the LHC", Eur.Phys.J. C **71** (2011) 1636. (arXiv:1103.1816).

[7] V. Khachatryan et al., "Transverse momentum and pseudorapidity distributions of charged hadrons in pp collisions at $\sqrt{s} = 0.9$ and 2.36 TeV", JHEP **1002** (2010) 041. (arXiv:1002.0621).

[8] V. Khachatryan et al., "Transverse-momentum and pseudorapidity distributions of charged hadrons in pp collisions at sqrt(s) = 7 TeV", Phys.Rev.Lett. **105** (2010) 022002. (arXiv:1005.3299).

[9] V. Khachatryan et al., "First Measurement of Hadronic Event Shapes in pp Collisions at sqrt(s)=7 TeV", Phys.Lett. B **699** (2011) 48–67. (arXiv:1102.0068).

[10] S. Chatrchyan et al., "Measurement of the Underlying Event Activity at the LHC with $\sqrt{s} = 7$ TeV and Comparison with $\sqrt{s} = 0.9$ TeV", JHEP **1109** (2011) 109. (arXiv:1107.0330).

[11] K. Aamodt et al., "Charged-particle multiplicity measurement in proton-proton collisions at sqrt(s) = 0.9 and 2.36 TeV with ALICE at LHC", Eur.Phys.J. C **68** (2010) 89–108. (arXiv:1004.3034).

[12] K. Aamodt et al., "Charged-particle multiplicity measurement in proton-proton collisions at sqrt(s) = 7 TeV with ALICE at LHC", Eur.Phys.J. C **68** (2010) 345–354. (arXiv:1004.3514).

[13] K. Aamodt et al., "Transverse momentum spectra of charged particles in proton-proton collisions at $\sqrt{s} =$ 900 GeV with ALICE at the LHC", Phys.Lett. B **693** (2010) 53–68. (arXiv:1007.0719).

[14] M. Bähr et al., "Herwig++ Physics and Manual", Eur. Phys. J. C **58** (2008) 639–707. (arXiv:0803.0883).

[15] T. Sjöstrand, S. Mrenna, P. Skands, "PYTHIA 6.4 Physics and Manual", JHEP **05** (2006) 026. (arXiv:hep-ph/0603175).

[16] T. Sjöstrand, S. Mrenna, P. Skands, "A Brief Introduction to PYTHIA 8.1", Comput. Phys. Commun. **178** (2008) 852–867. (arXiv:0710.3820).

[17] T. Gleisberg et al., "Event generation with SHERPA 1.1", JHEP **02** (2009) 007. (arXiv:0811.4622).

[18] T. Sjostrand, M. van Zijl, "A Multiple Interaction Model for the Event Structure in Hadron Collisions", Phys.Rev. D **36** (1987) 2019.

[19] J. Butterworth, J. R. Forshaw, M. Seymour, "Multiparton interactions in photoproduction at HERA", Z.Phys. C **72** (1996) 637–646. (arXiv:hep-ph/9601371).

[20] T. Sjöstrand, P. Z. Skands, "Multiple interactions and the structure of beam remnants", JHEP **03** (2004) 053. (arXiv:hep-ph/0402078).

[21] T. Sjöstrand, P. Z. Skands, "Transverse-momentum-ordered showers and interleaved multiple interactions", Eur. Phys. J. C **39** (2005) 129–154. (arXiv:hep-ph/0408302).

[22] M. Bähr, S. Gieseke, M. H. Seymour, "Simulation of multiple partonic interactions in Herwig++", JHEP **07** (2008) 076. (arXiv:0803.3633).

[23] M. Bähr, J. M. Butterworth, S. Gieseke, M. H. Seymour, "Soft interactions in Herwig++" arXiv:0905.4671.

[24] M. Bähr, J. M. Butterworth, M. H. Seymour, "The Underlying Event and the Total Cross Section from Tevatron to the LHC", JHEP **01** (2009) 065. (arXiv:0806.2949).

[25] P. Z. Skands, D. Wicke, "Non-perturbative QCD effects and the top mass at the Tevatron", Eur.Phys.J. C **52** (2007) 133–140. (arXiv:hep-ph/0703081).

[26] A. Siodmok, "Tuning of multiple partonic interactions model in Herwig++ using the early LHC data", these proceedings.