Lectures on new physics searches in $B \to D^{(*)}\tau\nu_\tau$

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The Standard model's predictions for the rates for $B \to D^*\tau\nu_\tau$ and $B \to D\tau\nu_\tau$ differ from the experimental results. The difference might be accounted by the presence of new physics. The understanding of the non-perturbative QCD dynamics in the meson transitions is crucial in order to refine the searches for new physics effects. We give short introduction to heavy quark effective theory (HQET) and then investigate the most general set of lowest dimensional effective operators leading to helicity suppressed modifications of $b \to c$ (semi)leptonic transitions. The contributions of these operators to $B \to D^{(*)}\tau\nu_\tau$ decay amplitudes can be found by determining the differential decay rate, longitudinal D^* polarization fraction, $D^* - \tau$ opening angle asymmetry and the τ helicity asymmetry. We identify the size of possible new physics contributions constrained by the present $B \to D^{(*)}\tau\nu_\tau$ rate measurements and find significant modifications are still possible in all these observables. Then we discuss few models of new physics scenarios which can contribute in both decay modes.

1 Introduction

In these lecture notes we present the short introduction to theoretical aspects of semileptonic decays [1] of B mesons, with an emphasis on the search for New physics (NP) in $B \to D^{(*)}\tau\nu_\tau$ processes. The semileptonic transitions are driven by the charged current interactions that originate from the exchange of the W boson in the Standard Model (SM). In theories beyond Standard Model (BSM), new particle could affect the physics of the decays as well. We explore these possibilities in subsequent sections.

The semileptonic decays have played significant role in the history of the particle physics, providing the basis for the construction of the SM. The four fermion interaction (Fermi's theory) was constructed as model of beta decays of nuclei. It has been further modified to include the parity violation through $V - A$ interactions [2], [3] and strangeness changing decays (Cabbibo mixing [4]). The theory breaks down at sufficiently high energies as it contains the dimensionful coupling parameter, and consequently a physical (electroweak) scale, v. The need for deeper, short distance understanding has been realized in a form of intermediate vector boson theory involving charged, massive spin one mediator. The developments that followed were leading towards the SM theory. The physics of the electroweak scale is currently probed at the Large Hadron Collider (LHC) experiments.

Following the observation of CP violation in weak interactions [5], Kobayashi and Maskawa (KM) [6] suggested that the CP violation can be explained with the introduction of the third generations of quarks. Consequently, the quark mixing matrix (known as Cabibbo-Kobayashi-

[1]involving lepton(s) and a hadron in the final state

Maskawa (CKM) matrix) can be parametrized in terms of three angles and one imaginary phase. The imaginary phase cannot be absorbed through the redefinitions of the quark fields and is source of all CP violation in the SM. This mechanism has been experimentally confirmed to be the dominant origin of the CP violation (see e.g. [7] and references therein), which led to the Nobel prize awarded to Kobayashi and Maskawa in 2008. The semileptonic decays are used for the extraction of the corresponding CKM elements, e.g. Wolfenstein's parameters λ and A [22] are precisely determined from the $K \to \pi\ell\nu$ and $b \to c\ell\nu$ transitions respectively. The underlying assumption is that these processes are fully described by the SM.

The CKM fits show impressive agreement with the KM mechanism, see e.g. [9], [10]. It is, however, worth to note that the fit gets significantly worse when the results of branching ratio of tauonic $B \to \tau\nu$ decay is included [1], [9], [10].

B physics provides some stringent tests of the SM at low energies. Recent measurements of $\mathcal{B}(B_s \to \mu^+\mu^-)$ and CP violation in $B_s \to J/\psi\phi$ decays considerably constrain contributions of NP to these observables. Semileptonic B decays play an important role in B physics, as their branching ratios are rather large and allow for extensive experimental studies. The decays are schematically represented by the diagram in the Fig. 1. The inclusive $B \to X_c\ell\nu$ and exclusive (with $D^{(*)}$ in the final state) processes are used for determination of V_{cb} matrix element. Inclusive determination uses the differential decay spectrum of final lepton's energy and hadronic, q^2 spectrum. It relies on operator product expansion (OPE) and Heavy Quark Effective Theory (HQET), [23], [24]. Average result of inclusive determinations is given in [1], $|V_{cb}| = (41.9 \pm 0.7) \cdot 10^{-3}$. Exclusive determinations also rely on HQET. In the infinite quark mass limit, all form factors are given by Isgur-Wise function, the function of product of four-velocities of B and $D^{(*)}$ mesons. Heavy Quark Symmetry defines normalization rate at $w = 1$ a point of maximal momentum transfer, $q^2 = (m_B - m_{D^{(*)}})^2$ and V_{cb} is obtained from extrapolation to $w = 1$. Exclusive determinations are less precise at the present and give [1] $|V_{cb}| = (39.6 \pm 0.9) \times 10^{-3}$.

Recently, there has been an increased interest in study of NP effects in semileptonic decays of B mesons involving tau leptons in the final state after the BaBar Colalboration published the results that show the excess in the following ratios [25]

$$\mathcal{R}^*_{\tau/\ell} = \mathcal{B}(B \to D^*\tau\nu)/\mathcal{B}(B \to D^*\ell\nu) = 0.332 \pm 0.030\,, \tag{1}$$

$$\mathcal{R}_{\tau/\ell} = \mathcal{B}(B \to D\tau\nu)/\mathcal{B}(B \to D\ell\nu) = 0.440 \pm 0.072\,. \tag{2}$$

Both results are consistent with measurements previously performed by Belle Collaboration [26]. The BaBar's results turned out to be larger than the SM predictions $\mathcal{R}^{*,\mathrm{SM}}_{\tau/\ell} = 0.252(3)$ [43] and $\mathcal{R}^{\mathrm{SM}}_{\tau/\ell} = 0.296(16)$ [13], [43] with $3.4\,\sigma$ significance when the two observables are combined [42]. The eventual confirmation of these result might point to effects of NP in $b \to c\ell\nu$ transitions. We interpret these results as signs of the NP and correspondingly study the NP contributions through the effective field theory formalism. Consequently we discuss several specific models that can produce the specific effective higher dimensional operators.

Some of leading questions in the flavour physics are related to so called SM *flavour puzzle*. The flavour parameters (masses, mixing angles and a KM phase) are hierarchical; the quark masses span several orders of magnitude and are all (except the top's mass) much smaller than the electroweak scale. Since the *SM* can be taken as an effective description of physics at low energies, its Lagrangian may be supplemented with dimension six quark flavour changing operators that parametrize the FCNCs which appear at subleading order in the SM. If the NP has generic flavour structure such that the dimensionless couplings in these operators are of order $\mathcal{O}(1)$ then the measurements of $FCNC$ observables constrain the scale of new physics to be bigger than $\sim 10^2 \cdots 10^4$ TeV [7]. However, the unnaturalness of the Higgs boson mass parameter is widely believed to be a problem which seeks for the resolution at the scale of around one TeV. Thus, if the new physics that solves the hierarchy problem exists at this scale, then it is constrained to have non-generic structure with couplings that resemble the SM. If this is the case, there is so called NP *flavour puzzle*.

There are several ways of dealing with the flavour violations in the NP models. One possibility is that of an Natural Flavour Conservation (NFC). As an example, the procedure of diagonalization of the mass matrices in the SM leaves the Yukawa couplings of quarks to Higgs boson flavour diagonal. Introduction of the second scalar doublet is theoretically well motivated for several reasons (see [12]). However it leads to appearance of two Yukawa matrices which are in general not simultaneously diagonalizable. Weinberg and Glashow [20] and Paschos [21] noted that if the quark of given helicity and charge has Yukawa interactions with only one Higgs doublet, FCNC Yukawa interactions are avoided at Lagrangian level. This can be achieved by the imposing the new discrete symmetries (for the most recent review of 2HDMs see e.g. [12]. The NP models in which all flavour and CP violation (in physical basis) originates from the CKM matrix belong to the class of models that satisfy Minimal Flavour Violation (MFV), [15], [16], see also lectures [17]. Within this criterion it is possible to relate the flavour violation for different sectors (e.g. FCNCs in processes involving B and K mesons [16]). We return to these scenarios through some specific examples later. The lectures are divided in the following sections: after introduction, sec. 2 describes parametrization of the amplitudes and form factors, sec. 3 introduces basic elements of heavy quark effective theory. In sec. 4 and 5 we consider the possible effects of the charged scalars from 2HDMs that couple more strongly to massive tau leptons and whose impact on the semileptonic processes involving the light lepton in final state is negligible. In sec. 6 we discuss the leptoquark model with the particular phenomenological ansatz for Yukawa couplings with leptons and quarks. The particular form of the ansatz can be consistently embedded into realistic GUT model.

2 Parametrization of the amplitudes and form factors

2.1 $B \to D\ell\nu_\ell$

The amplitude for the process $B \to D\ell\nu$ is given by product of matrix elements of the vector minus axial $(V-A)$ quark (hadronic) current $H_\mu \equiv \bar{c}\gamma_\mu(1-\gamma_5)b$ between the B and D meson states and leptonic $V-A$ current, $L_\mu = \bar{l}\gamma_\mu(1-\gamma_5)\nu_l$ between states of vacuum and $l-\nu$ pair,

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{cb} \langle l(k_1), \bar{\nu}_l | L_\mu | 0 \rangle \times \langle D(p') | H^\mu | \bar{B}(p) \rangle. \tag{3}$$

Both $\bar{B}(b\bar{q})$ and $D(c\bar{q})$ mesons are pseudoscalars $(J^P = 0^-)$, so the the matrix element of the axial current between states B and D vanishes, due to the conservation of parity in QCD. It is easy to understand this fact if we note that the axial current changes sign under parity transformations, so that overall matrix element changes sign. The matrix element of the vector current needs a non-perturbative QCD evaluation. We may use the Lorentz covariance to parametrize it in terms of *form factors*

$$\langle D(p') | V^\mu | \bar{B}(p) \rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu, \tag{4}$$

where the squared transferred momentum varies in the range $m_l^2 \leq q^2 \leq (m_B - m_D)^2$. Alternatively, the matrix element can be parametrized by:

$$\langle D(p_D)|\bar{c}\gamma^\mu b|\bar{B}(p_B)\rangle = \left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right) f_+(q^2) + \frac{m_B^2 - m_D^2}{q^2} q^\mu f_0(q^2)\,, \tag{5}$$

and the form factor $f_0(q^2)$ is suppressed in the case of light lepton, as can be seen from the formula for the decay rate:

$$\begin{aligned}\frac{d\Gamma}{dq^2}(B \to D\ell\bar{\nu}_\ell) =& \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_B^3}\left(1 - \frac{m_l^2}{q^2}\right)^2 \lambda^{1/2}\Bigg[\lambda\left(1 + \frac{m_l^2}{2q^2}\right) f_+(q^2)^2 \\ &+ \frac{3}{2}\frac{m_l^2}{q^2}(m_B^2 - m_D^2)^2 f_0(q^2)^2\Bigg],\end{aligned} \tag{6}$$

where function λ is given by $\lambda(m_B^2, m_D^2, q^2) = (m_B^2 - m_D^2 - q^2)^2 - 4m_D^2 q^2$. Also, in order to avoid the spurious pole at $q^2 = 0$, the kinematic constraint $f_+(0) = f_0(0)$ is implied.

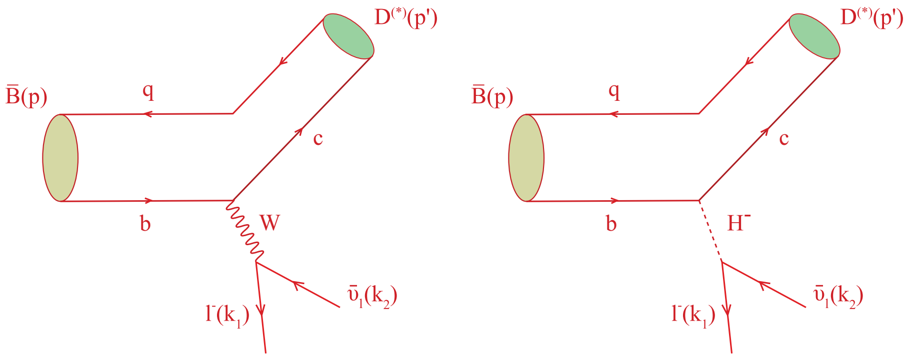

Figure 1: Diagrams contributing to the semileptonic B decays: SM exchange of W boson (left) and the exchange of the charged Higgs boson from the extended scalar sector (right)

In models that include charged scalar, the matrix element of scalar density is used $\langle D|\bar{c}b|B\rangle$. We may use the (anomalous Ward's) identity

$$q_\mu\langle D|\bar{c}\gamma^\mu b|B\rangle = (m_b - m_c)\langle D|\bar{c}b|B\rangle, \tag{7}$$

to derive the formula for scalar density:

$$\langle D|\bar{c}b|B\rangle = \frac{m_B^2 - m_D^2}{m_b - m_c} f_0(q^2). \tag{8}$$

In the formula for decay rate, this term is also suppressed by m_l^2/q^2, so that charged scalars do not influence the decays involving light leptons.

Recently Fermilab Lattice and MILC Collaborations [28] performed the calculation of $f_+(q^2)$

and $f_0(q^2)$ form factors in $2+1$ lattice QCD using the Fermilab's action [27]. The results are presented in the Fig. 1. in [28]. They calculate the following two observables:

$$\begin{aligned} R(D) &= 0.316(12)(7) \\ P_L(D) &= 0.325(4)(3). \end{aligned} \tag{9}$$

Also, authors of the Ref. [29] find the similair result ($R(D) = 0.31 \pm 0.02$) by combining the experimental and theoretical input.

2.1.1 $B \to D^* \ell \nu$

The matrix elements of the $V - A$ current between the pseudo-scalar $\bar{B}$ and vector D^* mesons depend on four independent form factors, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$ and $A_2(q^2)$

$$\langle D^*(p', \epsilon_\alpha)|\bar{c}\gamma_\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} p^\alpha p'^\beta , \tag{10a}$$

$$\begin{aligned} \langle D^*(p', \epsilon_\alpha)|\bar{c}\gamma_\mu\gamma_5 b|B(p)\rangle &= 2m_{D^*} A_0(q^2)\frac{\epsilon^* \cdot q}{q^2} q_\mu + (m_B + m_{D^*}) A_1(q^2) \left(\epsilon^*_\mu - \frac{\epsilon^* \cdot q}{q^2} q_\mu\right) \\ &\quad - A_2(q^2) \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} \left((p+p')_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu\right) . \end{aligned} \tag{10b}$$

The calculation of the matrix element of $V - A$ current in $B \to D^*$ transition turns out to be untrivial problem in Lattice QCD and results are unavailable at this moment. However, we may learn something about these form factors by using the HQET, which is the topic of the next section. We note that the form factor $A_0(q^2)$ does not enter the decay rates of the decays that involve the leptons of negligible mass (electron, muon). It is important to learn more about this form factor from the non-perturbative QCD, as it may hide the resolution for the current disagreement with the experiment.

3 Heavy quark effective theory and B decays

In this section we present the short introduction to Heavy quark symmetry and corresponding effective theory. More detailed and complete expositions of the subject can be found in numerous reviews. Clear exposition is given by [34], where also the higher order corrections are explained.

The degrees of freedom (fields) of Quantum Chromodynamics (QCD) at short distance are quarks and gluons. Lagrangians that describe the phenomena change through Renormalization Group (RG) transformations. This is the leading idea of Wilsonian effective field theory. The important feature of the QCD is asymptotic freedom[2]. At short distances, or equivalently in processes characterized by high momentum transfer, the effective gauge coupling becomes weak and the perturbative methods of calculation are well applicable. In deep infra-red, instead of gluons and quarks it is often more useful to define the theory in terms of another effective degrees of freedom. Such Lagrangians are based on some approximate symmetries of QCD. In principle, it is possible to match them to fundamental QCD Lagrangian, but because QCD is genuine strongly coupled theory at large distances, this is rarely possible in practice. At

[2] This property is in four dimensions unique to the non-Abelian theories, QCD being an example.

the energy scales smaller than approximately $\Lambda_{QCD} = 0.2\,GeV$, new complex structures arise which are intractable to analytical calculation tools. Such a phenomenon is confinement of gluons and quarks. The hadronic properties are therefore described within QCD using the numerical calculations on space-time lattices.

On the other hand, the progress has been made by discovering the approximate symmetries of the hadronic systems; the earliest example being the isospin symmetry. This is approximate symmetry that arises due to the difference in mass of up and down quarks, $m_u - m_d$ much smaller than the characteristic QCD scale. Another example is chiral symmetry $SU(2)_L \times SU(2)_R$, the approximate symmetry of QCD that originates from the observation that masses of both u and d quarks are much smaller than the QCD scale. The treatment simplifies by going to the effective theory where $m_{u,d}$ are set to zero. Chiral symmetry is spontaneously broken in reality, but the resulting effective theory (Chiral Perturbation Theory) allows systematic calculations of corrections of order $m_{u,d}/\Lambda_{QCD}$.

In this chapter we will explain the basic physical picture of HQET, which we construct when we recognize the new spin-flavour symmetry of QCD of the systems containing one heavy quark (c or b). We will be interested in the hadron that contains a heavy quark whose mass $m_Q \gg \Lambda_{QCD}$ and light degrees of freedom which we denote light cloud (complicated cloud of light quarks and soft gluons[3]). In such hadron, and here we will consider mesons, mass becomes rather irrelevant for the non-perturbative dynamics of the light cloud. Exchange of the momentum between light cloud and heavy quark are of the order of Λ_{QCD} and the changes in four-velocity[4] of heavy quark are of order Λ_{QCD}/m_Q, so quark can be modelled as static source of colour with conserved velocity. The creation of heavy quark-antiquark pairs is absent. This also means that light cloud does not probe the relativistic degrees of freedom of heavy quark, so its spin and colour magnetism decouple.

It is then instructive to construct the effective theory in which m_Q can be taken to infinity while keeping the velocity of heavy quark fixed. Flavour of heavy quark can be changed by the interaction with some external current (i.e. through W boson field) but as long as the new flavour is also heavier than the QCD scale, light cloud stays the same. This observations still does not allow us to calculate the properties of the light cloud, but is useful in finding the connections between the properties of different mesons containing heavy quarks. Also, the systematic method of obtaining the corrections of order $1/M_Q$ will be provided. The situation is reminiscent of the well known observation in atomic physics in which chemical properties of the atoms are independent on the isotope of the nucleus. The only parameters that matters is electric charge of nucleus, while its spin and mass decouple, up to some required precision.

3.1 HQET Lagrangian

The HQET is constructed to give simple description of processes in which heavy quark interacts with the light quark by the exchange of soft gluons. The high energy scale (cutoff) of this theory is of order m_Q. Momentum of heavy quark is

$$p_Q^\mu = p_M^\mu - q^\mu = m_Q v^\mu + k^\mu \tag{11}$$

where p_M is momentum of meson, q^μ is momentum of the light cloud, and we define the residual momentum as $k^\mu = (m_M - m_Q)v^\mu - q^\mu$, much smaller than the m_Q. Velocity is normalized

[3] Light cloud is also called "brown muck" in some literature.

[4] In the rest of the text the four-velocity is simply denoted as velocity.

to $v^2 = 1$ in our metric convention. The velocity of the heavy quark is $v_Q^\mu = \frac{p_Q^\mu}{m_Q} = v^\mu + \frac{k^\mu}{m_Q}$ and we notice that in the limit $m_Q \to \infty$ meson travels at the same four-velocity as heavy quark. This means that the interaction with the light cloud leaves the velocity of heavy quark conserved.
Let us approach to construction of HQET by writing the heavy quark field in the following form:

$$Q(x) = e^{-m_Q v\cdot x}[Q_v(x) + \mathcal{Q}_v(x)], \tag{12}$$

where new fields are defined as

$$\begin{aligned} Q_v(x) &= e^{m_Q v\cdot x}\frac{1+\not{v}}{2}Q(x), \\ \mathcal{Q}_v &= e^{m_Q v\cdot x}\frac{1-\not{v}}{2}Q(x). \end{aligned} \tag{13}$$

Notice that fields $Q_v(x)$ and $\mathcal{Q}_v(x)$ are constrained by:

$$\not{v}Q_v(x) = Q_v(x), \;\; \not{v}\mathcal{Q}_v(x) = -\mathcal{Q}_v(x). \tag{14}$$

Field $Q_v(x)$ produces the effects at leading order, whereas the field $\mathcal{Q}_v$ produces the $1/m_Q$ effects. We work in Dirac basis of gamma matrices, in which $Q(x)_v$ is upper component of quark Dirac spinor, as it can be seen from first equation in (13), because the matrix $(1+\not{v})/2$ becomes $(1+\gamma_0)/2$, in the rest frame of heavy quark.

Field $Q(x)$ annihilates heavy quark with velocity v, but does not create antiquark. This field is called "large component". Since in the HQET the creation of heavy quark-antiquark pairs is absent, quark and antiquark live in totally different regions in momentum space, infinitely far away in the limit $m_Q \to \infty$. For simplicity, we will now deal with one quark field only, although everything can be done with antiquark also; the only changes are $v \to -v$ and $Q_v \to \mathcal{Q}_v$. In that case the effects of the quark component are absent [31].

Let us insert the expansion (12) into the relevant kinetic part of QCD Lagrangian to get:

$$\begin{aligned} \mathcal{L} &= \bar{Q}(i\not{D} - m_Q)Q(x) \\ &= \bar{Q}_v iv\cdot DQ_v - \bar{\mathcal{Q}}_v(iv\cdot D + 2m_Q)\mathcal{Q}_v + \bar{\mathcal{Q}}_v i\not{D}_\perp Q_v + Q_v i\not{D}_\perp \bar{\mathcal{Q}}_v, \end{aligned} \tag{15}$$

where $D_\perp^\mu = D^\mu - v^\mu v\cdot D$. For illustration we give explicit derivation of the first term in above formula:

$$\begin{aligned} &\bar{Q}_v e^{m_Q\, v\cdot x}(i\not{D} - m_Q)e^{m_Q\, v\cdot x}Q_v \\ &= \bar{Q}_v[m_Q(\not{v} - 1) + i\not{D}]Q_v \\ &= \bar{Q}_v iQ_v\not{D}Q_v \\ &= \bar{Q}_v\frac{1+i\not{v}}{2}i\not{D}\frac{1+i\not{v}}{2}Q_v \\ &= \bar{Q}_v iv\cdot DQ_v. \end{aligned} \tag{16}$$

where third and fourth row are obtained by use of constraints (14). A remark is in order at this point. The QCD Lagrangian (15) is the effective Lagrangian whose parameters are defined at the scale of order of heavy quark mass, while the effects of short distance gluons are integrated out. It is the fact that after integrating out short distance field modes a la Wilson, besides the running of dimensionless couplings, the tower of higher dimensional operators appears. We neglect these operators in the Lagrangian(15), but their effects can be introduced through radiative α_S corrections, which are perturbative due to the asymptotic freedom.

From the Lagrangian (15) we see that field $Q_v(x)$ is massless, while the field component $\mathcal{Q}_v$ has mass $2m_Q$. These massive degrees of freedom we integrate out. We could proceed by writing the generating functional for QCD with the Lagrangian (16), and then explicitly solve the path integral for field $\mathcal{Q}_v$, which would lead to the generating functional determined by the action functional containing the HQET Lagrangian. This method of derivation has been achieved in Ref. [39].
Instead of performing this procedure, we will use the observation that the integrating out the dynamical degree of freedom is equivalent up to overall normalization constant to solving the equation of motion for the variable and then substituting back to the Lagrangian, under the condition that the integral is of Gaussian type. It turns out that the renormalization constant is functional determinant that can be absorbed into the normalization of generating functional in gauge invariant manner, (see Ref. [39]).
We first insert the expansion (12) of the quark field into the QCD equation of motion that follows from the Lagrangian (15) and get the:

$$i\not{D}Q_v + (i\not{D} - 2m_Q)\mathcal{Q}_v = 0 \tag{17}$$

Multiplying the above equation by $P_- = \frac{1-\not{v}}{2}$ and solving for $\mathcal{Q}_v$ one gets:

$$\mathcal{Q}_v = \frac{1}{iv\cdot D + 2m_Q - i\epsilon} i\not{D}_\perp Q_v. \tag{18}$$

One can see that small component is really suppressed by powers of order $1/m_Q$, and after the insertion of this relation to the starting Lagrangian (15), one obtains the Lagrangian of HQET:

$$\mathcal{L}_{eff.} = \bar{Q}_v iv\cdot DQ_v + \bar{Q}_v i\not{D}_\perp \frac{1}{iv\cdot D + 2m_Q - i\epsilon} i\not{D}_\perp Q_v. \tag{19}$$

We can now expand the non-local second term from the above Lagrangian in powers of $1/m_Q$. We use the following identity

$$\bar{Q}_v \not{D}_\perp \not{D}_\perp = \bar{Q}_v D^2 Q_v - \bar{Q}_v (D\cdot v)^2 Q_v + \frac{g}{2}\bar{Q}_v \sigma^{\mu\nu}F_{\mu\nu}Q_v. \tag{20}$$

The resulting expansion up to first order in $1/m_Q$ is then the following one

$$\mathcal{L}_{HQET} = \bar{Q}_v\, i\, vDQ_v + \frac{1}{2m_Q}\bar{Q}_v D^\mu(\eta_{\mu\nu} - v_\mu v_\nu)D^\nu Q_v + \frac{g}{4m_Q}\bar{Q}_v\sigma^{\mu\nu}F_{\mu\nu}Q_v. \tag{21}$$

In the heavy quark rest frame ($\vec{v} = 0$) $1/m_Q$ terms correspond to non-relativistic kinetic energy term and QCD version of Pauli's term, respectively. In the infinite mass limit, only the first term in the Lagrangian is present and the HQ symmetries are evident. In the presence of two heavy quarks (b and c) the Lagrangian contains the sum of two corresponding terms. Since there is no dependence in m_Q, the $SU(2)$ flavour symmetry emerges. Coupling of the gluons to spin of the heavy quark is also contained in higher order term - we get the spin symmetry. One can conclude that the heavy quark symmetry group is then $SU(4)$. The new symmetry implies some immediate application in spectroscopy of heavy mesons, see e.g. [36].

3.2 Weak matrix elements and HQ

Let us now introduce the basic physical picture that allows the derivation of the relations between different matrix elements of electroweak currents.
In HQ limit the state of the meson can be factorized as a product of states corresponding to the state of heavy meson and light cloud

$$|M, j_Q, j_l\rangle \simeq |Q, j_Q\rangle|light\ cloud, j_l\rangle. \tag{22}$$

Imagine that we want to calculate the matrix element of any covariant weak current between two (not necessarily the same) states of pseudoscalar heavy mesons $\langle P', j_Q, j_l'|\Gamma|P, j_Q, j_l\rangle$, in the kinematical point in which there is no change in velocity, $v = v'$. Using the factorization we get

$$\begin{aligned}\langle P', j_Q, j_l'|\Gamma|P, j_Q, j_l\rangle &\simeq \langle Q, j_Q\,|\Gamma|Q', j_Q'\rangle\langle light\ cloud', j_l'\,|\,light\ cloud, j_l\rangle \\ &= \langle Q, j_Q\,|\Gamma|Q', j_Q'\rangle\delta_{j_l,j_l'}.\end{aligned} \tag{23}$$

While there is no velocity change, the state of the light cloud is left unchanged and the overlap (scalar product) of light cloud states is equal to 1. Also, if the flavour of the final heavy quark state is changed, the overlap of light clouds is still the same. In this way it is possible to connect matrix elements of the weak currents between different heavy mesons. In more general situation the overlap is not trivial but can be parametrized by the function of the product of velocities $w = v \cdot v'$.
The HQ symmetry alone will not let us discover anything about this function and some non-perturbative method of calculation, like QCD sum rules will be needed, but nevertheless relation (23) contains great deal of information [31].

3.3 Isgur-Wise function

In this section we use the basic idea from previous section and study the matrix elements of weak hadronic vector and axial currents between the meson states B and $D^{(*)}$. Following ref. [34], usual relativistic normalization of meson states is given by:

$$\langle M(p')|M(p)\rangle = 2E(2\pi)^3\delta^3(\vec{p} - \vec{p'}). \tag{24}$$

Since HQ symmetry relates heavy quarks at equal velocities, and the dependence of the mass of heavy quark is absent, it is more suitable to use the following mass independent normalization:

$$\langle M(v')|M(v)\rangle = \frac{2E}{m_M}(2\pi)^3\delta^3(\vec{p} - \vec{p'}), \tag{25}$$

with trivial relation to the conventional definition.
In HQ limit $|M(v)\rangle$ is only characterized by configuration of its light degrees of freedom.

Let us consider the elastic scattering of pseudoscalar meson $P(v) \to P(v')$ by an external vector current. Action of the current is to replace $v \to v'$ and the corresponding change in the momentum of the light cloud is:

$$q^2 \simeq \Lambda^2_{QCD}(v' - v)^2 \simeq \Lambda^2_{QCD}(v \cdot v' - 1). \tag{26}$$

Lorentz covariance imposes the parametrization of the current matrix element by the functions $h_\pm$ in the following way:

$$\langle P(v')|\bar{Q}_{v'}\gamma^\mu Q(v)|P(v)\rangle = h_+(w)\,(v+v')^\mu + h_-(w)(v-v')^\mu, \tag{27}$$

where $w = v \cdot v'$ is conveniently chosen Lorentz invariant variable. By contracting the both sides of above definition with $(v-v')_\mu$ and using the constraints (14), one finds that $h_-(w) = 0$. Let us now switch the notation and give function h_+ special name, $h_+(w) \equiv \xi(w)$. Due to the HQ flavour symmetry, the dynamics of light cloud does not differentiate between two different heavy quarks, so the following relation is also true:

$$\langle P'(v')|\bar{Q}_{v'}\gamma^\mu Q(v)|P(v)\rangle = \xi(w)\,(v+v')^\mu, \tag{28}$$

where P' is a different psudoscalar meson. The universal function $\xi(w)$ is called Isgur-Wise function, [38], and in HQ limit it describes any matrix element of the type $\langle M', j'_Q, j'_l|\Gamma|M, j_Q, j_l\rangle$, where Γ is arbitrary Dirac's covariant current.

For equal velocities, $j^\mu = \bar{Q}'_v\gamma^\mu Q_v$ is conserved current of heavy quark symmetry.[5] The corresponding conserved charges are the generators of this flavour symmetry:

$$N_{Q'Q} = \int d^3x j^0(x). \tag{29}$$

The diagonal elements are number operators and off-diagonal terms change one heavy quark to another $N_{Q'Q}|P(v)\rangle = |P'(v)\rangle$. It then follows that:

$$\langle P'(v)|N_{Q'Q}|P(v)\rangle = \langle P(v)|P(v)\rangle = 2v^0(2\pi)^3\delta^3(0), \tag{30}$$

and comparing to the relation (28) one concludes:

$$\xi(1) = 1, \tag{31}$$

which can be understood in terms of the heuristic physical picture we gave in the previous section. Isgur-Wise function can be visualized as the overlap of the light clouds boosted relative to each other by $v \cdot v'$.
The recoil energy of the meson P' in its rest frame is given by:

$$E = m_{P'}(v \cdot v' - 1) \tag{32}$$

so the kinematical point $v \cdot v' = 1$ is called zero recoil point. Now let us apply the above results to the usual parametrization of $B \to D$ form factors in the relativistic normalization of meson states (24):

$$\begin{aligned}\langle D(p')|\bar{c}\gamma^\mu b|B(p)\rangle = f_+(q^2)\Big[(p+p')^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\Big] \\ + f_0(q^2)\frac{m_B^2 - m_D^2}{q^2}q^\mu,\end{aligned} \tag{33}$$

[5] which can be checked from the leading term of the Lagrangian (21)

where the transferred momentum $q = p - p'$. Comparing (33) to (28) we get the following relations:

$$\begin{aligned} \xi(v\cdot v') &= \lim_{m\to\infty} R f_+(q^2) \\ &= \lim_{m\to\infty} R\Big[1 - \frac{q^2}{(m_B+m_D)^2}\Big]^{-1} f_0(q^2), \end{aligned} \tag{34}$$

where the constant R is given as $R = \frac{m_B^2+m_D^2-q^2}{2m_Bm_D}$ and

$$v\cdot v' = \frac{m_B^2+m_D^2-q^2}{2m_Bm_D}. \tag{35}$$

The limit $m\to\infty$ is taken in such a way that $v\cdot v'$ is kept fixed. The relations (34) are valid as long as the momentum of the light cloud is not large enough to probe the scale m_Q. This condition is fairly satisfied in the case of $B\to D$ transition, for which

$$\Lambda_{QCD} \ll m_{b,c}, \tag{36}$$

due to smallness of the factor

$$(v\cdot v' - 1)_{max} = \frac{(m_B^2-m_D^2)^2}{2m_Bm_D} = 0.6. \tag{37}$$

The new spin symmetry leads to relation between pseudoscalar and vector meson matrix elements, as well. The vector meson with longitudinal polarization ϵ_3 is related to pseudoscalar meson in the effective theory through the action of the spin operator:

$$|V(v,\epsilon_3)\rangle = 2S_Q^3|P(v)\rangle. \tag{38}$$

It then follows that

$$\langle V'(v',\epsilon_3)|\bar{Q}'_{v'}\Gamma Q_v|P(v)\rangle = \langle P'(v')|\bar{Q}'_{v'}(2S^3\Gamma)Q_v|P(v)\rangle. \tag{39}$$

We can evaluate the above expression in the rest frame of the final meson:

$$\begin{aligned} v'^\mu &= (1,0,0,0), \\ \epsilon_3^\mu &= (0,0,0,1), \\ S^3 &= \frac{1}{2}\gamma_5\gamma^0\gamma^3. \end{aligned} \tag{40}$$

We then obtain the following commutation relations, for vector current:

$$\begin{aligned} 2[S_{Q'}^3, V^0 - A^0] &= A^3 - V^3\,, \\ 2[S_{Q'}^3, V^3 - A^3] &= A^0 - V^0\,, \\ 2[S_{Q'}^3, V^1 - A^1] &= i(A^2 - V^2)\,, \\ 2[S_{Q'}^3, V^2 - A^2] &= -i(A^1 - V^1)\,. \end{aligned} \tag{41}$$

Combining (39) and (41) one relates the matrix elements of weak vector minus axial current between the psudoscalars meson to the Isgur-Wise function:

$$\begin{aligned}\langle V'(v',\epsilon)|\bar{Q}_v\gamma^\mu(1-\gamma_5)Q_v|P(v)\rangle = i\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v_\alpha v_\beta\,\xi(v\cdot v') - \\ [\epsilon^{*\mu}(v\cdot v'+1) - v^\mu\epsilon^*\cdot v]\,\xi(v\cdot v').\end{aligned} \tag{42}$$

where the completely antisymmetric Levi-Civita symbol is normalized as $\epsilon^{0123} = -1$. In more conventional parametrization vector minus axial matrix element is given by:

$$\begin{aligned}\langle D(p',\epsilon)|\bar{c}\gamma^\mu(1-\gamma_5)b|B(p)\rangle = {} & \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_B+m_D*}\epsilon^*_\nu p'_\alpha p_\beta V(q^2) \\ & - \Big[(m_B+m_D*)\epsilon^{*\mu}A_1(q^2) - \frac{\epsilon^*\cdot q}{m_B+m^*_D}(p'+p)^\mu A_2(q^2)\Big] \\ & - 2m_{D^*}\frac{\epsilon^*\cdot q}{q^2}q^\mu A_0(q^2).\end{aligned} \tag{43}$$

The function $A_3(q^2)$ is given as linear combination of form factors $A_1(q^2)$ and $A_2(q^2)$, subject to a constraint $A_3(0) = A_0(q^2)$ to cancel unphysical pole at $q^2 = 0$:

$$A_3(q^2) = \frac{m_B+m_D}{2m_{D^*}}A_1(q^2) - \frac{m_B-m_{D^*}}{2m_{D^*}}A_2(q^2) \tag{44}$$

Comparing (42) to (43) one finds the following relations [40]:

$$\begin{aligned}\xi(v\cdot v') = \lim_{m\to\infty} R^*V(q^2) = \lim_{m\to\infty} R^*A_0(q^2) = \lim_{m\to\infty} R^*A_2(q^2) \\ \lim_{m\to\infty} R^*\Big[1 - \frac{q^2}{(m_B+m_{D^*})^2}\Big]^{-1}A_1(q^2),\end{aligned} \tag{45}$$

where $R^* = 0.9$ is the D^* version of constant R in the case of D meson.
Caprini, Lellouch and Neubert obtained dispersive constraints on the form factors in $B \to (D^*)$ transitions fully exploiting the HQS including the $1/m$ corrections, [41]. The full expressions for form factors can be found in Appendix of Ref. [43], where the form factor $A_0(q^2)$ is also estimated. The recent results on the extraction of Isgur-Wise function from experimental results, as well as several other useful results are found in [58].

4 New physics and helicity amplitudes

Let us introduce the following effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}}J_{bc,\mu}\sum_{\ell=e,\mu,\tau}\big(\bar{\ell}\gamma^\mu P_L\nu_\ell\big) + \text{h.c.}\,, \tag{46}$$

where $P_{L,R} \equiv (1\mp\gamma_5)/2$, while J^μ_{bc} is $b \to c$ charged current that includes the $V-A$ current and additional beyond SM current given by the derivative of (pseudo)scalar density, and contributes to the helicity suppressed amplitude and becomes important in the process with the tau lepton is in final state:

$$J^\mu_{bc} = \bar{c}\gamma^\mu P_L b + g_{SL}i\partial^\mu(\bar{c}P_L b) + g_{SR}i\partial^\mu(\bar{c}P_R b)\,. \tag{47}$$

The leptonic current has the structure as in the SM, however, using the equation of motion it can be shown that the above Hamiltonian can be realized in Two Higgs Doublet Models (2HDM) of the type II [12], where the dimensionful coupling $g_{SR} \sim -m_b \frac{\tan^2\beta}{m_{H^+}^2}$.
It is convenient to introduce the helicity amplitudes formalism for a simpler calculations of decay distributions. Let the momenta of the initial B meson, final M (D or D^*) meson, the final final charged lepton and (anti)neutrino be p, p', k_1, k_2, respectively. The angle θ_l is defined as the angle between three momenta[6] of D^* and ℓ in the $\ell - \nu$ center of mass (CM) frame (see Fig. 2).
Let us introduce the lepton helicity amplitude, where m_l is the helicity of the lepton in $\ell\nu$ CM frame

$$L_{\lambda_l,m}(q^2, \cos\theta_\ell) = \tilde{\epsilon}_\mu(m)\langle l(k_1, m_l)\nu_\ell(k_2)|\bar{l}\gamma^\mu(1-\gamma_5)\nu_\ell|0\rangle, \tag{48}$$

and hadronic helicity amplitude:

$$H^{m_M,m}(q^2, \cos\theta_\ell) = \tilde{\epsilon}^*_\mu(m)\langle M(p', \epsilon)|\bar{l}\gamma^\mu(1-\gamma_5)\nu_\ell|\bar{B}(p)\rangle, \tag{49}$$

where $\tilde{\epsilon}(m)$ are polarization vectors of virtual W boson (in the case of SM) or, equivalently, of the lepton-neutrino pair. The m_M labels the polarization of the vector meson in the final state. The $\tilde{\epsilon}(m)$ satisfy the normalization and completeness relations

$$\tilde{\epsilon}^*(m)\tilde{\epsilon}(m') = g_{mm'}, \;\; \sum_{mm'} \tilde{\epsilon}_\mu(m)\tilde{\epsilon}^*(m') = g_{\mu\nu}. \tag{50}$$

The polarization vectors of the meson M satisfy the analogous relations:

$$\epsilon^*_\alpha(m)\epsilon^\alpha(m') = -\delta_{mm'}, \;\; \sum_{mm'} \tilde{\epsilon}_\alpha(m)\tilde{\epsilon}^*(m')\delta_{mm'} = -g_{\alpha\beta} + \frac{p_\alpha p'_\beta}{m_M^2}. \tag{51}$$

Then the SM amplitudes can be expressed as the following sum of products of helicity amplitudes, where m takes values $(t, 0, \pm)$

$$\mathcal{A}_{SM}^{\lambda_\tau, m_M} = \frac{G_F}{\sqrt{2}} \sum_m \eta_m H_{\lambda_M, m} L_{\lambda_\ell, m}. \tag{52}$$

The factor η_m takes the values $\eta_{\pm,0} = 1$ and $\eta_s = -1$. In the same way we can calculate the hadronic helicity amplitudes for effective vector current (47) and observe that the additional terms in the current affect only H_{0t} helicity amplitude:

$$H_{0t} = H_{0t}^{SM}\Big[1 + (g_{SR} - g_{SL})\frac{q^2}{m_b + m_c}\Big]. \tag{53}$$

In Ref. [43] several observables were explored and the contributions from the Hamiltonian (46) is possible in all of them (for updated results see also [44]).

[6] for masses of the particles we reserve the explicit labels written in subscript

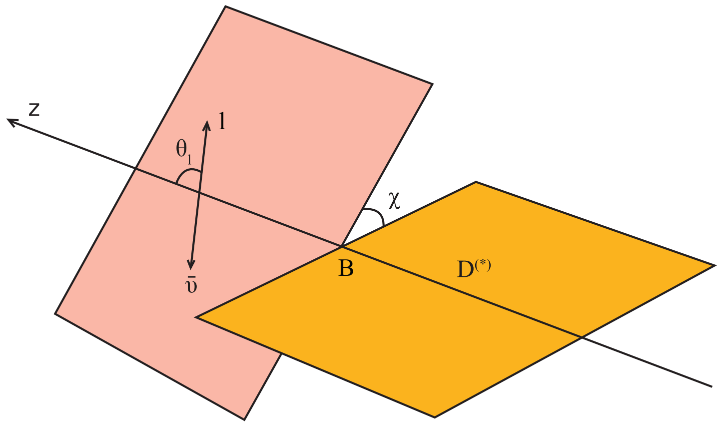

Figure 2: The relevant kinematical variables in the semileptonic B decay

In the following we shortly summarize the findings from the paper [43]. The helicity amplitudes H_{00} and H_{0t} contribute to amplitudes involving D^*_L's, leading to a prediction for the longitudinal decay rate. One can also study the singly differential longitudinal rate ratio $R^*_L(q^2)$ defined analogously to $R^*(q^2)$ as described in [43]. A simple angular (opening angle) asymmetry is defined as the difference between partial rates where the angle θ between the D^* and τ three-momenta in the $\tau - \bar{\nu}_\tau$ rest-frame is bigger or smaller than $\pi/2$. In the decay modes with light leptons, this asymmetry (A^ℓ_θ) can be used to probe for the presence of right-handed $b \to c$ currents, since these contribute with opposite sign to $H_{\pm\pm}$ relative to the SM. In the tau modes, it is sensitive only to the real part of NP $g_{SR} - g_{SL}$ contributions and thus provides complementary information compared to the total rate (or R^*). On the other hand, the inclusive asymmetry A_θ integrated over q^2 is very small in the SM with $A_{\theta,\mathrm{SM}} = -6.0(8)\%$; for our NP benchmark point we obtain $A_{\theta,\mathrm{NP}} = 3.4\%$, but even values as low as -30% are still allowed. In [43] it was found that the tau spin asymmetry, defined as $A_\lambda(q^2) = [d\Gamma_\tau/dq^2(\lambda_\tau = -1/2) - d\Gamma_\tau/dq^2(\lambda_\tau = 1/2)]/[d\Gamma_\tau/dq^2]$, where $\lambda_\tau = \pm 1/2$ are tau helicities defined in $\tau\nu_\tau$ center of mass frame, can provide additional useful information.

5 Lepton flavour universality violation in B decays

During the last three years there has been a systematic disagreement between the experimental and SM predicted theoretical values for the branching ratio of $B \to \tau\nu$. The latest Belle collaboration result $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = (0.72^{+0.27}_{-0.25} \pm 0.11) \times 10^{-4}$ [45] ameliorates somewhat the enduring tension with the measured value of $\sin 2\beta$ in the global CKM fit. However, the current world average experimental value still deviates from the SM prediction by 2.6σ significance if Gaussian errors are assumed [9].

The B meson coupling constant is the only hadronic parameter entering the theoretical branching ratio prediction. The errors of the most recent lattice QCD results are at the level of 5% [46] and already sub leading compared to the dominant parametric uncertainty due to $|V_{ub}|$. One can eliminate the V_{ub} dependence completely by introducing the LFU probing ratio $\mathcal{R}^\pi_{\tau/\ell} \equiv [\tau(B^0)/\tau(B^-)][\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell)] = 0.73 \pm 0.1$. This is to be compared to the SM prediction of $\mathcal{R}^{\pi,\mathrm{SM}}_{\tau/\ell} = 0.31(6)$ [42]. The measured value is more than a factor of 2

bigger - a discrepancy with 2.6σ significance if Gaussian errors are assumed.
The τ lepton in the final state of the (semi)leptonic B meson decays is particularly interesting due to the large τ mass which allows to probe parts of amplitudes in B meson (semi)leptonic decays which are not accessible if the final state contains only light leptons. Possible NP effects in the ratios $\mathcal{R}^{(*)}_{\tau/\ell}$ and $\mathcal{R}^{\pi}_{\tau/\ell}$ can be approached by using the effective Lagrangian approach [42], [43].

We interpret the above anomalies as a possible sign of lepton flavour universality violation (LFUV). The lepton flavour universality is one of the key predictions of the SM and is strongly constrained in the pion and kaon sectors, where it was found to be in excellent agreement with the SM. The signs of LFUV in B decays might be signs of NP which we parametrize through the following extension of the SM Lagrangian with a set of higher dimensional operators ($\mathcal{Q}_i$) that are generated at a NP scale Λ above the electroweak symmetry breaking scale $v = (\sqrt{2}/4G_F)^{1/2} \simeq 174$ GeV

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_a \frac{z_a}{\Lambda^{d_a-4}} \mathcal{Q}_i + \text{h.c.} \tag{54}$$

The label d_a stands for the dimensions of the operators $\mathcal{Q}_a$, while z_a are the dimensionless Wilson coefficients (below we also use $c_a \equiv z_a(v/\Lambda)^{d_a-4}$). Two restrictions are enforced: (i) dangerous down-type flavour changing neutral currents (FCNCs) and (ii) LFU violations in the pion and kaon sectors are not to be generated at the tree level. The lowest dimensional operators that can modify $R^{(*)}_{\tau/\ell}$ and $\mathcal{R}^{\pi}_{\tau/\ell}$ are then

$$\mathcal{Q}_L = (\bar{q}_3\gamma_\mu\tau^a q_3)\mathcal{J}^\mu_{3,a}\,, \qquad \mathcal{Q}^i_R = (\bar{u}_{R,i}\gamma_\mu b_R)(H^\dagger\tau^a\tilde{H})\mathcal{J}^\mu_{3,a}\,, \tag{55a}$$

$$\mathcal{Q}_{LR} = i\partial_\mu(\bar{q}_3\tau^a H b_R)\textstyle\sum_j \mathcal{J}^\mu_{j,a}\,, \qquad \mathcal{Q}^i_{RL} = i\partial_\mu(\bar{u}_{R,i}\tilde{H}^\dagger\tau^a q_3)\textstyle\sum_j \mathcal{J}^\mu_{j,a}\,, \tag{55b}$$

where $\tau_a = \sigma_a/2$, $\mathcal{J}^\mu_{j,a} = (\bar{l}_j\gamma^\mu\tau_a l_j)$, $\tilde{H} \equiv i\sigma_2 H^*$ and i,j are generational indices.

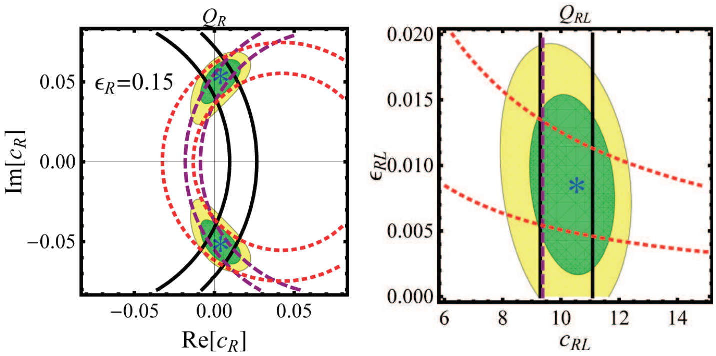

Figure 3: Preferred parameter regions for effective operators $\mathcal{Q}^i_R$ (left plot, as a funciton of complex c_R Wilson coefficient, and ϵ_R fixed to the best fit value), and for $\mathcal{Q}^i_{RL}$ (right plot, as a function of real c_{RL} Wilson coefficient and the mixing ratio ϵ_{RL}). The best fit points are marked with an asterisk.

We work in the down quark mass basis, where $q_i = (V^{ji*}_{CKM} u_{L,j}, d_{L,i})^T$, and charged lepton mass basis, $l_i = (V^{ji*}_{PMNS}\nu_{L,j}, e_{L,i})^T$. The requirement that there are no down-type tree-level FCNCs imposes flavour alignment in the down sector for the operators $\mathcal{Q}_L, \mathcal{Q}_{LR}$ and $\mathcal{Q}^i_{RL}$. An additional possibility is to assume [43] the presence of new light invisible fermions, imitating the missing energy signature of SM neutrinos in the $b \to u_i \tau\nu$ decays. In the presence of general flavour violating NP, contributions to $b \to u$ transitions are not generally related to $b \to c$ transitions. In the case of $\mathcal{Q}^i_R$ for example, the SM expectations are rescaled by $|1 - c_R/2V_{cb}|^2$ in the case of $\mathcal{R}_{\tau/\ell}$ and by $|1 + \epsilon_R c_R/2V_{ub}|^2$ for $\mathcal{R}^\pi_{\tau/\ell}$. The parameters (c_i, ϵ_i) can be obtained by fitting the data using CKM inputs from the global fit as given in [42]. The results are presented in Fig. 3. Among existing NP models the two-Higgs doublet models (2HDMs) are obvious candidates to induce the $\mathcal{Q}^i_{RL}$ operators. Unfortunately, none of the 2HDMs with natural flavour conservation can simultaneously account for the three considered LFU ratios, while in ref. [42] a 2HDM with more general flavour structure has been considered explaining all the observed deviations. The results of the fits to 2HDM of Type III (this model has general flavour structure) are given in Ref. [61].

6 The leptoquark and $B \to D^{(*)}\tau\nu$

In this section we shortly describe effects of the leptoquark which residues in the SM representation $(3, 2, 7/6)$ on the $B \to D^{(*)}$ transition. This field can be embedded into **45** dimensional representation of $SU(5)$ which can help in providing the unification of the SM gauge couplings in non-supersymmetric framework. Also, this representation may correct the mass relations between the down-type quarks and charged leptons [47]. Out of four scalar leptoquarks that couple to leptons and quarks through renormalizable couplings, the two are viable: $(3, 2, 1/6)$ and $(3, 2, 7/6)$. Other possibilities, $(3, 1, -1/3)$ and $(3, 3, -1/3)$, can destabilize proton, [54]. Since the $(3, 2, 1/6)$ couples to right-handed neutrino, for minimality reasons we stick to the analysis of the impact of $\Delta \equiv (3, 2, 7/6)$ leptoquark [48], which couples to the SM fermions through the following interaction Lagrangian

$$\mathcal{L} = \bar{\ell}_R\, Y\, \Delta^\dagger Q + \bar{u}_R\, Z\, \tilde{\Delta}^\dagger L + h.c. \tag{56}$$

In the mass diagonal basis of the down-type quarks and charged leptons, the two isospin components of the scalar couple to quarks and leptons as following:

$$\begin{aligned} \mathcal{L}^{(2/3)} &= (\bar{\ell}_R Y d_L)\, \Delta^{(2/3)*} + (\bar{u}_R[ZV_{PMNS}]\nu_L)\, \Delta^{(2/3)} + h.c\,, \\ \mathcal{L}^{(5/3)} &= (\bar{\ell}_R[YV^\dagger_{CKM}]u_L)\, \Delta^{(5/3)*} - (\bar{u}_R Z \ell_L)\, \Delta^{(5/3)} + h.c. \end{aligned} \tag{57}$$

In the first two generations, the flavour violation is well fitted with the parameters CKM and PMNS, so in order to explain the BaBar's anomaly with the leptoquark contribution we may require the couplings of Δ to $\bar{b}\tau$ and not to $\bar{b}e$ and $\bar{b}\mu$ bilinear. Also, we require that only c quark and not u or t couple to neutrinos. This can be achieved by demanding the following Yukawa couplings *ansatz*:

$$Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{33} \end{pmatrix}, \qquad ZV_{PMNS} = \begin{pmatrix} 0 & 0 & 0 \\ z_{21} & z_{22} & z_{23} \\ 0 & 0 & 0 \end{pmatrix}. \tag{58}$$

The $\Delta^{(5/3)}$ Yukawa couplings are related to these by the CKM and $PMNS$ rotations as follows:

$$YV^\dagger_{CKM} = y_{33} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V^*_{ub} & V^*_{cb} & V^*_{tb} \end{pmatrix}, \qquad Z = \begin{pmatrix} 0 & 0 & 0 \\ \tilde{z}_{21} & \tilde{z}_{22} & \tilde{z}_{23} \\ 0 & 0 & 0 \end{pmatrix}. \tag{59}$$

After integrating out heavy Δ field and performing appropriate Fierz transformations, we derive the following Hamiltonian relevant for $b \to c\ell\nu$ transition

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} V_{cb} \Big[(\bar{\tau}_L \gamma^\mu \nu_L)(\bar{c}_L \gamma_\mu b_L) + g_S (\bar{\tau}_R \nu_L)(\bar{c}_R b_L) + g_T (\bar{\tau}_R \sigma^{\mu\nu} \nu_L)(\bar{c}_R \sigma_{\mu\nu} b_L) \Big], \tag{60}$$

including the SM contribution, where the dimensionless couplings $g_{S,T}$ are introduced through the definition: $g_S(m_\Delta) = 4 g_T(m_\Delta) \equiv \frac{1}{4} \frac{y_{33} z_{23}}{2m_\Delta^2} \frac{\sqrt{2}}{G_F V_{cb}}$. This relation between Wilson's coefficients is valid at matching scale m_Δ which we set to the reference mass of $m_\Delta = 500$ GeV and changes due to the QCD anomalous dimensions of scalar and tensor operators. The scale dependence of the operators is cancelled in the leading logarithm approximation by the scale dependence of corresponding Wilson's coefficients:

$$\begin{aligned} g_S(m_b) &= \left(\frac{\alpha_S(m_b)}{\alpha_S(m_t)} \right)^{-\frac{\gamma_S}{2\beta_0^{(5)}}} \left(\frac{\alpha_S(m_t)}{\alpha_S(m_\Delta)} \right)^{-\frac{\gamma_S}{2\beta_0^{(6)}}} g_S(m_\Delta)\,, \\ g_T(m_b) &= \left(\frac{\alpha_S(m_b)}{\alpha_S(m_t)} \right)^{-\frac{\gamma_T}{2\beta_0^{(5)}}} \left(\frac{\alpha_S(m_t)}{\alpha_S(m_\Delta)} \right)^{-\frac{\gamma_T}{2\beta_0^{(6)}}} g_S(m_\Delta). \end{aligned} \tag{61}$$

Anomalous dimension coefficients are $\gamma_S = -8$, $\gamma_T = 8/3$ and coefficient $\beta_0^{(f)} = 11 - 2/3 n_f$, where n_f is a number of active quark flavours. The coefficients are then run to the beauty quark scale, $\mu = m_b = 4.2$ GeV, at which the matrix elements of hadronic currents are calculated. Difference between running of g_S and g_T modifies the original matching scale relation to

$$g_T(m_b) \simeq 0.14\, g_S(m_b). \tag{62}$$

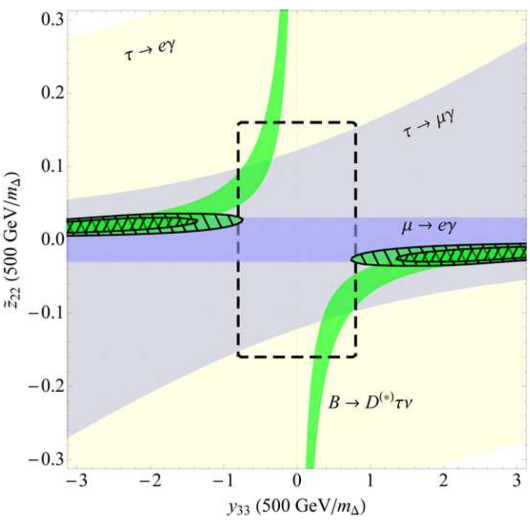

Figure 4: Constraints on the couplings to $b\tau$ (y_{33}) and to $c\mu$ ($\tilde{z}_{22}$) coming from the $1\,\sigma$ region of $\mathcal{R}^{(*)}_{\tau/\ell}$ (thin hyperbolic region), 90 % CL upper bounds on $\tau \to \mu\gamma$ (dark band) and $\tau \to e\gamma$ (bright band). Muon magnetic moment upper bound is denoted by horizontal dashed lines. Vertical dashed lines are the perturbativity cuts in the y_{33} direction. Doubly (singly) hatched region is the 1σ (2σ) region

The presence of both (pseudo)scalar and tensor operators requires the calculation of matrix elements of the (pseudo)scalar and tensor matrix elements. The matrix element of the (pseudo)scalar operator can be readily calculated by using the identity (7).
Further, the matrix elements of tensor operator between the B and D states can be parametrized with the single function $T(q^2)$ [29]

$$\langle D(p_D)|\bar{c}\sigma^{\mu\nu}b|\bar{B}(p_B)\rangle = -i(p_B^\mu p_D^\nu - p_D^\mu p_B^\nu)\frac{2f_T(q^2)}{m_B + m_D}, \tag{63}$$

while the matrix element of tensor operator between B and vector D^* state has the form [58]

$$\begin{aligned}\langle D^*(p_{D^*},\epsilon)|\bar{c}\sigma_{\mu\nu}(1-\gamma_5)b|\bar{B}(p_B)\rangle &= T_0(q^2)\frac{\epsilon^*\cdot q}{(m_B+m_{D^*})^2}\epsilon_{\mu\nu\alpha\beta}\,p_B^\alpha\,p_{D^*}^\beta + T_1(q^2)\epsilon_{\mu\nu\alpha\beta}p_B^\alpha\epsilon^{*\beta}\\ &+ i\Big[T_3(q^2)(\epsilon^*_\mu p_{B,\nu} - \epsilon^*_\nu p_{B,\mu}) + T_4(q^2)(\epsilon^*_\mu p_{D^*,\nu} - \epsilon^*_\nu p_{D^*,\mu})\\ &+ T_5(q^2)\frac{\epsilon^*\cdot q}{(m_B+m_{D^*})^2}(p_{B,\mu}p_{D^*,\nu} - p_{B,\nu}p_{D^*,\mu})\Big].\end{aligned} \tag{64}$$

The scalar and tensor helicity amplitudes can be readily calculated by using:

$$\begin{aligned}\mathcal{A}_S^{m_M,m_\ell} &= -g_S H_S^{m_M} L_l^{m_\tau}\\ \mathcal{A}_T^{m_M,m_\ell} &= -g_T\sum_{m,m'}\eta_m\eta_{m'}H^{m_M}_{m,m'}L^{m_\ell}_{m,m'}.\end{aligned} \tag{65}$$

where the tensor helicity amplitudes are given in analog to (48) and (49), with the only difference that we now calculate contractions of the tensorial matrix elements with two polarization vectors of the $\ell - \nu$ pair, see e.g. [53].
The corresponding form factors are discussed in [29], [58], [48] and are out of scope of the present text.
It turns out that multitude of the processes constrain the above Yukawa couplings ansatz, particularly $l' \to l\gamma$ lepton flavour changing processes. More details on relevant calculations can be found in [48]. Here we insert only the final graph with all constraints, see Fig. 4. It turns out that the contribution of Δ can fit the BaBar's anomaly through the minimal predictive couplings ansatz, and also have interplay with the physics of GUT scale.
The Yukawa ansatz (58) can be consistently implemented in $SU(5)$ GUT model and the preferable ratios of the couplings $\tilde{z}_i$ can be evaluated. This shows remarkable potential of low energy precision flavour constraints for physics of very high energies.

7 Summary

We investigated possibilities to observe NP contributions in $B \to D^*\tau\nu_\tau$ and $B \to D^\tau\nu_\tau$. In addition to the ratio of $B \to D^*\tau\nu_\tau$ and $B \to D^\tau\nu_\tau$, the NP might modify a number of new variables. The study is performed within most general framework of the effective Lagrangian, as well as within few models of NP. The complete study of a particular leptoquark contribution in $B \to D^*\tau\nu_\tau$ and $B \to D^\tau\nu_\tau$ has been performed, accompanied by the constraints coming from from low energy phenomenology. The existing discrepancy can be well explained within a proposed model.

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