Rare Semileptonic $B^+ \to \pi^+ \ell^+ \ell^-$ Decay

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We present a precise calculation of the dilepton invariant-mass spectrum and branching fraction for $B^+ \to \pi^+ \ell^+ \ell^-$ ($\ell^\pm = e^\pm, \mu^\pm$) in the Standard Model (SM) based on the effective Hamiltonian approach for the $b \to d\ell^+\ell^-$ transitions. Theoretical estimates strongly depend on the form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$. Of these, $f_+(q^2)$ is well measured in the semileptonic decays $B \to \pi \ell \nu_\ell$ and we use the B-factory data to parametrize it. Using an $SU(3)_F$-breaking Ansatz and Lattice-QCD data, we calculate the $B \to \pi$ form factors. The resulting total branching fraction $\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$ is in good agreement with the experimental value obtained by the LHCb collaboration.

1 Introduction

Recently, the LHCb collaboration has reported the first observation of the $B^+ \to \pi^+\mu^+\mu^-$ decay with 5.2σ significance, using 1.0 fb^{-1} integrated luminosity in proton-proton collisions at the Large Hadron Collider (LHC) at $\sqrt{s} = 7$ TeV [1]. The measured branching ratio $\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-) = [2.3 \pm 0.6(\text{stat}) \pm 0.1(\text{syst})] \times 10^{-8}$ [1] is in good agreement with the SM expectated rate [2], which, however, is based on model-dependent input for the $B \to \pi$ form factors. Hence, it is very desirable to calculate the form factors from first principles, such as the Lattice-QCD, which have their own range of validity restricted by the recoil energy. With improved lattice technology, one can use the lattice form factors to predict the decay rates in the $B \to \pi$ transitions in the low-recoil region, where the lattice results apply without any extrapolation, in a model-independent manner. We describe such a framework, which makes use of the methods based on the Heavy-Quark Symmetry (HQS) in the large-recoil region, data on the charged-current processes $B^0 \to \pi^-\ell^+\nu_\ell$ and $B^+ \to \pi^0\ell^+\nu_\ell$, to determine one of the form factors, $f_+(q^2)$, and the available lattice results for the form factors in the low-recoil region. The details of the analysis are presented in our recent paper [3] and the main steps are summarized in this contribution.

2 Theory of $B^+ \to \pi^+ \ell^+ \ell^-$ Decay

The effective weak Hamiltonian encompassing the transitions $b \to d\,\ell^+\ell^-$ ($\ell = e$, μ, or τ), in the Standard Model (SM) can be written as follows [4]:

$$\mathcal{H}_{\text{eff}}^{b\to d} = \frac{4G_F}{\sqrt{2}} \left[V_{ud}V_{ub}^* \left(C_1\,\mathcal{O}_1^{(u)} + C_2\,\mathcal{O}_2^{(u)} \right) + V_{cd}V_{cb}^* \left(C_1\,\mathcal{O}_1 + C_2\,\mathcal{O}_2 \right) - V_{td}V_{tb}^* \sum_{i=3}^{10} C_i\,\mathcal{O}_i \right], \quad (1)$$

where G_F is the Fermi constant, $V_{q_1q_2}$ are the CKM matrix elements which satisfy the unitary condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ (it can be used to eliminate one combination). In contrast to the $b \to s$ transition, all three terms in the unitarity relation are of the same order in λ ($V_{ub}^* V_{ud} \sim V_{cb}^* V_{cd} \sim V_{tb}^* V_{td} \sim \lambda^3$), with $\lambda = \sin\theta_{12} \simeq 0.2232$ [5]. The local operators appearing in (1) are the dimension-six operators defined at an arbitrary scale μ as in [6]. The Wilson coefficients $C_i(\mu)$ ($i = 1, \ldots, 10$) depending on the renormalization scale μ are calculated at the matching scale $\mu_W \sim M_W$, the W-boson mass, as a perturbative expansion in the strong coupling constant $\alpha_s(\mu_W)$ [6] and can be evolved to a lower scale $\mu_b \sim m_b$ using the anomalous dimensions of the above operators to NNLL order [6].

The hadronic matrix elements of the operators $\mathcal{O}_i$ between the B- and π-meson states are expressed in terms of three independent form factors [7]:

$$\langle \pi(p_\pi)|\bar{b}\gamma^\mu d|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}\, q^\mu\right] + f_0(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2}\, q^\mu, \quad (2)$$

$$\langle \pi(p_\pi)|\bar{b}\sigma^{\mu\nu} q_\nu d|B(p_B)\rangle = \frac{i f_T(q^2)}{m_B + m_\pi}\left[q^2\left(p_B^\mu + p_\pi^\mu\right) - \left(m_B^2 - m_\pi^2\right) q^\mu\right], \quad (3)$$

where p_B^μ and p_π^μ are the four-momenta of the B- and π-mesons, respectively, m_B and m_π are their masses, and $q^\mu = p_B^\mu - p_\pi^\mu$ is the momentum transferred to the lepton pair. The $B \to \pi$ transition form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ are scalar functions whose shapes are determined by using non-perturbative methods.

The differential branching fraction in the dilepton invariant mass q^2 can be expressed as follows:

$$\frac{d\mathcal{B}\left(B^+ \to \pi^+\ell^+\ell^-\right)}{dq^2} = \frac{G_F^2 \alpha_{\rm em}^2 \tau_B}{1024\pi^5 m_B^3}\,|V_{tb}V_{td}^*|^2 \sqrt{\lambda(q^2)}\sqrt{1 - \frac{4m_\ell^2}{q^2}}\, F(q^2), \quad (4)$$

where $\alpha_{\rm em}$ is the fine-structure constant, m_ℓ is the lepton mass, τ_B is the B-meson lifetime, $\lambda(q^2) = \left(m_B^2 + m_\pi^2 - q^2\right)^2 - 4m_B^2 m_\pi^2$ is the kinematical function encountered in three-body decays (triangle function), and $F(q^2)$ is a dynamical function encoding the Wilson coefficients and the form factors:

$$\begin{aligned} F(q^2) &= \frac{2}{3}\,\lambda(q^2)\left(1 + \frac{2m_\ell^2}{q^2}\right)\left|C_9^{\rm eff}(q^2)\, f_+(q^2) + \frac{2m_b}{m_B + m_\pi}\, C_7^{\rm eff}(q^2)\, f_T(q^2)\right|^2 \\ &+ \frac{2}{3}\,\lambda(q^2)\left(1 - \frac{4m_\ell^2}{q^2}\right)\left|C_{10}^{\rm eff}\, f_+(q^2)\right|^2 + \frac{4m_\ell^2}{q^2}\left(m_B^2 - m_\pi^2\right)^2\left|C_{10}^{\rm eff}\, f_0(q^2)\right|^2. \end{aligned} \quad (5)$$

The dynamical function (5) contains the effective Wilson coefficients $C_7^{\rm eff}(q^2)$, $C_9^{\rm eff}(q^2)$ and $C_{10}^{\rm eff}$ which are specific combinations of the Wilson coefficients entering the effective Hamiltonian (1). To the NNLO approximation, the effective Wilson coefficients given in [6, 8, 9].

To perform a numerical analysis one needs to know the $B \to \pi$ transition form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ in the entire kinematic range: $4m_\ell^2 \leq q^2 \leq (m_B - m_\pi)^2$. Their model-independent determination is the main aim of this paper, which is described in detail in subsequent sections. Several parametrizations of the semileptonic form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ have been proposed in the literature. We especially outline the Boyd-Grinstein-Lebed (BGL) parametrization because namely this parametrization was used in our analysis. In the framework of BGL parametrization the shape for the form factors $f_i(q^2)$ with $i = +, 0, T$ is presented as follows [10]:

$$f_i(q^2) = \frac{1}{P(q^2)\phi_i(q^2, q_0^2)} \sum_{k=0}^{k_{\max}} a_k(q_0^2)\left[z(q^2, q_0^2)\right]^k, \quad z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}, \quad (6)$$

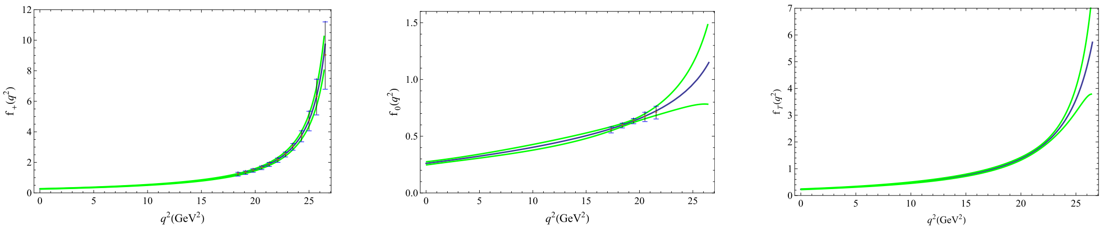


Figure 1: (Color online) The vector, scalar and tensor $B \to \pi$ transition form factors $f_p(q^2)$, $f_0(q^2)$ and $f_T(q^2)$, respectively, in the entire kinematical region using the BGL parametrization. The solid green lines show the uncertainty in the form factors. The vertical bars in the left and middle plots are the Lattice-QCD data [12]

with the pair-production threshold $m_+^2 = (m_B + m_\pi)^2$ and a free parameter q_0^2. In our analysis we make the choice $q_0^2 = 0.65 m_-^2$. The proposed shapes (6) for the form factors contains the so-called Blaschke factor $P(q^2)$ which accounts for the hadronic resonances in the sub-threshold region $q^2 < m_+^2$. For the semileptonic $B \to \pi \ell \nu_\ell$ decay, where ℓ is an electron or a muon, there is only B^*-meson with the mass $m_{B^*} = 5.325$ GeV satisfying the sub-threshold condition and producing the pole in the form factor at $q^2 = m_{B^*}^2$. In this case, the Blaschke factor is simply $P(q^2) = z(q^2, m_{B^*}^2)$ for $f_{+,T}(q^2)$ and $P(q^2) = 1$ for $f_0(q^2)$.

The coefficients a_k ($k = 0, 1, \ldots, k_{\max}$) entering the Taylor series in Eq. (6) are the parameters, which are determined by fits of the data. The outer function $\phi_i(q^2, q_0^2)$ is an arbitrary analytic function, whose choice only affects particular values of the coefficients a_k and are given in [11]. Having relatively small values of $z(q^2, q_0^2)$ in the physical region of q^2, the shape of the form factor can be well approximated by the truncated series at $k_{\max} = 2$ or 3.

3 Shapes of Form Factors

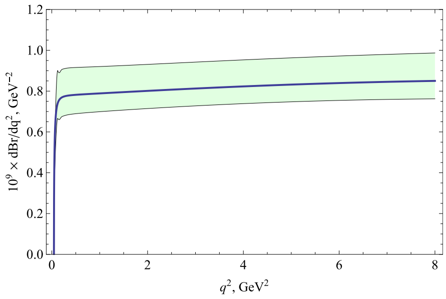
Measurements of the $B^0 \to \pi^- \ell^+ \nu_\ell$ and $B^+ \to \pi^0 \ell^+ \nu_\ell$ decays, where $\ell = e$, μ, allow to extract both the CKM matrix element V_{ub} and the shape of the $f_+(q^2)$ form factor. The differential branching fractions of the above processes can be written in the form [5]:

$$\frac{d\Gamma(B \to \pi \ell^+ \nu_\ell)}{dq^2} = C_P \, \frac{G_F^2 |V_{ub}|^2}{192 \pi^3 m_B^3} \, \lambda^{3/2}(q^2) f_+^2(q^2), \tag{7}$$

where C_P is the isospin factor with $C_P = 1$ for the π^+-meson and $C_P = 1/2$ for the π^0-meson, $q = p_\ell + p_\nu$ is the lepton-pair four-momentum bounded by $m_\ell^2 \le q^2 \le (m_B - m_\pi)^2$, and p_ℓ and p_ν are the four-momenta of the charged lepton and neutrino, respectively.

The partial branching fraction of $B^0 \to \pi^- \ell^+ \nu_\ell$ has been measured by the BaBar and Belle collaborations and of $B^0 \to \pi^- \ell^+ \nu_\ell$ by the Belle collaboration [13, 14, 15, 16]. Using these data we extracted the $f_+(q^2)$ form factor shape using the standard minimization procedure of the χ^2-distribution function [5]. The resulting form factor $f_+(q^2)$ from the combined analysis of the BaBar and Belle datasets is shown in the left plot in Fig. 1. In this analysis we have assumed that the experimental points are all uncorrelated.

The parameters of $f_0(q^2)$ can be obtained from the existing results of the $B \to \pi$ transition form factor calculated by the HPQCD collaboration [12]. In addition one can use the exact



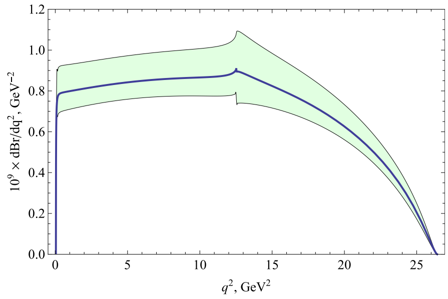


Figure 2: The dilepton invariant-mass distributions in the $B^+ \to \pi^+\ell^+\ell^-$ decay in the range $0 \leq q^2 \leq 8$ GeV^2 (left plot) and in the entire range $0 \leq q^2 \leq 26.4$ GeV^2 (right plot).

relation $f_+(0) = f_0(0)$, where $f_+(q^2)$ is extracted from the experimental data. The form-factor parametrization we use for $f_0(q^2)$ follows our default choice from the analysis of $f_+(q^2)$ — the BGL expansion in $z(q^2, q_0^2)$ truncated at $k_{\rm max} = 2$. The resulting $f_0(q^2)$ form factor shape is shown in Fig. 1 (middle plot), where we also present the Lattice-QCD data [12].

One should mention that there is only scant information about the $f_T^{B\pi}(q^2)$ form factor at present. So, one needs to find a reliable method to extract it from the existing model-independent data. We use an $SU(3)_F$-symmetry-breaking Ansatz involving the $B \to K$ and $B \to \pi$ form factors. We recall that all three $B \to K$ transition form factors $f_+^{BK}(q^2)$, $f_0^{BK}(q^2)$ and $f_T^{BK}(q^2)$ have been calculated recently by the HPQCD collaboration [17, 18] and the two $B \to \pi$ transition form factors $f_+^{B\pi}(q^2)$ and $f_0^{B\pi}(q^2)$ are also known [12]. With this knowledge, we first estimate the $SU(3)_F$-breaking corrections in the already known vector and scalar form factors and use these corrections for estimating the $B \to \pi$ tensor form factor $f_T^{B\pi}(q^2)$ from the corresponding $B \to K$ transition form factor $f_T^{BK}(q^2)$. The resulting $f_T^{B\pi}(q^2)$ form factor obtained is shown in the right plot in Fig. 1.

As all the form factors in the $B \to \pi$ transition are now known, we can make predictions for the dilepton invariant-mass spectrum and decay width in the semileptonic $B \to \pi\,\ell^+\ell^-$ decays for $\ell^\pm = e^\pm, \mu^\pm$.

4 Predictions for $B^+ \to \pi^+\ell^+\ell^-$ Decay

The B^+-meson is a bound state of the heavy $\bar{b}$- and light u-quarks, hence one can apply the so-called Heavy-Quark Symmetry (HQS), which is valid in the large-recoil limit (at small values of q^2). Using the HQS allows one to simplify significantly the description of the $B^+ \to \pi^+\ell^+\ell^-$ decay at small q^2 ($q^2 \leq 8$ GeV^2), namely, applying the HQS results in reducing the number of independent form factors of the $B \to \pi$ transition from three to one. The relations between the three form-factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ in the HQS limit with taking into account symmetry-breaking corrections are worked out in Ref. [7]. With use of these relations the dimuon invariant mass spectrum was obtained and is presented in the left plot in Fig. 2.

In the low hadronic-recoil region (large-q^2) there is no heavy-quark symmetry relations among form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ any more and they should be considered as three independent quantities. All of them were extracted by us in the entire kinematic range and used for further calculations.

The invariant-mass spectrum in the entire range of q^2 ($4m_\ell^2 < q^2 < 26.4$ GeV^2) is presented in the right plot in Fig. 2. We get the following prediction for the total branching fraction [3]:

$$\mathcal{B}(B^+ \to \pi^+ \mu^+\mu^-) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}, \tag{8}$$

where the resulting average uncertainty about 15% and is coming from the scale dependence μ_b of the Wilson coefficients, the CKM matrix element $|V_{td}|$ and form factors (FF).

5 Summary and Outlook

We have presented a theoretically improved calculation of the branching fraction for the $B^\pm \to \pi^\pm\mu^+\mu^-$ decay, measured recently by the LHCb collaboration [1]. The combined accuracy on the branching ratio is estimated as $\pm 15\%$, and the resulting branching fraction $\mathcal{B}(B^\pm \to \pi^\pm\mu^+\mu^-) = (1.88^{+0.32}_{-0.21}) \times 10^{-8}$ [3] is in agreement with the LHCb data [1].

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