Pinning down the self-similar gluon distribution from momentum sum rule

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The concept of self-similarity in the contemporary physics of Deep Inelastic Scattering (DIS) was introduced in 2002 when Lastovicka proposed a functional form of the structure function $F_2(x, Q^2)$ at small x. In this paper, we use the original Lastovicka's model to compute the momentum sum rule $\int_0^1 \left(F_2(x, Q^2) + G(x, Q^2) \right) dx = 1$, which relates the fraction of momentum carried by quarks and gluons inside the proton. There exists a singularity at $x \approx 0.019$ in this model. Therefore, we use Cauchy's principal value integration method to construct the fraction of momentum carried by quarks and gluons defined as $\langle x \rangle_q = \int_0^1 F_2(x, Q^2) dx$ and $\langle x \rangle_g = \int_0^1 G(x, Q^2) dx$ respectively. We suggest that the relation between quarks and gluons is given as $G(x, Q^2) = x^{-\lambda(Q^2)} F_2(x, Q^2)$, where $\lambda(Q^2)$ is the function of Q^2.

1 Introduction

Self-similarity is a possible feature of multi-partons inside a proton at small Bjorken-x, first suggested by Lastovicka of DESY, Germany in the year 2002 [1]. Based on this notion, a form of structure function $F_2(x, Q^2)$ was proposed which could explain the H1 and ZEUS data for $6.2 \times 10^{-7} \leqslant x \leqslant 0.01$. In the present work, we use the momentum sum rule [2]

$$\int_0^1 \left(F_2(x, Q^2) + G(x, Q^2) \right) dx = 1 \tag{1}$$

and explore the possibility of pinning down the gluon distribution $G(x, Q^2)$ from it. The momentum sum rule is to be satisfied by any reasonable model of structure function. However, such requirement calls for information about the entire physical regime of $x \, (0 \leqslant x \leqslant 1)$. It is also well-known that the gluon PDFs are, on the other hand, not directly measurable although there are several indirect ways of measuring like the longitudinal structure function or the slope and curvature of the structure function. We summarize the preliminary results of our analysis.

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2   Formalism

A description of the proton structure function $F_2(x, Q^2)$ reflecting self-similarity was proposed with a few parameters which were fitted from the HERA data [3, 4]. The concept of self-similarity, when applied to proton structure, leads to a simple parameterization of quark densities within the proton. The structure function (using two magnification factors $\dfrac{1}{x}$ and $\left(1 + \dfrac{k_t^2}{Q_0^2}\right)$) is subsequently obtained as:

$$F_2(x, Q^2) = \frac{e^{D_0} Q_0^2 \, x^{-D_2+1}}{1 + D_3 - D_1 \log x} \left(x^{-D_1 \log\left(1 + \frac{Q^2}{Q_0^2}\right)} \left(1 + \frac{Q^2}{Q_0^2}\right)^{D_3+1} - 1 \right) \tag{2}$$

where the parameters are

$$
\begin{aligned}
D_0 &= 0.339 \pm 0.145 \\
D_1 &= 0.073 \pm 0.001 \\
D_2 &= 1.013 \pm 0.01 \\
D_3 &= -1.287 \pm 0.01 \\
Q_0^2 &= 0.062 \pm 0.01 \text{ GeV}^2
\end{aligned}
\tag{3}
$$

Here D_1, D_2 and D_3 are the parameters identified as the relevant fractal dimensions [1].

This specific parameterization provides an excellent description of the data in the region of four momentum transfer squared, $0.045 \leqslant Q^2 \leqslant 120$ GeV^2 and Bjorken-x, $6.2 \times 10^{-7} \leqslant x \leqslant 0.01$. We assume, for simplicity, the following relation between the structure function and the gluon distribution [5]:

$$G(x, Q^2) = c(x, Q^2) \cdot F_2(x, Q^2) \tag{4}$$

where the function $G(x, Q^2)$ is to be determined from momentum sum rule. The momentum sum rule is given as:

$$
\begin{aligned}
1 &= \int_0^1 \left\{ F_2(x, Q^2) + G(x, Q^2) \right\} dx \\
&= \int_0^1 \left\{ F_2(x, Q^2) + c(x, Q^2) F_2(x, Q^2) \right\} dx \\
\Rightarrow & \int_0^1 \left\{ c(x, Q^2) F_2(x, Q^2) \right\} dx = 1 - \int_0^1 F_2(x, Q^2) dx
\end{aligned}
\tag{5}
$$

Thus, knowing the value of $\int_0^1 F_2(x, Q^2) dx$, the gluon content of the proton can be determined. For simplicity, we assume that $c(x, Q^2)$ has the following form:

$$c(x, Q^2) = x^{-\lambda(Q^2)} \tag{6}$$

where $\lambda(Q^2)$ is to be determined from momentum sum rule. Eq. (6) conforms to the expectation that for small x gluon dominates.

Integrating the structure function of Eq. (2), we have:

$$\int_0^1 F_2(x, Q^2)dx = \int_0^1 \frac{e^{D_0} Q_0^2 x^{-D_2+1}}{1 + D_3 - D_1 \log x} \left(x^{-D_1 \log\left(1+\frac{Q^2}{Q_0^2}\right)} \left(1 + \frac{Q^2}{Q_0^2}\right)^{D_3+1} - 1 \right) dx \quad (7)$$

Substituting the corresponding values of the parameters in the above equation, we get:

$$\int_0^1 F_2(x, Q^2)dx = \int_0^1 \frac{e^{0.339} \, 0.062 \left(\frac{1}{x}\right)^{0.013}}{0.073 \log(\frac{1}{x}) - 0.287} \left(\left(\frac{1}{x}\right)^{0.073 \log\left(1+\frac{Q^2}{0.062}\right)} \left(1 + \frac{Q^2}{0.062}\right)^{-0.287} - 1 \right) dx \tag{8}$$

Using Eq. (2) and after a few steps following Cauchy's principal value integration method [6], we arrive at the following form of the integrated structure function (for any Q^2):

$$\int_0^1 F_2(x, Q^2)dx = \frac{e^{0.339} \, 0.062}{0.073} \left(1 + \frac{Q^2}{0.062}\right)^{-0.287} e^{-\mu y_0} .$$

$$\left(\log\left|\frac{y_{max} - y_0}{y_0}\right| - \mu y_{max} + \sum_{n=2}^{\infty} \frac{(-1)^n \mu^n}{n \cdot n!} \left\{ (y_{max} - y_0)^n + (-1)^{n+1} y_0^n \right\} \right)$$

$$- \frac{e^{0.339} \, 0.062}{0.073} e^{-\rho y_0} .$$

$$\left(\log\left|\frac{y_{max} - y_0}{y_0}\right| - \rho y_{max} + \sum_{n=2}^{\infty} \frac{(-1)^n \rho^n}{n \cdot n!} \left\{ (y_{max} - y_0)^n + (-1)^{n+1} y_0^n \right\} \right) \quad (9)$$

where

$$\mu = 1 - \left(0.013 + 0.073 \log\left(1 + \frac{Q^2}{0.062}\right)\right)$$

$$\rho = 0.987$$

$$y_0 = 3.93$$

$$y_{max} = \log\left(\frac{1}{x_{min}}\right) \tag{10}$$

and x_{min} is introduced to take care of the end-point singularity $x = 0$ in the model. Taking only the first term in the infinite series of RHS of Eq. (9), we get [7, 8]

$$\int_0^1 F_2(x, Q^2)dx = 0.024607 \cdot \log\left|\frac{y_{max} - y_0}{y_0}\right| \cdot \frac{Q^2}{0.062} \tag{11}$$

leading to

$$x_{min} \approx 3.58 \times 10^{-4}. \tag{12}$$

Using this value of x_{min}, we calculate

$$\langle x \rangle_q = \int\limits_{x_{min}}^{1} F_2(x, Q^2)dx \tag{13}$$

and

$$\langle x \rangle_g = 1 - \langle x \rangle_q \tag{14}$$

for any Q^2 using the RHS of Eq. (9), where $\langle x \rangle_q$ and $\langle x \rangle_g$ are fraction of momentum carried by quarks and gluons respectively.

3 Results

In Table 1, we record the values of $\langle x \rangle_q$ and $\langle x \rangle_g$ for several representative values of Q^2:

Q^2 (GeV^2)	$\langle x \rangle_q$	$\langle x \rangle_g$
10	0.0716	0.9283
20	0.1433	0.8567
30	0.2149	0.7850
35	0.2508	0.7492
40	0.2867	0.7133
45	0.3225	0.6775
50	0.3583	0.6418
55	0.3942	0.6058
60	0.4299	0.5700
65	0.4658	0.5342
70	0.5017	0.4983
75	0.5375	0.4625
80	0.5733	0.4267
85	0.6092	0.3908
90	0.6449	0.3550

Table 1: Values of $\langle x \rangle_q$ and $\langle x \rangle_g$ for a few representative values of Q^2.

In Figure 1, it is shown that within the leading term approximation used, $\langle x \rangle_q$ increases with Q^2 while $\langle x \rangle_g$ decreases. At $Q^2 \approx 70$ GeV^2, both of them become nearly equal, i.e. both quarks and gluons share momentum equally.

It is to be noted that one usually expects the other pattern, i.e. $\langle x \rangle_g = \int\limits_{0}^{1} G(x, Q^2)dx$ should increase with Q^2 and $\langle x \rangle_q = \int\limits_{0}^{1} F_2(x, Q^2)dx$ should decrease [9]. This feature is presumably due to the overestimation of the large x quarks in the original Eq. (2) used and the crude approximation of taking only the first term of the infinite series in Eq. (9). The effects of large

x and higher order terms are currently under study. It will then lead to proper evaluation of the exponent λ in the definition of gluon in Eq. (6).
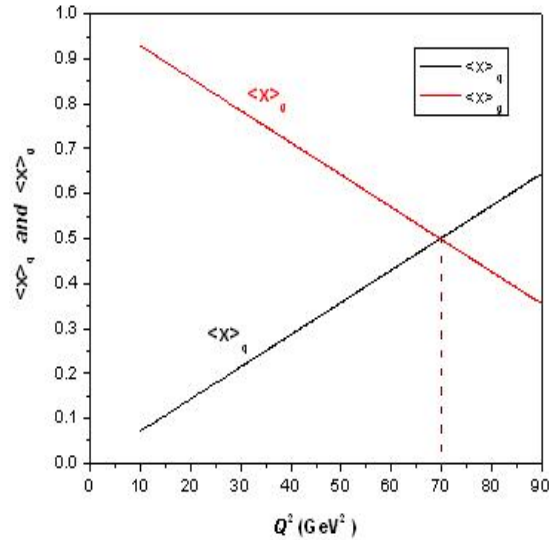

Figure 1: $\langle x \rangle_q$ and $\langle x \rangle_g$ vs Q^2.

4   Conclusion

In this paper, we have reformulated the gluon distribution function based on momentum sum rule. We also report some preliminary results of how quark and gluon momenta are shared in the proton. Inclusion of higher order terms in the infinite series and the large x effect will hopefully pin down the gluon accurately.

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