On possible role of scalar glueball-quarkonia mixing in the f(0)(1370,1500,1710) resonances produced in charmonia decays

S.B. Gerasimov[1]

[1]Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141980, Russia

The next to lowest mass scalar multiplet is treated as the $q\bar{q}$, P-wave nonet, weakly mixed with the lower - mass, presumably $qq\bar{q}\bar{q}$ S-wave nonet and, in principle, with the $J^{PC} = 0^{++}$ -glueball. The modified Gell-Mann-Okubo-type mass-formulas are used to derive and discuss the quark-gluon configuration structure of the obtained meson states which are then used to obtain the relations between the decay ratios $Br(J/\psi \to \omega f_0)/Br(J/\psi \to \phi f_0)$, where $f_0(1720) \cong G$ is the glueball, the $f_0(1370)$ and $f_0(1506)$ are quarkonium states. Some other relations between the radiative and radiationless decays of the lowest mass charmonium states into scalar resonances are presented and discussed.

1. As is known, the precise understanding of mass and dynamics of the glueball decays is problematic up to now in spite of very large number of works devoted to the problems mentioned. We concentrate on the mass region $1.3 \div 1.7$ GeV occupied by the spin-zero 0^{++} mesons. In this group of mesons there are three isoscalar mesons with similar masses which, in the presence of the nearly lying isotriplet and isodoublet ones, suggest the overpopulated nonet where a possible glueball is hidden within structures of the three isoscalar states. Whether this idea is right or wrong one should deduce from data on the reactions creating them as well as from the relations between the branching ratios of their decays. With this in mind, we present the results of a simple approach enabling one to discuss an acute problem of the existence and properties of glueballs with quantum numbers $I^G J^{PC} = 0^+0^{++}$ (for the different approaches, see [1] and references mentioned therein).

2. We define the 3×3 mass-matrix $\hat{V(i)}$ as acting on the basis vectors N, S, G to transform them into one of three vectors of the physical meson states $f_0(i)$:

$$(f_0(i)) = \hat{V}(i) \cdot \begin{pmatrix} N \\ G \\ S \end{pmatrix} \tag{1}$$

where

$$N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad S = s\bar{s},$$

and G is the glueball.

We consider the mass-matrices $\hat{V}(i)$ taking into account explicitly the different appearance of the two types of gluon effects in mixing states of the differing flavor. In a certain sense, we follow the way proposed in old works by Isgur [2] to connect the strong "non-ideality" of the $SU(3)$-singlet-octet mixing angle in the lowest pseudoscalar and scalar meson nonet with the overwhelmingly strong, as compared to the respective term in the vector or tensor meson nonet, annihilation term in the mass-matrix inducing the non-diagonal $q\bar{q} \leftrightarrow s\bar{s}, (q = u, d)$ transitions. We remind that the celebrated Gell-Mann–Okubo formula

$$3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$$

follows as the mass sum rule after exclusion of parameters introduced into the general mass term of the phenomenological meson lagrangian

$$\Delta L = M^2 \cdot Tr(V_8 V_8) - \mu^2 \cdot Tr(V_8 V_8 \lambda_8) \tag{2}$$

Okubo [3] proposed to replace $V_8 \to V_9$ in the GMO mass operator and drop the term proportional to $Tr(V_9)$. The well-known "ideal mixing " mass relations

$$m^2(\rho) = m^2(\omega), \quad 2m^2(K^*) - m^2(\rho) = m^2(\phi)$$

are fulfilled for the vector and reasonably well for tensor $q\bar{q}$ nonets but poor for the pseudoscalar one.

The standard hierarchy of meson masses following from the effective lagrangian with the $SU(3)$ breaking is

$$m^2(s\bar{s}) \geq m^2(q\bar{s}) \geq m^2(q\bar{q}).$$

The idea to relate the apparently specific situation for the pseudoscalar meson sector with additional strong annihilation mechanism transforming the given flavor quark field combinations into each other was put forward phenomenologically by Isgur [2] and is now interpreted as mediated by short-range fluctuations in the quark-gluon vacuum.

We follow these ideas in the further generalized form via introducing the "bare" scalar glueball mass and nondiagonal glueball-quarkonium transition-mass into the spin-zero meson mass-matrices.

Hence, in the $N^{'} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})^{'}, S^{'} = (s\bar{s})^{'}$ basis our symmetric real mass-matrix acquires the following form:

$$\hat{M}^2 = \begin{pmatrix} M_{N'}^2 + 2A_Q & \sqrt{2}A_G & \sqrt{2}A_Q \\ \sqrt{2}A_G & M_G^2 & A_G \\ \sqrt{2}A_Q & A_G & M_{S'}^2 + A_Q \end{pmatrix} \tag{3}$$

After reducing it to the diagonal form we should get the matrix of the eigenvalues $\hat{M}^2{}_{ph}$:

$$\hat{M}^2{}_{ph} = \begin{pmatrix} M_{f_0}^2(1) & 0 & 0 \\ 0 & M_{f_0}^2(2) & 0 \\ 0 & 0 & M_{f_0}^2(3) \end{pmatrix}$$

We start treating the mass relations with the higher-mass scalar 0^{++}-sector:

$$M_{a_0} = 1474 \pm 19, M_{K^*{}_0} = 1425 \pm 50$$

$$M_{f_0}(1) = 1370 \pm 50, M_{f_0}(2) = 1505 \pm 6,$$

$$M_{f_0}(3) = 1720 \pm 7$$

where all values are in MeV [4].

We define the "bare" mass values $M_{N'}$ and $M_{S'}$ devoid of the strong annihilation contributions via

$$M_{N'} = M_{a_0}, {M_{S'}}^2 = 2{M_{K^*{}_0}}^2 - {M_{a_0}}^2,$$

A short digression: the second relation is alike of the S-wave vector quarkonia, but we would like to note the opposite mass hierarchy sequence

$$M^2(s\bar{s})^{'} \leq M^2(q\bar{s})^{'} \leq M^2(q\bar{q})^{'}$$

which can follow from the suggested [5] mixing of two scalar nonets composed of light (u,d,s)-quarks: the low-mass, presumably the two-quark-two-antiquark-nonet with the total orbital angular moment $L = 0$ and $(mass^2) \leq 1GeV^2$ and the higher-mass, quark-antiquark states with orbital moment $L = 1$.

Phenomenologically, we can indicate this just by changing the sign of the constant μ^2 in the general $SU(3)$ mass formula (2) and by formally introducing the "primed" quark amplitudes including the mentioned 4-quark admixtures and satisfying the inverted mass sequence.

The physical meaning of terms in the mixing-mass matrix is following. The term A_Q in the mass matrix represents the dynamical self-mass term determined by the short-ranged, quark-flavor changing processes, while M_G and A_G are the "bare" gluon mass and the non-diagonal self-mass term arising in the course of quarkonium-glueball transitions. These three unknown terms have to be found by solution of the system of three non-linear equations representing the equalities of three invariants of the diagonalization process: the trace, the determinant and the sum of main minors of the matrices before and after diagonalization. The diagonalized mass-matrix is assumed to contain only experimentally defined masses of scalar meson resonances. Successively excluding unknown variables A_Q and A_G in favor of M_G, we solve numerically the resulting equation by varying the remaining unknown M_G under constraint ${A_G}^2 \geq 0$. There is trivial "decoupling-solution" $A_G = 0$ and $M_G \simeq M_{f_0(3)}$ and none for the "postulated" ${A_G}^2 > 0$ representing, by convention, the nonzero mixing of the glueball G with the remaining quarkonia. Therefore, we have to accept for the physical glueball mass our solution practically coinciding with the mass of $M_{f_0}(3)$ and to derive conclusion about the decoupling of the gluon from two near-by f_0-quarkonium:

$$M_G(ph) \simeq 1730\ MeV \iff M_{f_0}(3) = 1720 \pm 7\ MeV \tag{4}$$

The state vectors of $f_0(1506)$ and $f_0(1370)$ are obtained then by the diagonalization of the rest 2×2 matrix:

$$f_0(1506) = 0.868 \cdot N^{'} \pm 0.496 \cdot S^{'}(5) \tag{5}$$

$$f_0(1370) = \mp 0.496 \cdot N^{'} + 0.868 \cdot S^{'}.(6) \tag{6}$$

The choice of signs (upper→ (u),or lower→ (l)) remains to be done on the physics ground.

3. The sensitive check of our results can be obtained from the radiative and hadronic decays of the lowest mass charmonium states. In the radiative transitions, it is natural to accept the dominance of diagrams of the annihilation of bound $c\bar{c}$-quarks to photon and the pair of intermediate gluon followed by the hadronization process, for instance, $J/\psi \to \gamma gg \to \gamma + hadrons$. We accept the following approximations. First, we drop the 4-quark admixtures to the $f_0(1370; 1500)$ quarkonium state vectors following a kind of the minimal "quark-counting" approximation for the "hard" annihilation processes. In the processes with the two quarkoniums in the final state we keep only the singlet $SU(3)$ projection for the final total hadron state in the transitions $gluons \rightarrow hadrons(n_{q\bar{q}} \geq 2)$. Further, the final phase volume factors will be taken into account as the only explicitly taken into account momentum-dependent characteristics of the processes. Firstly we apply the simple $SU(3)$-symmetry approach to ratio of the branching ratio $J/\psi \rightarrow \gamma f_0(1370)$ *vs* $J/\psi \rightarrow \gamma f_0(1506)$ to fix the signs in (5)-(6). Rewriting (5)-(6) in terms of the $SU(3)$-basis vectors and leaving only coefficients referring to the transition $gg \rightarrow q\bar{q}|_{singlet}$ in the matrix elements, we obtain

$$\frac{\Gamma(J/\psi \to \gamma + f_0(1370))}{\Gamma(J/\psi \to \gamma + f_0(1506))} = \frac{|\vec{k}(f_0(1370))|}{|\vec{k}(f_0(1506))|} \cdot \begin{pmatrix} \frac{.904^2}{.426^2} \\ \\ \frac{.092^2}{.996^2} \end{pmatrix} = \begin{pmatrix} 4.74 \\ \\ .009 \end{pmatrix} \quad \begin{matrix} (5;6u) \\ \\ (5;6l) \end{matrix}$$

From this point onwards we accept the lower signs in (5)-(6) because the $Br(J/\psi \rightarrow \gamma f_0(1370))$ remains unknown and is presumably lower or much lower than measured radiative transitions to $f_0(1506)$ and $f_0(1720)$ resonances. The ratio of the 3-momenta stands for the ratio of the phase space factors accepted for the decays of the type $A \rightarrow B + C$ and giving a minimal kinematic dependence in terms of particle masses in the initial and final states. Under these assumptions, we turn to several processes with participation of the vector $\phi(1.02)$- and $\omega(.783)$-mesons that serving to be good flavor "filters" for the state vectors of scalars participating in a particular reaction. The matrix elements of the process $(J/\psi \rightarrow V + f_0(1720))$ includes a series of the virtual transitions, symbolically, $J/\psi \rightarrow 3g \rightarrow V + gg \rightarrow V + f_0(1720)$, that are proportional to the well-known SU(3)-singlet component of the ω- and ϕ- meson, and to the form-factors of the $3gV$-vertex. As gluons are assumed to be effectively-hard vector quanta we replace approximately the ratios of the full ω- and $\phi-$ form-factors by the respective ratios of their radial "functions-at-zero-distance", entering the ratios of the widths of $V \rightarrow e^-e^+$ -decays, according to the Van Royen - Weisskopf [6] relation. Thus

$$R_{\omega\phi}(f_0(1720)) = \frac{|\vec{k}_{\omega f_0(1720)}|}{|\vec{k}_{\phi f_0(1720)}|} \cdot (tan\theta_V)^{-2} \cdot \frac{V_\omega^2(0)}{V^2{}_\phi(0)} \simeq 1.1; (Exp: 1.33 \pm .34)[4] \quad (7)$$

To get ratios $R_{\omega\phi}(f_0(1370))$ and $R_{\omega\phi}(f_0(1506))$ we need to consider the contributions of both $(1)_V \times (1)_f$- and $(8)_V \times (8)_f$ - terms in the ratios. We apply the $SU(3)$ isoscalar factor $-(1/\sqrt{8})$ corresponding to the singlet content of each $(8)_V \times (8)_{f_0}$ (see,*e.g* [4]) and will treat both terms as acting either coherently or incoherently in all ratios. Comparing the results we found that the incoherent squaring option seem applies better for $R_{\omega\phi}(f_0(1370))$ where the BES2 Collaboration has observed the $J\psi \rightarrow f_0(1370)\phi \rightarrow 2\pi\phi$ -decay channel [7] and does not report the similar channel with ω-meson presumably due to its lower probability. Therefore

$$R_{\omega\phi}(f_0(1370)) = \frac{|\vec{k}_{\omega f_0(1370)}|}{|\vec{k}_{\phi f_0(1370)}|} \cdot \frac{[(1)_\omega \times (1)_{f_0(1370)}]^2 + [(8)_\omega \times (8)_{f_0(1370)}]^2 \cdot (1/\sqrt{8})^2}{[(1)_\phi \times (1)_{f_0(1370)}]^2 + [(8)_\phi \times (8)_{f_0(1370)}]^2 \cdot (1/\sqrt{8})^2} \simeq .76 \quad (8)$$

$$R_{\omega\phi}(f_0(1506)) = \frac{|\vec{k}_{\omega f_0(1506)}|}{|\vec{k}_{\phi f_0(1506)}|} \cdot \frac{[(1)_\omega \times (1)_{f_0(1506)}]^2 + [(8)_\omega \times (8)_{f_{0(1506)}}]^2 \cdot (1/\sqrt{8})^2}{[(1)_\phi \times (1)_{f_0(1506)}]^2 + [(8)_\phi \times (8)_{f_0(1506)}]^2 \cdot (1/\sqrt{8})^2} \simeq 2.04 \quad (9)$$

The approximate or even moderate-broken $SU(3)_{flav}$-symmetry seems, however, not applicable to pair-wise decays of more heavy scalar charmonium $\chi_{c0}(3.414)$ to $f_0(1370, 1506, 1720)$.

As was measured by BES Collaboration [8], the branching ratios upper-bounds for the transitions $\chi_{c0}(3.414) \rightarrow f_0(1370)f_0(1506); f_0(1720)f_0(1506)$ are much less than the branching ratio $\chi_{c0}(3.414) \rightarrow f_0(1370)f_0(1720)$ assuming the dominant contribution of the strange quarks in the intermediate processes $gg \rightarrow s\bar{s}$, due to larger values their momentum-scale dependent mass as compared to the masses of the non-strange quarks [9]. To estimate the relevance of this idea we apply the extremal case of taking into account the contribution of only strange quarks to ratio of the transitions $\chi_{c0}(1P; 3.414) \rightarrow f_0(1.720) + f_0(1370))$ and $\chi_{c0}(1P; 3.414) \rightarrow f_0(1.720) + f_0(1506))$. Returning to the quark-flavor basis (5)-(6) with the fixed lower signs, using the needed values of $Br(f_0 \rightarrow \pi\pi(K\bar{K}))$ from [4] to exclude them from the experimentally measured ratios, we obtain

$$R_{f_0(1.37),f_0(1.50)}(\chi_{c0}(3414)) = \frac{|\vec{k}_{f_0(1370),f_0(1720)}|}{|\vec{k}_{f_0(1506),f_0(1720)}|} \cdot \frac{[.868^2]}{[.496^2]} \simeq 3.9 \tag{10}$$

while the lower limit of the experimental value is $13 \pm 4.6 + 6.8(-4.4)$ [8]. It is seen that even with the use of the extreme assumption about strange quark domination in the mechanism of the $\chi_{c0} \rightarrow f_0(1370; (1506)) + f_0(1720)$ decays the accord with experimental ratio is a marginal one.

To conclude, the further accumulation of more precise data on the decays of more massive scalar, tensor, etc., charmonium states into the meson resonances including the scalar glueball $G(0^{++}) \cong f_0(1720)$ decoupled off nearby quarkonium scalars , will provide more constrained and unequivocal way to study the dynamics of the gluon degrees of freedom in QCD.

The author is grateful to Organizers of the Heavy Quark School-2013 (JINR, Dubna) for invitation to take part in it.

References

[1] E. Gregory, A. Irving, S. Lucini, *et al.*, JHEP, **1210**, 170, (2012).

[2] N. Isgur, Phys. Rev., D 13, 122, (1976).

[3] S. Okubo, Phys. Lett., **5**, 165, 1963.

[4] J. Beringer, *et al.* (PDG), Phys. Rev. **D86** 010001 (1012).

[5] D. Black, A.H. Fariborz and J. Schechter, Phys. Rev. **D 61**, 074001, (2000).

[6] R. Van Royen and V.F. Weisskopf, Nuovo Cim., **A50**, 617, (1967).

[7] M. Ablikim, *et al.*, (BES Collaboration), Phys. Lett. **B607**, 243, (2005).

[8] M. Ablikim, *et al.*, (BES Collaboration), Phys. Rev., **D72**, 092002, (2005).

[9] M. Chanowitz, Phys. Rev. Lett., **95**, 172001, (2005).