Mesons with open charm and beauty: an overview

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The spectrum of mesons with open charm and beauty is analyzed using heavy quark symmetry arguments. A classification of the newly observed states is presented, together with predictions for several unobserved resonances.

1 Heavy meson doublets

A QCD framework for the analysis of hadrons containing a single heavy quark can be set up using the heavy quark (HQ) limit, and is formalized in the heavy quark effective theory (HQET) [1]. This is an effective theory of QCD formulated for N_f heavy quarks Q with mass $m_Q \gg \Lambda_{QCD}$, with the four-velocity of Q fixed. The theory displays heavy quark spin-flavour symmetries, i.e. the invariance under $SU(2N_f)$ transformations, which are symmetries of the QCD Lagrangian in the heavy quark limit. Within this framework, several heavy hadron properties can be studied, with important results represented, for instance, by the relations among semileptonic transition form factors in weak heavy hadron matrix elements [2]. The heavy meson spectrum can also be studied from the point of view of the heavy quark limit [3]: this is particularly interesting, due to the numerous recently discovered charm and beauty resonances needing to be recognized [4].

The classification of heavy $Q\bar{q}$ mesons (q is a light quark) in the HQ limit relies on the decoupling of the heavy quark spin s_Q from the spin of the light antiquark and gluons. The separate conservation in strong interaction processes of s_Q and of the total angular momentum s_ℓ of the light degrees of freedom permits a classification of the heavy mesons according to the value of s_ℓ. Mesons can be collected in doublets: the two states in each doublet (*spin partners*) have total spin $J = s_\ell \pm \frac{1}{2}$ and parity $P = (-1)^{\ell+1}$, with ℓ the orbital angular momentum of the light degrees of freedom and $\vec{s}_\ell = \vec{\ell} + \vec{s}_q$ (s_q is the light antiquark spin). Within each doublet the two states are degenerate in the HQ limit and, due to flavour symmetry, the properties of the states in a doublet can be related to those of the corresponding states differing for the heavy quark flavour. Corrections can be systematically included considering next-to-leading terms in an expansion in the inverse heavy quark mass.

We focus on the meson doublets with $\ell = 0, 1, 2$ ($s-$, $p-$ and $d-$wave states in the quark model), discussing their properties in the HQ limit and considering a few next-to-leading corrections. This allows us to study how the observed charmed and beauty mesons fit in the theoretical classification. Moreover, using data in the charm sector, the properties of the corresponding beauty mesons can be predicted.

Important information for a proper identification comes from the heavy meson decays to light pseudoscalar mesons, whose features depend on the quantum numbers of the decaying resonances. An effective Lagrangian approach, with the heavy meson doublets represented by effective fields and the octet of light pseudo Goldstone mesons grouped in a single field, can be formulated imposing the invariance under heavy quark spin-flavour transformations and chiral transformations of the light pseudo Goldstone boson fields. This allows to infer the properties of the heavy meson doublets in the HQ limit, namely that the two degenerate states in a doublet have the same full width, that the sum of the partial widths of a state in a doublet to another heavy state in another doublet with emission of a light meson is the same for the two members of a doublet, that the ratios of partial decay widths for a given state are related, that the partial decay widths are independent of the heavy quark flavour [3].

The lightest $Q\bar{q}$ mesons correspond to $\ell = 0$, hence $s_\ell^P = \frac{1}{2}^-$. This doublet consists of two states with $J^P = (0^-, 1^-)$, denoted as (P, P^*). For $\ell = 1$ one has $s_\ell^P = \frac{1}{2}^+$ with $J^P = (0^+, 1^+)$ (the states are (P_0^*, P_1')), and $s_\ell^P = \frac{3}{2}^+$ with $J^P = (1^+, 2^+)$ $((P_1, P_2^*))$. $\ell = 2$ corresponds to either $s_\ell^P = \frac{3}{2}^-$ (states (P_1^*, P_2)) or $s_\ell^P = \frac{5}{2}^-$ ($(P_2'^*, P_3)$). An analogous notation holds for the radial excitations with $n = 2$ (denoted by a tilde: $\tilde{P}$, $\tilde{P}^*$, ...). The effective fields describing the various doublets in the HQ limit are listed below, with $a = u, d, s$ light flavour index. The fields, defined including a factor $\sqrt{m_Q}$, have dimension 3/2 and annihilate mesons of four velocity v which is conserved in strong interaction processes.

$$
\begin{aligned}
s_\ell^P = \frac{1}{2}^- &: \quad H_a = \frac{1+\not{v}}{2}\left[P^*_{a\mu}\gamma^\mu - P_a\gamma_5\right] \\
s_\ell^P = \frac{1}{2}^+ &: \quad S_a = \frac{1+\not{v}}{2}\left[P'^{\mu}_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\right] \\
s_\ell^P = \frac{3}{2}^+ &: \quad T_a^\mu = \frac{1+\not{v}}{2}\left[P^{\mu\nu}_{2a}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\right] \\
s_\ell^P = \frac{3}{2}^- &: \quad X_a^\mu = \frac{1+\not{v}}{2}\left[P^{*\mu\nu}_{2a}\gamma_5\gamma_\nu - P'^*_{1a\nu}\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{3}{2}\gamma^\nu(\gamma^\mu + v^\mu)\right]\right] \\
s_\ell^P = \frac{5}{2}^- &: \; X'^{\mu\nu}_a = \frac{1+\not{v}}{2}\left[P^{\mu\nu\sigma}_{3a}\gamma_\sigma - P^{*'\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5\left[g^\mu_\alpha g^\nu_\beta - \frac{1}{5}\gamma_\alpha g^\nu_\beta(\gamma^\mu - v^\mu) - \frac{1}{5}\gamma_\beta g^\mu_\alpha(\gamma^\nu - v^\nu)\right]\right].
\end{aligned} \tag{1}
$$

The octet of light pseudoscalar mesons is introduced defining $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$ and $\Sigma = \xi^2$, with $\mathcal{M}$ incorporating the fields of π, K and η ($f_\pi = 132$ MeV):

$$
\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} . \tag{2}
$$

Imposing invariance under heavy quark spin-flavour and light quark chiral transformations, an effective QCD Lagrangian can be constructed [5, 6], with kinetic terms of the heavy meson doublets and of the Σ field reading:

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{8} Tr[\partial^\mu \Sigma \partial_\mu \Sigma^\dagger] + i\, Tr[\bar{H}_b v^\mu D_{\mu ba} H_a] \\ &+ Tr[\bar{S}_b\, (i\, v^\mu D_{\mu ba} \;-\; \delta_{ba}\, \Delta_S) S_a] + Tr[\bar{T}_b^\alpha\, (i\, v^\mu D_{\mu ba} \;-\; \delta_{ba}\, \Delta_T) T_{a\alpha}] \\ &+ Tr[\bar{X}_b^\alpha\, (i\, v^\mu D_{\mu ba} \;-\; \delta_{ba}\, \Delta_X) X_{a\alpha}] + Tr[\bar{X}_b^{\prime\alpha\beta}\, (i\, v^\mu D_{\mu ba} \;-\; \delta_{ba}\, \Delta_{X'}) X'_{a\alpha\beta}] \;,\end{aligned} \tag{3}$$

with $D_{\mu ba} = -\delta_{ba}\partial_\mu + \mathcal{V}_{\mu ba} = -\delta_{ba}\partial_\mu + \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right)_{ba}$ and $\mathcal{A}_{\mu ba} = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right)_{ba}$. The parameters Δ_F (with $F = S,\, T,\, X,\, X'$) correspond to the mass splittings between the higher mass doublets and the lightest one described by H:

$$\Delta_F = \overline{M}_F - \overline{M}_H \;, \tag{4}$$

with $\overline{M}_{(F)}$ the spin-averaged masses of the doublets:

$$\begin{aligned}\overline{M}_H &= \frac{3M_{P^*} + M_P}{4} \;, \qquad \overline{M}_S = \frac{3M_{P'_1} + M_{P^*_0}}{4} \;, \qquad \overline{M}_T = \frac{5M_{P^*_2} + 3M_{P_1}}{8} \;, \\ \overline{M}_X &= \frac{5M_{P_2} + 3M_{P^*_1}}{8} \;, \quad \overline{M}_{X'} = \frac{7M_{P_3} + 5M_{P'^*_2}}{12} \;.\end{aligned} \tag{5}$$

Corrections to the heavy quark limit involve symmetry breaking terms suppressed by powers of $1/m_Q$ [7]. For instance, the Lagrangian terms

$$\begin{aligned}\mathcal{L}_{1/m_Q} &= \frac{1}{2m_Q}\Big\{\lambda_H Tr[\bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] + \lambda_S Tr[\bar{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}] + \lambda_T Tr[\bar{T}_a^\alpha \sigma^{\mu\nu} T_a^\alpha \sigma_{\mu\nu}] \\ &\quad + \lambda_X Tr[\bar{X}_a^\alpha \sigma^{\mu\nu} X_{a\alpha} \sigma_{\mu\nu}] + \lambda_{X'} Tr[\bar{X}_a^{\prime\alpha\beta} \sigma^{\mu\nu} X_a^{\prime\alpha\beta} \sigma_{\mu\nu}]\Big\}\end{aligned} \tag{6}$$

break the mass degeneracy between the members of the various doublets. The constants λ_H, λ_S, λ_T, λ_X and $\lambda_{X'}$ are related to the hyperfine splittings:

$$\begin{aligned}\lambda_H &= \frac{1}{8}\left(M^2_{P^*} - M^2_P\right) \;, \qquad \lambda_S = \frac{1}{8}\left(M^2_{P'_1} - M^2_{P^*_0}\right) \;, \qquad \lambda_T = \frac{3}{16}\left(M^2_{P^*_2} - M^2_{P_1}\right) \;, \\ \lambda_X &= \frac{3}{16}\left(M^2_{P_2} - M^2_{P^*_1}\right) \;, \quad \lambda_{X'} = \frac{5}{24}\left(M^2_{P_3} - M^2_{P'^*_2}\right) \;.\end{aligned} \tag{7}$$

The transitions $F \to HM$ (with $F = H, S, T, X, X'$ and M a light pseudoscalar meson), at the leading order in the light meson momentum and heavy quark mass expansion, can be described by the Lagrangian interaction terms [5]:

$$\begin{aligned}\mathcal{L}_H &= g\, Tr\Big[\bar{H}_a H_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\Big] \\ \mathcal{L}_S &= h\, Tr\Big[\bar{H}_a S_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\Big] \;+\; h.c. \\ \mathcal{L}_T &= \frac{h'}{\Lambda_\chi} Tr\Big[\bar{H}_a T^\mu_b (iD_\mu \not{\mathcal{A}} + i\not{D}\, \mathcal{A}_\mu)_{ba} \gamma_5\Big] + h.c. \\ \mathcal{L}_X &= \frac{k'}{\Lambda_\chi} Tr\Big[\bar{H}_a X^\mu_b (iD_\mu \not{\mathcal{A}} + i\not{D}\, \mathcal{A}_\mu)_{ba} \gamma_5\Big] + h.c. \\ \mathcal{L}_{X'} &= \frac{1}{{\Lambda_\chi}^2} Tr\Big[\bar{H}_a X'^{\mu\nu}_b \big[k_1 \{D_\mu, D_\nu\} \mathcal{A}_\lambda + k_2 (D_\mu D_\lambda \mathcal{A}_\nu + D_\nu D_\lambda \mathcal{A}_\mu)\big]_{ba} \gamma^\lambda \gamma_5\Big] + h.c.;\end{aligned} \tag{8}$$

these terms involve the coupling constants g, h, h', k_i (we set $k = k_1 + k_2$, and the chiral symmetry-breaking scale Λ_χ to $\Lambda_\chi = 1$ GeV). $\mathcal{L}_S$ and $\mathcal{L}_T$ describe positive parity heavy meson transitions with the emission of light pseudoscalar mesons in $s-$ and $d-$ wave, respectively, $\mathcal{L}_X$ and $\mathcal{L}_{X'}$ the transitions of higher mass mesons of negative parity, belonging to the X and X' doublets, with the emission of light pseudoscalar mesons in $p-$ and $f-$ wave. At the same order in the expansion in the light meson momentum, the structure of the Lagrangian terms for radial excitations of the various doublets is unchanged, since it is dictated only by the spin-flavour and chiral symmetries, but the coupling constants are replaced by new ones, $\tilde{g}$, $\tilde{h}$, etc.

In this basic framework all data for mesons with open charm and beauty can be analyzed, and a classification scheme for the observed resonances can be elaborated. In Table 1 we propose the assignment for the observed charmed $c\bar{q}$, $c\bar{s}$, and beauty $b\bar{q}$, $b\bar{s}$ (with $q = u, d$) mesons to the various doublets [4], justified by the arguments presented below.[1]

Table 1: Observed open charm and open beauty mesons organized in HQ doublets. States denoted by ($\star$) have uncertain assignment; they are classified according to the scheme proposed in this study.

s_ℓ^P	J^P	$c\bar{q}$ (n=1)	$c\bar{q}$ (n=2)	$c\bar{s}$ (n=1)	$c\bar{s}$ (n=2)	$b\bar{q}$ (n=1)	$b\bar{s}$ (n=1)
H $\frac{1}{2}^-$	0^-	$D(1869)$	$D(2550)$ $\star$	$D_s(1968)$		$B(5279)$	$B_s(5366)$
	1^-	$D^*(2010)$	$D^*(2600)$ $\star$	$D_s^*(2112)$	$D_{s1}^*(2700)$	$B^*(5325)$	$B_s^*(5415)$
S $\frac{1}{2}^+$	0^+	$D_0^*(2400)$		$D_{s0}^*(2317)$			
	1^+	$D_1'(2430)$		$D_{s1}'(2460)$	$D_{sJ}(3040)$ $\star$		
T $\frac{3}{2}^+$	1^+	$D_1(2420)$		$D_{s1}(2536)$	$D_{sJ}(3040)$ $\star$	$B_1(5721)$	$B_{s1}(5830)$
	2^+	$D_2^*(2460)$		$D_{s2}^*(2573)$		$B_2^*(5747)$	$B_{s2}^*(5840)$
X $\frac{3}{2}^-$	1^-						
	2^-						
X' $\frac{5}{2}^-$	2^-	$D(2750)$ $\star$					
	3^-	$D(2760)$ $\star$		$D_{sJ}(2860)$ $\star$			

2 Arguments for the classification

The analysis of the doublets with either $\ell = \frac{1}{2}^+, \frac{3}{2}^\pm, \ldots$, or $n > 1$ is based on the mass and width experimental data collected in Tables 2 and 3. The $s_\ell^P = \frac{3}{2}^+$ charmed doublets are filled by ($D_1(2420)$, $D_2^*(2460)$) and ($D_{s1}(2536)$, $D_{s2}^*(2573)$) in the non-strange and strange sector, respectively; their widths are quite narrow, as expected for mesons with d-wave decays.

($D_0^*(2400)$, $D_1'(2430)$) and ($D_{s0}^*(2317)$, $D_{s1}'(2460)$) can be identified with the members of the $s_\ell^P = \frac{1}{2}^+$ charm doublet, although they present intriguing features. The non-strange states follow the expectation of being broad, due to their s-wave strong decays. After the first evidences of broad $c\bar{q}$ states [11], the separate identification of the two states and the measurement of their masses and widths is due to Belle [12]. On the contrary, the strange partners, first observed in 2003 [13], are very narrow: they are below the DK (for $D_{s0}^*(2317)$) and D^*K (for $D_{s1}'(2460)$) thresholds, their isospin-conserving decays are kinematically forbidden, and

[1] The recently observed structures $D_J(3000)$ and $D_J^*(3000)$, mentioned in the text, are not included in the Table.

Table 2: Measured mass and width of the observed excited open charm mesons, as reported by the PDG [8] (with the states denoted by † omitted from summary tables), excluding the data on $D^{*0,+}(2600)$, $D^0(2750)$ and $D^{*0,+}(2760)$ which are from BaBar [9]; new experimental results on these states have also been provided by LHCb [10]. The widths of $D^{*+}(2600)$ and $D^{*+}(2600)$ are kept fixed in the experimental BaBar analysis [9]. The bounds are at 95% CL.

$c\bar{q}$	mass (MeV)	Γ (MeV)	$c\bar{s}$	mass (MeV)	Γ (MeV)
$D_0^{*0}(2400)$	2318 ± 29	267 ± 40			
$D_0^{*\pm}(2400)^\dagger$	$2403 \pm 14 \pm 35$	$283 \pm 24 \pm 34$	$D^*_{s0}(2317)$	2317.8 ± 0.6	< 3.8
$D_1^{\prime 0}(2430)^\dagger$	$2427 \pm 26 \pm 25$	$384 \pm^{107}_{75} \pm 74$			
			$D'_{s1}(2460)$	2459.6 ± 0.6	< 3.5
$D_1^0(2420)$	2421.4 ± 0.6	27.4 ± 2.5			
$D_1^\pm(2420)$	2423.2 ± 2.4	25 ± 6	$D_{s1}(2536)$	2535.12 ± 0.13	$0.92 \pm 0.03 \pm 0.04$
$D_2^{*0}(2460)$	2462.6 ± 0.6	49.0 ± 1.3			
$D_2^{*\pm}(2460)$	2464.3 ± 1.6	37 ± 6	$D^*_{s2}(2573)$	2571.9 ± 0.8	17 ± 4
$D^0(2550)^\dagger$	$2539.4 \pm 4.5 \pm 6.8$	$130 \pm 12 \pm 13$			
$D^{*0}(2600)$	$2608.7 \pm 2.4 \pm 2.5$	$93 \pm 6 \pm 13$			
$D^{*+}(2600)$	$2621.3 \pm 3.7 \pm 4.2$	93 (fixed)	$D^*_{s1}(2700)$	2709 ± 4	117 ± 13
$D^0(2750)$	$2752.4 \pm 1.7 \pm 2.7$	$71 \pm 6 \pm 11$			
$D^{*0}(2760)$	$2763.3 \pm 2.3 \pm 2.3$	$60.9 \pm 5.1 \pm 3.6$			
$D^{*+}(2760)$	$2769.7 \pm 3.8 \pm 1.5$	60.9 (fixed)	$D_{sJ}(2860)$	$2863.2 \pm^{4.0}_{2.6}$	58 ± 11
			$D_{sJ}(3040)^\dagger$	$3044 \pm 8 \pm^{30}_{5}$	$239 \pm 35 \pm^{46}_{42}$

the observed strong decays $D_s\pi^0$ and $D_s^*\pi^0$ violate isospin conservation. Their identification with the doublet (D^*_{s0}, D'_{s1}) is supported by analyses of the radiative decays [14] and by lattice QCD studies [15]. A puzzling aspect is the mass degeneracy between the strange states and their non-strange partners. Another issue is the possible mixing between the two 1^+ states: in the case of non-strange mesons, Belle has determined a small mixing angle: $\theta \simeq -0.10$ rad [12].

$D_{sJ}(2860)$ and $D^*_{s1}(2700)$ in Table 1 were observed in the DK final state at the B factories [16, 17], and confirmed in pp collisions at the LHC [18]. The spin-parity $J^P = 1^-$ of $D^*_{s1}(2700)$ has been established studying the production in B decays. $D^*_{s1}(2700)$ and $D_{sJ}(2860)$ are also seen to decay to D^*K [19], hence they have natural parity $J^P = 1^-, 2^+, 3^-, \cdots$; the D^*K mode excludes the assignment $J^P = 0^+$ for $D_{sJ}(2860)$. Additional information comes from the ratios of decay rates [19]

$$\frac{\mathcal{B}(D^*_{s1}(2700) \to D^*K)}{\mathcal{B}(D^*_{s1}(2700) \to DK)} = 0.91 \pm 0.13 \pm 0.12\,, \quad \frac{\mathcal{B}(D_{sJ}(2860) \to D^*K)}{\mathcal{B}(D_{sJ}(2860) \to DK)} = 1.10 \pm 0.15 \pm 0.19\ , \quad (9)$$

where $D^{(*)}K = D^{(*)0}K^+ + D^{(*)^+}K^0_S$. As discussed below, for $D^*_{s1}(2700)$ the ratio coincides with the result in the heavy quark limit if $D^*_{s1}(2700)$ is identified with the first radial excitation of $D^*_s(2112)$ [20]. The classification of $D_{sJ}(2860)$ is more uncertain. The resonance decays to both DK and D^*K, hence it may be identified with the lowest lying $n = 1$ state with either $J^P_{s_\ell} = 1^-_{3/2}$, i.e. D^*_{s1} in the X doublet, or $J^P_{s_\ell} = 3^-_{5/2}$, i.e. the state D_{s3} in the X' doublet. Another possibility is the identification with the radial excitation with $n = 2$ and $J^P_{s_\ell} = 2^+_{1/2}$, i.e. the state $\tilde{D}^*_{s2}$ in the $\tilde{T}$ doublet. Allowed decay modes are into DK, $D_s\eta$, D^*K and $D^*_s\eta$. Considering the ratios of strong decay rates in the three possible cases, the identification of

$D_{sJ}(2860)$ with D_{s3} was proposed [21], which explains the quite narrow width as due to the f-wave decays. On the other hand, D^*_{s1} and $\tilde{D}^*_{s2}$ decay in p- and d- wave, respectively; therefore, the first one is expected to be broader, while a larger mass, $M(\tilde{D}^*_{s2}) \simeq 3.157$ GeV, is predicted by the quark model for the second one [22]. We shall return below to $D_{sJ}(2860)$.

A broad structure in the D^*K distribution was also observed, $D_{sJ}(3040)$ [19]. Absence of signal in the DK distribution suggests unnatural parity $J^P = 1^+, 2^-, 3^+, \cdots$. The lightest not yet observed states with these quantum numbers are the two $J^P = 2^-$ states of the $\ell = 2$ doublets, D_{s2} with $s_\ell^P = \frac{3}{2}^-$ and D'^*_{s2} with $s_\ell^P = \frac{5}{2}^-$. $J^P = 3^+$ corresponds to a doublet with $s_\ell^P = \frac{7}{2}^+$, the mass of which is expected to be larger. In the case of radial excitations, the identification with the states with $n = 2$, $J^P = 1^+$, and $s_\ell^P = \frac{1}{2}^+$ (the meson $\tilde{D}'_{s1}$) or $s_\ell^P = \frac{3}{2}^+$ (the meson $\tilde{D}_{s1}$) is possible. In the heavy quark limit, the two $J^P = 1^+$ are expected to be broader than the two $J^P = 2^+$ states, hence $D_{sJ}(3040)$ is likely to be identified with one of the two axial-vector mesons. This justifies the classification of $D_{sJ}(3040)$ as one of the two states with $J^P = 1^+$, $n = 2$, proposed in Table 1. The properties of the corresponding spin and non-strange partners can be predicted accordingly [23].

The last four states in Table 1 are the non-strange $c\bar{q}$ mesons discovered by BaBar in $e^+e^- \to c\bar{c} \to D^{(*)}\pi X$ [9], with measured mass and width in Table 2, recently confirmed by LHCb [10]. The ratios

$$\frac{\mathcal{B}(D^{*0}(2600) \to D^+\pi^-)}{\mathcal{B}(D^{*0}(2600) \to D^{*+}\pi^-)} = 0.32 \pm 0.02 \pm 0.09 \;, \quad \frac{\mathcal{B}(D^{*0}(2760) \to D^+\pi^-)}{\mathcal{B}(D^{*0}(2750) \to D^{*+}\pi^-)} = 0.42 \pm 0.05 \pm 0.11 \tag{10}$$

measured by BaBar can be used for the classification. Moreover, for the $D^{*+}\pi^-$ mode, information comes from the $\cos\theta_H$ distribution, with θ_H the angle between the primary pion π^- and the slow pion π^+ from the D^{*+} decay. For $D^*(2600)$, this distribution suggests natural parity, consistent with the observation in both $D\pi$ and $D^*\pi$. The $\sim \cos^2\theta_H$ distribution for $D^0(2550)$ is compatible with a $J^P = 0^-$ state. Babar suggested that ($D(2550)$, $D^*(2600)$) compose the $\tilde{H}$, $J^P = (0^-, 1^-)$ doublet of $n = 2$ radial excitations of (D, D^*) mesons, while ($D(2750)$, $D^*(2760)$), can be identified with the $\ell = 2$, $n = 1$ states [9], mainly from comparison with quark model results [24]. Since there are two possible doublets with $\ell = 2$, the identification with the $J^P = (2^-, 3^-)$ doublet would come together with the assignment $D_{sJ}(2860) = D_{s3}$, and in this case $D_{sJ}(2860)$ and $D^*(2760)$ represent corresponding states with and without strangeness. Other classifications have been proposed [25] and discussed [4].

Finally, other broad states, denoted as $D_J(3000)$ and $D^*_J(3000)$, have been recently observed in the region around 3000 MeV by LHCb in the final states $D^{*+}\pi^-$, $D^+\pi^-$ and $D^0\pi^+$ [10]. They are not included in this overview, as their assignment deserves a dedicated study.

The masses and widths of the beauty excited states, observed at LEP [26], Tevatron [27] and LHCb [28], are collected in Table 3.

Table 3: Mass and width (in MeV) of the observed open beauty excited mesons [8].

$b\bar{q}$	mass	Γ	$b\bar{s}$	mass	Γ
$B_1^0(5721)$	5723.5 ± 2.0		$B_{s1}^0(5830)$	5828.7 ± 0.4	
$B_2^{*0}(5747)$	5743 ± 5	$22.7^{+3.8+3.2}_{-3.2-10.2}$	$B_{s2}^{*0}(5840)$	5839.7 ± 0.6	$1.56 \pm 0.13 \pm 0.47$

3 Mass parameters

The assignments proposed in Table 1 are supported by the values of the HQ parameters: the average masses $\overline{M}_F$ in Eq. (5), the mass splitting Δ_F and the hyperfine splitting λ_F parameters in Eq. (7) that we collect in Table 4. Flavour symmetry implies that the mass splitting Δ_F is the same regardless of the heavy quark flavour of the doublets, and that the mass splitting λ_F between spin partners in a doublet is independ of the heavy flavour. Indeed, from the Lagrangian (3) and (6), one has:

$$\Delta_F^{(c)} = \Delta_F^{(b)} \quad , \quad \lambda_F^{(c)} = \lambda_F^{(b)} \quad .$$

The observed deviations, due to both light flavour and heavy quark mass effects, suggest the size of the higher order symmetry breaking terms: as an example, the strange quark mass effect is visible in $\overline{M}_F$.

Table 4: Spin averaged masses $\bar{M}_F$ (in MeV), mass splittings Δ_F (in MeV) and hyperfine splitting parameters λ_F (in MeV^2) defined in Eq.(5) and (7).

	$c\bar{u}$	$c\bar{d}$	$c\bar{s}$	$b\bar{u}$	$b\bar{d}$	$b\bar{s}$
$\overline{M}_H$	1971.45 ± 0.12	1975.12 ± 0.10	2076.4 ± 0.4	5313.7 ± 0.3	5313.8 ± 0.3	5403 ± 2
$\overline{M}_{\tilde{H}}$	2591.4 ± 3.3					
$\overline{M}_S$	2400 ± 28		2424.1 ± 0.5			
$\overline{M}_T$	2447.1 ± 0.5	2449.0 ± 1.6	2558.1 ± 0.5		5735.7 ± 3.2	5834.7 ± 0.5
$\overline{M}_{X'}$	2758.8 ± 2.3					
Δ_S	429 ± 28		347.7 ± 0.6			
Δ_T	475.7 ± 0.5	473.9 ± 1.6	481.7 ± 0.6		421.9 ± 3.2	431.7 ± 2.1
$\Delta_{X'}$	787.4 ± 2.3					
λ_H	$(262.3 \pm 0.2)^2$	$(261.2 \pm 0.2)^2$	$(270.9 \pm 0.6)^2$	$(246.8 \pm 1.2)^2$	$(245.9 \pm 1.2)^2$	$(256.3 \pm 6.4)^2$
$\lambda_{\tilde{H}}$	$(211.2 \pm 13.4)^2$					
λ_S	$(254 \pm 54)^2$		$(290.9 \pm 0.9)^2$			
λ_T	$(195 \pm 2)^2$	$(193 \pm 7)^2$	$(187.7 \pm 2.1)^2$		$(205 \pm 28)^2$	$(149.9 \pm 6.7)^2$
$\lambda_{X'}$	$(112 \pm 24)^2$					

Using the input from Table 1, predictions can be worked out for the masses of unobserved states, namely the missing $n = 1$ and $n = 2$, $J^P_{s_\ell} = (0^-, 1^-)_{1/2}$ charmed mesons, see Table 5. Moreover, in the HQ limit and using charm data, the beauty meson properties can be computed. For $F = \tilde{H}$, S, T, X' and $\tilde{T}$, with the data in Table 4 predictions for beauty doublets can be worked out, Table 6. Noticeably, B^*_{s0} and B'_{s1} turn out to be below the BK and B^*K thresholds; they are expected to be very narrrow, with main $B_s\pi^0$ and $B^*_s\pi^0$ decay modes [29, 30]. The masses of the resonances recently observed by CDF [31] and LHCb [32] in the $B\pi$ channel follow the expectations.

Table 5: Predicted mass and width (in MeV) of two not yet observed charm mesons, together with their spin partners.

		$\tilde{D}_{(s)}$ $(0^-, n=2)$	$\tilde{D}^*_{(s)}$ $(1^-, n=2)$	$D'^*_{(s)2}$ (2^-)	$D_{(s)3}$ (3^-)
$c\bar{q}$		$D(2550)$	$D^*(2600)$	$D(2750)$	$D(2760)$
$c\bar{s}$	mass	2643 ± 13	$D^*_{s1}(2700)$	2851 ± 7	$D_{sJ}(2860)$
	Γ	33.5 ± 3.3		20.5 ± 2.4	

Table 6: Predicted mass and width (in MeV) of doublets of excited beauty mesons. For the decay widths of B^*_{s0} and B'_{s1} see the text.

		$\tilde{B}_{(s)}(0^-, n=2)$	$\tilde{B}^*_{(s)}(1^-, n=2)$	$B^*_{(s)0}(0^+)$	$B'_{(s)1}(1^+)$	$B'^*_{(s)2}(2^-)$	$B_{(s)3}(3^-)$
$b\bar{q}$	M	5911 ± 5	5941 ± 3	5708 ± 23	5753 ± 31	6098 ± 2	6103 ± 3
	Γ	149 ± 15	186 ± 18	269 ± 58	268 ± 70	103 ± 8	129 ± 10
$b\bar{s}$	M	5997 ± 6	6027 ± 8	5707 ± 1	5766 ± 1	6181 ± 5	6186 ± 5
	Γ	76 ± 9	118 ± 14			57 ± 6	78 ± 7

4 Strong decays

Two-body heavy meson decays in final states comprising a light pseudoscalar meson can be analyzed using the Lagrangian (8). A prime role is played by the effective strong coupling constants, for which the following information is available.

g governs the strong transition among states in the H doublet. The measurement $\Gamma(D^{*\pm}) = 96{\pm}4{\pm}22$ KeV [8], recently improved by BaBar: $\Gamma(D^{*\pm}) = 83.5{\pm}1.7{\pm}1.2$ KeV [33], corresponds to the value in Table 7; it is larger than a set of theoretical results in the HQ limit and at finite m_Q [34, 35, 36], and agrees with more recent calculations [37].

h controls the decays $S \to HM$, and can be obtained using data on the $c\bar{q}$ doublet S, with $q = u, d$. From the widths of ($D^*_0(2400)$, $D'_1(2430)$) in Table 2, the value in Table 7 can be derived, which agrees with QCD sum rule [35] and lattice QCD determinations [36]. The predicted widths of the corresponding beauty mesons are in Table 6.

h' is involved in $T \to HM$ decays, and can be determined from Table 2. The obtained value in Table 7 translates into a prediction for the $D_{s1}(2536)$ decay width: $\Gamma(D_{s1}(2536)) = 0.305{\pm}0.002$ MeV. The BaBar determination in Table 2 [38], is larger than this result, a possible consequence of the mixing with the axial-vector state $D'_{s1}(2460)$ [39]. In the case of the beauty T doublet, the width of the B^{*0}_2 meson has been measured, giving $h' = 0.36 \pm 0.09$, with a $\mathcal{O}(30\%)$ deviation form the charm value. The computed widths of the $s^P_\ell = 3/2^+$ beauty states are: $\Gamma(B_1) = 13.6 \pm 0.6$ MeV, $\Gamma(B_{s1}) = 0.016 \pm 0.002$ MeV and $\Gamma(B^*_{s2}) = 0.9 \pm 0.1$ MeV, the last one compatible with the recent LHCb result [28].

$\tilde{g}$ governs the decays $\tilde{H} \to HM$, with $\tilde{H}$ the radial excitations of H. Observed states that fit in such a doublet, with and without strangeness, are $D(2550)$, $D^*(2600)$ and the strange one $D^*_{s1}(2700)$. From their measured widths we obtain the value in Table 7. The predicted width of the spin partner of $D^*_{s1}(2700)$ using the mass fixed in Sec. 3, is in Table 5, and the expected widths of the corresponding beauty resonances are in Table 6.

k**.** In the classification of $D_{sJ}(2860)$ as the $J^P = 3^-$ state of the X' doublet, the resonances ($D(2750)$, $D^*(2760)$) fill the corresponding non strange doublet. From their mass and width we obtain the coupling $k = k_1 + k_2$ in Table 7. This allows to predict the width of the D'^*_{s2}, the spin partner of $D_{sJ}(2860)$, and of the analogous beauty state, see Tables 5 and 6. The results from other assignments to $D_{sJ}(2860)$ are discussed in [4].

Information comes from ratios of decay rates in which the dependence on the strong cou-

Table 7: Coupling constants in the effective Lagrangian (8), obtained from the experimental data and using the classification in Table 1.

g	h	h'	$\tilde{g}$	k
0.64 ± 0.075	0.56 ± 0.04	0.43 ± 0.01	0.28 ± 0.015	0.42 ± 0.02

plings cancels out. For a meson $F_{(s)}$ decaying to $P_{(s)}\,M$ and $P^*_{(s)}\,M$, these ratios are relevant:

$$\begin{aligned} R_\pi^{(F)} &= \frac{\mathcal{B}(F \to D^*\pi)}{\mathcal{B}(F \to D\pi)} \quad , \\ R_K^{(F_s)} &= \frac{\mathcal{B}(F_s \to D^*K)}{\mathcal{B}(F_s \to DK)} \;,\; R_\eta^{(F_s)} = \frac{\mathcal{B}(F_s \to D_s\eta)}{\mathcal{B}(F_s \to DK)} \;,\; R_\eta^{*(F_s)} = \frac{\mathcal{B}(F_s \to D_s^*\eta)}{\mathcal{B}(F_s \to DK)} \quad . \end{aligned} \tag{11}$$

$D^{(*)}\pi(K)$ indicates $D^{(*)0}\pi^+(K^+) + D^{(*)+}\pi^0(K_S)$ for charged states and to $D^{(*)0}\pi^0(K_S) + D^{(*)+}\pi^-(K^-)$ for neutral ones. Table 8 reports the predictions for $D^*(2600)$ and $D^*_{s1}(2700)$, identified with $\tilde{D}^*$ and $\tilde{D}^*_s$, respectively; for $D_2^{*0}(2460)$ and $D^*_{s2}(2573)$, and for $D^*(2760)$ and $D_{sJ}(2860)$ identified with D_3 and D_{s3}. A detailed discussion is in [4]. Here we only mention a few issues.

- For $D^*_{s1}(2700)$, the results in Table 8 agree with the measurement in Eq.(9) [19], supporting the classification of this state as $\tilde{D}^*_{s1}$.
- Identifying $D^*(2760)$ with D_3 and $D(2750)$ with its spin partner $D_2'^*$, one obtains the ratio $\frac{\mathcal{B}(D^{*0}(2760) \to D^+\pi^-)}{\mathcal{B}(D^{*0}(2750) \to D^{*+}\pi^-)}\Big|_{X'} = 0.660 \pm 0.001$. On the other hand, in the hypothesis that $(D(2750),\, D^*(2760))$ fill the $(\tilde{D}_1',\, \tilde{D}_2^*)$ doublet, the result is $\frac{\mathcal{B}(D^{*0}(2760) \to D^+\pi^-)}{\mathcal{B}(D^{*0}(2750) \to D^{*+}\pi^-)}\Big|_{\tilde{T}} = 0.563 \pm 0.001$. The measurement (10) does not discriminate between the two possibilities.
- If $D_{sJ}(2860)$ is identified with D_{s3}, the ratios in Table 8 do not compare favorably with the measurement in Eq.(9) [21]. A possible reason is the existence of the spin partner with very close mass, $M(D^{*\prime}_{s2}) = 2851 \pm 7$ MeV, difficult to resolve in the common D^*K decay mode. If the signal measured to give Eq.(9) includes the decay $D^{*\prime}_{s2} \to D^*K$, the actual measurement is the $D^{(*)}K$ sample produced from both the states, hence

  $$\bar{R}(2860) = \frac{\Gamma(D_{sJ}(2860) \to D^*K) + \Gamma(D^{*\prime}_{s2}(2851) \to D^*K)}{\Gamma(D_{sJ}(2860) \to DK)} \quad ,$$

  whose prediction is: $\bar{R}(2860) = 0.99 \pm 0.05$, compatible with (9).
- For the beauty system, the computed ratio R_K for B^*_{s2} is confirmed by the LHCb measurement: $R_K = (9.1 \pm 1.3 \pm 1.2) \times 10^{-2}$ [28].

5 Conclusions

Using the heavy quark symmetry as a guideline, the observed $c\bar{q}$ and $b\bar{q}$ mesons can be classified in doublets. Of course, finite heavy quark mass effects, such as those inducing a mixing between states with the same J^P belonging to different doublets, could distort the picture: their

Table 8: Computed ratios $R_M^{(F)}$.

$c\bar{q}$	R_π	$c\bar{s}$	R_{K^0}	R_η	R^*_η
$D^{*0}(2600)$	1.22 ± 0.01	$D^*_{s1}(2700)$	0.91 ± 0.03	0.195 ± 0.006	0.05 ± 0.01
$D_2^{*0}(2460)$	0.440 ± 0.001	$D^*_{s2}(2573)$	0.086 ± 0.002	0.018 ± 0.001	-
$D^{*0}(2760)$	0.514 ± 0.004	$D_{sJ}(2860)$	0.39 ± 0.01	0.132 ± 0.003	0.025 ± 0.001
$b\bar{q}$	R_π	$b\bar{s}$	R_K	R_η	R^*_η
$\tilde{B}^*$	1.63 ± 0.005	$\tilde{B}^*_s$	1.43 ± 0.015	0.132 ± 0.008	0.11 ± 0.015
B_2^*	0.87 ± 0.01	B^*_{s2}	0.07 ± 0.005	-	-
B_3	0.92 ± 0.005	B_{s3}	0.815 ± 0.006	0.103 ± 0.002	0.063 ± 0.003

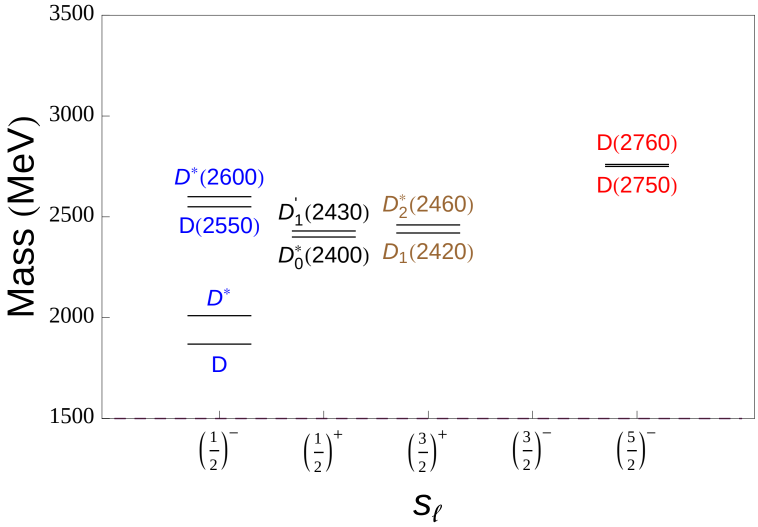

Figure 1: Spectrum of $c\bar{q}$ mesons organized in spin doublets. The structures denoted as $D_J(3000)^0$ and $D^*_J(3000)^0$ [10] are not included in the plot.

description in terms of the effective theory would require additional parameters. *A posteriori*, looking at data, one can estimate the size of such effects, and check the scaling rules and the main features determined by the heavy quark symmetry.

A comprehensive assignment is proposed in Table 1; in the case of charm, the spectrum is depicted in Figs.1 and 2. The properties of missing states are predicted accordingly. A wealth of new interesting information is expected from the ongoing experiments.

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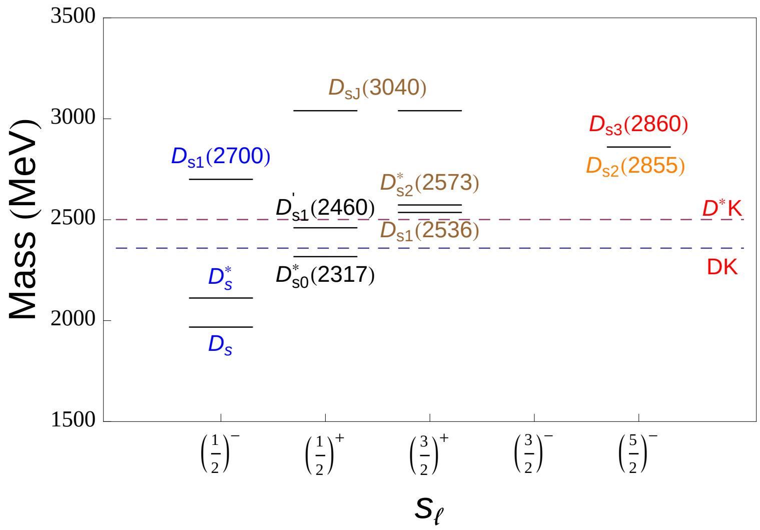

Figure 2: Spectrum of $c\bar{s}$ mesons organized in spin doublets. The horizontal dashed lines correspond to the DK and D^*K threshold. Two possible classifications for $D_{sJ}(3040)$ are shown. The predicted $D_{s2}(2855)$ is included in the plot.

References

[1] M. Neubert, Phys. Rept. **245**, 259 (1994); T. Mannel, In *St. Goar 1996, Heavy quarks at fixed target* 107-129; arXiv:hep-ph/9611411; F. De Fazio, in *At the Frontier of Particle Physics/Handbook of QCD*, ed. by M. Shifman (World Scientific, Singapore, 2001), page 1671, arXiv:hep-ph/0010007.

[2] N. Isgur and M. B. Wise, Phys. Lett. B **237**, 527 (1990).

[3] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991); M. Lu, M. B. Wise and N. Isgur, Phys. Rev. D **45**, 1553 (1992).

[4] P. Colangelo, F. De Fazio, F. Giannuzzi and S. Nicotri, Phys. Rev. D **86**, 054024 (2012).

[5] M. B. Wise, Phys. Rev. D **45**, 2188 (1992); G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992); P. L. Cho, Phys. Lett. B **285**, 145 (1992); Phys. Rev. D **46**, 1148 (1992) [Erratum-ibid. D **55**, 5851 (1997)]; R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B **299**, 139 (1993).

[6] A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **10**, 1 (2000).

[7] A. F. Falk and T. Mehen, Phys. Rev. D **53** (1996) 231.

[8] J. Beringer et al. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012) and http://pdg.lbl.gov/.

[9] P. del Amo Sanchez *et al.* [The BABAR Collaboration], Phys. Rev. D **82**, 111101 (2010).

[10] R. Aaij *et al.* [LHCb Collaboration], JHEP **1309**, 145 (2013).

[11] S. Anderson *et al.* [CLEO Collaboration], Nucl. Phys. **A663**, 647 (2000); J. M. Link *et al.* [FOCUS Collaboration], Phys. Lett. **B586**, 11 (2004).

[12] K. Abe *et al.* [Belle Collaboration], Phys. Rev. D **69**, 112002 (2004).

[13] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **90**, 242001 (2003); Y. Mikami *et al.* [Belle Collaboration], Phys. Rev. Lett. **92**, 012002 (2004); K. Abe *et al.*, Phys. Rev. Lett. **92**, 012002 (2004); D. Besson *et al.* [CLEO Collaboration], Phys. Rev. D **68**, 032002 (2003); L. Benussi [FOCUS Collaboration], Int. J. Mod. Phys. A **20**, 549 (2005); B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. **D69**, 031101 (2004); P. Krokovny *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262002 (2003); P. Krokovny, AIP Conf. Proc. **717**, 475 (2004).

[14] P. Colangelo, F. De Fazio and A. Ozpineci, Phys. Rev. D **72**, 074004 (2005).

[15] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek and R. M. Woloshyn, Phys. Rev. Lett. **111**, 222001 (2013).

[16] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97**, 222001 (2006).

[17] J. Brodzicka *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 092001 (2008).

[18] R. Aaij *et al.* [LHCb Collaboration], arXiv:1207.6016 [hep-ex].

[19] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **80**, 092003 (2009).

[20] P. Colangelo, F. De Fazio, S. Nicotri and M. Rizzi, Phys. Rev. D **77**, 014012 (2008).

[21] P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. B **642**, 48 (2006).

[22] M. Di Pierro and E. Eichten, Phys. Rev. D **64**, 114004 (2001).

[23] P. Colangelo and F. De Fazio, Phys. Rev. D **81**, 094001 (2010).

[24] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[25] E. van Beveren and G. Rupp, Phys. Rev. D **81**, 118101 (2010); Z. G. Wang, Phys. Rev. D **83**, 014009 (2011).

[26] P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. B **345**, 598 (1995); D. Buskulic *et al.* [ALEPH Collaboration], Z. Phys. C **69**, 393 (1996); R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. B **425**, 215 (1998); M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **465**, 323 (1999); R. Akers *et al.* [OPAL Collaboration], Z. Phys. C **66**, 19 (1995).

[27] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **99**, 172001 (2007). T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **102**, 102003 (2009). T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100**, 082001 (2008). V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **100**, 082002 (2008).

[28] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **110**, 151803 (2013).

[29] P. Colangelo and F. De Fazio, Phys. Lett. B **570**, 180 (2003).

[30] P. Colangelo, F. De Fazio and R. Ferrandes, Mod. Phys. Lett. A **19**, 2083 (2004); P. Colangelo, F. De Fazio and R. Ferrandes, Phys. Lett. B **634**, 235 (2006).

[31] T. A. Aaltonen *et al.* [CDF Collaboration], arXiv:1309.5961 [hep-ex].

[32] [LHCb Collaboration], LHCb-CONF-2011-053.

[33] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. Lett. 111, **111801** (2013) [Phys. Rev. Lett. **111** (2013) 111801]; J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **88**, 052003 (2013).

[34] P. Colangelo, G. Nardulli, A. Deandrea, N. Di Bartolomeo, R. Gatto and F. Feruglio, Phys. Lett. B **339**, 151 (1994); P. Colangelo, F. De Fazio and G. Nardulli, Phys. Lett. B**334**, 175 (1994); V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D **51**, 6177 (1995); D. Becirevic, B. Blossier, E. Chang and B. Haas, Phys. Lett. B **679**, 231 (2009).

[35] P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo and R. Gatto, Phys. Rev. D **52** (1995) 6422; P. Colangelo and F. De Fazio, Eur. Phys. J. C **4** (1998) 503.

[36] D. Becirevic, E. Chang and A. L. Yaouanc, arXiv:1203.0167 [hep-lat].

[37] D. Becirevic and A. L. Yaouanc, JHEP **9903** (1999) 021; B. El-Bennich, M. A. Ivanov and C. D. Roberts, Phys. Rev. C **83**, 025205 (2011); K. U. Can, G. Erkol, M. Oka, A. Ozpineci and T. T. Takahashi, Phys. Lett. B **719**, 103 (2013); D. Becirevic and F. Sanfilippo, Phys. Lett. B **721**, 94 (2013).

[38] J. P. Lees *et al.* [The BABAR Collaboration], Phys. Rev. D **83**, 072003 (2011).

[39] V. Balagura *et al.* [Belle Collaboration], Phys. Rev. D **77**, 032001 (2008).