Helicity Amplitudes and Angular Decay Distributions

J.G. Körner[1]

[1]PRISMA Cluster of Excellence, Institut für Physik, Johannes-Gutenberg-Universität,

Staudinger Weg 7, 55099 Mainz, Germany

I discuss how to obtain angular decay distributions for sequential cascade decays using helicity methods. The angular decay distributions follow from a reasonably simple master formula involving bilinear forms of helicity amplitudes and Wigner's d functions. I discuss in some detail the issue of gauge invariance for off-shell gauge bosons. As a technical exercise I calculate the linear relation between the helicity amplitudes and the invariant amplitudes of semileptonic and rare baryon decays. I discuss two explicit examples of angular decay distributions for (i) the decay $t \to b + W^+(\to \ell^+\nu_\ell)$ (which leads to the notion of the helicity fractions of the W^+), and (ii) the sequential decay $\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)$.

1 Introductory remarks

In these lectures I want to discuss some examples of sequential cascade decays and their corresponding angular decay distributions. The angular decay distributions follow from a reasonably simple master formula involving bilinear forms of helicity amplitudes and Wigner's d functions. Some sample cascade decay processes are

- Polarized top quark decay [1, 2] $t(\uparrow) \to b + W^+(\to \ell^+\nu_\ell)$
- Rare $\Lambda_b(\uparrow)$ decays [3] $\Lambda_b(\uparrow) \to \Lambda_s(\to p\pi^-) + j_{\rm eff}(\to \ell^+\ell^-)$
- Higgs decay to gauge bosons [4] $H \to W^+(\to \ell^+\nu_\ell) + W^{-*}(\to \ell^-\bar{\nu}_\ell)$
  $H \to Z(\to \ell^+\ell^-) + Z^*(\to \ell^+\ell^-)$
- Rare B decays [5] $B \to D + j_{\rm eff}(\to \ell^+\ell^-)$
  $B \to D^*(\to D\pi) + j_{\rm eff}(\to \ell^+\ell^-)$
- Semileptonic Λ_b decays [6] $\Lambda_b \to \Lambda_c(\to \Lambda\pi^+) + W^{-*}(\to \ell^-\bar{\nu}_\ell)$
- Semileptonic B decays [7, 8, 9] $B \to D + W^*(\to \ell\nu)$
  $B \to D^*(\to D\pi) + W^*(\to \ell\nu)$
- Nonleptonic Λ_b decays [10] $\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)$
- Semileptonic hyperon decays ($\ell^- = e^-, \mu^-$) [11] $\Xi^0(\uparrow) \to \Sigma^+(\to p+\pi^0)+W^{-*}(\to \ell^-\bar{\nu}_\ell)$

In our treatment of these decay processes we have accounted for lepton mass effects whenever this is warranted for by the decay kinematics.

The generic form of most of the above cascade decays is $H_1 \to H_2(\to H_3 + H_4) + W, W^*$, $j_{\rm eff}(\to \ell + \bar{\ell})$ where the H_i can be mesons, baryons or quarks, and the W and W^* denote either on-shell or off-shell charged W's. For neutral current transitions, $j_{\rm eff}$ denotes an effective four-vector and/or four-axial vector current relevant for the desription of rare decays. The interest in deriving angular decay distributions via helicity methods is two-fold. First it facilitates the theoretical analysis of a decay distribution in terms of parity or CP violating contributions. Second it allows one to generate experimental decay distributions via a suitable Monte Carlo program (see e.g. Ref. [11]).

Take as an example the semileptonic hyperon decay $\Xi^0(\uparrow) \to \Sigma^+(\to p + \pi^0) + \ell^- + \bar{\nu}_\ell$ ($\ell^- = e^-, \mu^-$). The decay process is described by three polar angles θ, θ_B and θ_P (as e.g. in Fig. 2) and two azimuthal angles ϕ_B and ϕ_ℓ which describe the relative azimuthal orientation of the two planes that characterize the cascade decay process.

As we shall learn in this lecture, the angular decay distribution can be derived from the master formula [11]

$$\begin{aligned} W(\theta,\theta_P,\theta_B,\phi_B,\phi_\ell) \;\propto\; & \sum_{\lambda_\ell,\lambda_W,\lambda'_W,J,J',\lambda_2,\lambda'_2,\lambda_3} (-1)^{J+J'} |h^{V-A}_{\lambda_l \lambda_{\nu=\pm 1/2}}|^2 e^{i(\lambda_W - \lambda'_W)\phi_\ell} \\ & \times \rho_{\lambda_2-\lambda_W,\lambda'_2-\lambda'_W}(\theta_P) d^J_{\lambda_W,\lambda_\ell-\lambda_\nu}(\theta) d^{J'}_{\lambda'_W,\lambda_\ell-\lambda_\nu}(\theta) H_{\lambda_2\lambda_W} H^*_{\lambda'_2\lambda'_W} \\ & \times e^{i(\lambda_2-\lambda'_2)\phi_B} d^{1/2}_{\lambda_2\lambda_3}(\theta_B) d^{1/2}_{\lambda'_2\lambda_3}(\theta_B) |h^B_{\lambda_3 0}|^2 \end{aligned} \tag{1}$$

where

$h^{V-A}_{\lambda_\ell\lambda_{\nu=\pm1/2}}$	:	helicity amplitudes for the transition $W^* \to \ell + \nu_\ell$:
		$\lambda_{\bar{\nu}} = 1/2$ for $(\ell^-\bar{\nu}_\ell)$; $\lambda_\nu = -1/2$ for $(\ell^+\nu_\ell)$
$\rho_{\lambda_1\lambda'_1}$	:	density matrix for the polarized parent baryon B_1
$H_{\lambda_2\lambda_W}$	:	helicity amplitudes for the transition $B_1 \to B_2 + W^*$
$h^B_{\lambda_3 0}$	:	helicity amplitudes for the transition $B_2 \to B_3 + \pi$
$d^J_{mm'}$	:	Wigner's d functions

The λ_ℓ, λ_W, … are helicity labels of the baryons, leptons and the W^* that participate in the process. They take the values

$$\begin{aligned} \lambda_\ell,\, \lambda_1,\, \lambda_2,\, \lambda_3 &= \pm 1/2 \\ \lambda_W &= 1,\, 0, -1 \ (J=1); \quad t \ (J=0) \\ \lambda_{\bar{\nu}} &= +1/2;\ \lambda_\nu = -1/2 \end{aligned}$$

We shall see in these lectures that the off-shell gauge boson W has a spin-1 and a spin-0 component. Thus we have to sum over $J = 0, 1$. The phase factor $(-1)^{J+J'} = \pm 1$ is associated with the Minkowski metric of our world. The angular decay distribution (1) covers both final lepton states $(\ell^-\bar{\nu}_\ell)$ and $(\ell^+\nu_\ell)$ which are distinguished through the labelling $\lambda_\nu = \pm 1/2$ ($\lambda_{\bar{\nu}} = +1/2$, $\lambda_\nu = -1/2$). This covers the charge conjugated process or also the semileptonic decay $\Sigma^+ \to \Lambda + e^+\nu_e$.

The master formula (1) is quite general. After appropiate angular integrations over θ_B and ϕ_B the master formula also applies to the three-fold angular decay distribution of polarized top decay $t(\uparrow) \to b + \ell^+\nu_\ell$, etc., etc.. The summation over helicities can be quite elaborate if done by hand. However, the summation can be done by computer. A FORM package doing the summation automatically is available from M.A. Ivanov.

1.1 Polarization of the lepton

In the master formula (1) I have summed over the helicities of the lepton. To obtain the polarization of the lepton leave the lepton helicity unsummed, i.e.

$$\sum_{\lambda_\ell,\ldots} \quad \to \quad \sum_{\ldots}$$

For example, the longitudinal polarization of the charged lepton is then given by

$$P^z(\ell) = \frac{W_{\lambda_\ell=+1/2} - W_{\lambda_\ell=-1/2}}{W_{\lambda_\ell=+1/2} + W_{\lambda_\ell=-1/2}} \tag{2}$$

In the same vein the transverse polarization components $P^x(\ell)$ and $P^y(\ell)$ can be obtained from the nondiagonal elements of the W^* density matrix. Note that the longitudinal polarization of the lepton in Eq. (2) refers to the lepton-neutrino cm system, and *not* to the Ξ^0 rest system.

2 Gauge boson off-shell effects

2.1 Off-shell effects and scalar degrees of freedom

When the gauge boson is off its mass shell $q^2 \neq m^2_{W,Z}$ one has to take into account the scalar degree of freedom of the gauge boson. Take the unitary gauge and write out the numerator of the W gauge boson propagator as

$$H_{\mu\nu}L^{\mu\nu} = H_{\mu\nu}\, g^{\mu\mu'} g^{\nu\nu'}\, L_{\mu'\nu'} \quad \longrightarrow \quad H_{\mu\nu}\Big(g^{\mu\mu'} - \frac{q^\mu q^{\mu'}}{m_W^2}\Big)\Big(g^{\nu\nu'} - \frac{q^\nu q^{\nu'}}{m_W^2}\Big)\, L_{\mu'\nu'}.$$

The term $q^\mu q^{\mu'}/m_W^2$ is usually dropped in low energy applications such as μ-decay and also in the charm and bottom sector. Split the propagator numerator into a spin-1 and a spin-0 piece

$$\Big(\underbrace{-g^{\mu\mu'} + \frac{q^\mu q^{\mu'}}{q^2}}_{\text{spin 1}} - \underbrace{\frac{q^\mu q^{\mu'}}{q^2}(1 - \frac{q^2}{m_W^2})}_{\text{spin 0}}\Big)\Big(\underbrace{-g^{\nu\nu'} + \frac{q^\nu q^{\nu'}}{q^2}}_{\text{spin 1}} - \underbrace{\frac{q^\nu q^{\nu'}}{q^2}(1 - \frac{q^2}{m_W^2})}_{\text{spin 0}}\Big).$$

There are three contributions *i*) $\text{spin}\,1 \otimes \text{spin}\,1$, *ii*) $-\ (\text{spin}\,1 \otimes \text{spin}\,0 + \text{spin}\,0 \otimes \text{spin}\,1\,)$ and *iii*) $\text{spin}\,0 \otimes \text{spin}\,0$.

Note the minus sign in case ii) which results from the Minkowski metric. This extra minus sign can be readily incorporated into the master formulas for angular decay distributions by introducing the factor $(-1)^{J+J'}$ and summing over $J, J' = 0, 1$. The scalar contributions are $O(m_\ell^2)$ since $q^\mu L_{\mu\nu} \sim O(m_\ell)$. Note, however, that q^2 can be small since the range of off-shellness is

$$(m_{\ell\,1} + m_{\ell\,2})^2 \leq q^2 \leq (M_1 - M_2)^2$$

for the decay $H_1(M_1) \to H_2(M_2) + \ell_1(m_{\ell_1}) + \bar{\ell}_2(m_{\ell_2})$.

2.2 The issue of gauge invariance

Consider the gauge boson propagator in the general R_ξ gauge and rewrite it into a convenient form. For definiteness we consider the decay $t \to b + W^+$ where we shall also consider gauge boson off-shell effects which allows one to calculate finite width effects as will be done in Sec. 2.3.

$$\begin{aligned} D^{\mu\nu} &= \frac{i}{q^2 - m_W^2}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu (1-\xi_W)}{q^2 - \xi_W m_W^2}\right) \\ &= \frac{i}{q^2 - m_W^2}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2} - \frac{q^\mu q^\nu}{m_W^2} + \frac{q^\mu q^\nu (1-\xi_W)}{q^2 - \xi_W m_W^2}\right) \end{aligned} \tag{3}$$

resulting in

$$D^{\mu\nu} = \frac{i}{q^2 - m_W^2}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}\right) - i\,\frac{q^\mu q^\nu}{m_W^2}\,\frac{1}{q^2 - \xi_W m_W^2}. \tag{4}$$

The first term in Eq. (4) is referred to as the unitary propagator. The second gauge-dependent term in Eq. (4) can be seen to exactly cancel the contribution of the charged Goldstone ϕ^+ exchange if fermion lines are attached to the gauge boson and the charged Goldstone boson contribution. One uses the Dirac equation to convert the q^μ and q^ν contributions in the second term of Eq. (4) to fermion masses. In our case one would have

$$\begin{aligned} q^\nu \bar{u}_f \gamma_\nu (1-\gamma_5) v_{\bar{f}} &= m_f \bar{u}_f (1-\gamma_5) v_{\bar{f}} + m_{\bar{f}} \bar{u}_f (1+\gamma_5) v_{\bar{f}}, \\ q^\mu \bar{u}_b \gamma_\mu (1-\gamma_5) u_t &= m_t \bar{u}_b (1+\gamma_5) u_t - m_b \bar{u}_b (1-\gamma_5) u_t. \end{aligned}$$

One can then see that the second term in Eq. (4) is exactly cancelled by the corresponding ϕ^+-exchange contribution (with the same fermion pair attached). This exercise shows that it does not make sense to talk of an external off-shell gauge boson in isolation. One must include the coupling to a final state fermion pair if one wants to obtain a gauge invariant result.

2.3 Off-shell effects in the decay $t \to b + W^+$

In the zero width approximation and using the unitary gauge the differential rate for $t \to b+W^+$ is given by

$$\frac{d\Gamma}{dq^2} \sim H_{\mu\nu}\left(g^{\mu\mu'} - \frac{q^\mu q^{\mu'}}{m_W^2}\right)\left(g^{\nu\nu'} - \frac{q^\nu q^{\nu'}}{m_W^2}\right) L_{\mu'\nu'}\,\delta(q^2 - m_W^2).$$

On shell one has $q^2 = m_W^2$, and it makes no difference whether one uses the Landau gauge ($\xi = 0$) with $(g^{\mu\nu} - q^\mu q^\nu / q^2)$ or the unitary gauge ($\xi = \infty$) with $(g^{\mu\nu} - q^\mu q^\nu / m_W^2)$. Since we want to account for off-shell effects the use of the unitary gauge is mandatory as explained in Sec. 2.2. Finite width effects can be accounted for by smearing the zero-width formula with the replacement

$$\delta(q^2 - m_W^2) \quad \longrightarrow \quad \frac{m_W \Gamma_W}{\pi}\frac{1}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2}.$$

One then integrates in the limits

$$m_\ell^2 \leq q^2 \leq (m_t - m_b)^2.$$

Numerically the finite width corrections amount to -1.55% in $\Gamma_{t\to b+W^+}$ [12, 13]. Curiously, the negative finite width corrections are almost completely cancelled by the positive first order electroweak corrections [13].

2.4 Scalar contributions in some sample decay processes

Scalar contributions are of $O(m_\ell^2)$. They are therefore important for decay processes where the lepton mass is comparable to the scale of the decay process. For semileptonic and rare processes the characteristic scale would be given by the mass difference $M_1 - M_2$. A more symmetric scale is used in the PDG tables, namely the largest momentum of any of the decay products in the rest frame of the decaying particle. Sample decay processes are

- Decays involving the τ

$$\begin{array}{rcl} B \to D + \tau\nu_\tau & : & \Gamma_S/\Gamma \approx 58\,\% \quad [8,\ 9] \\ B \to D^* + \tau\nu_\tau & : & \Gamma_S/\Gamma \approx 7\,\% \quad [8,\ 9] \\ B \to \pi + \tau\nu_\tau & : & \Gamma_S/\Gamma \approx (30 \div 50)\,\% \quad [14] \\ H \to W^+W^{-*}(\to \tau^-\nu_\tau) & : & \Gamma_S/\Gamma = 0.73\,\% \quad [4] \\ H \to ZZ^*(\to \tau^+\tau^-) & : & \Gamma_S/\Gamma = 1.19\,\% \quad [4] \end{array}$$

  The decays $B \to D^{(*)} + \tau\nu_\tau$ and $B \to \pi + \tau\nu_\tau$ have been widely discussed in the literature because the scalar contribution can be augmented by charged Higgs exchange [15, 16].

- Hadronic semi-inclusive decays $H \to ZZ^*(\to b\bar{b})$

$$H \to ZZ^*(\to b\bar{b}) \quad : \quad \Gamma_S/\Gamma = 7.9\,\% \quad [17]$$

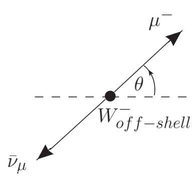
Since the ratio $m_e/(m_n - m_p) = 0.395$ is not small it comes of no surprise that there is a sizeable scalar contribution to the neutron β decay $n \to p + e^-\bar{\nu}_e$. In fact one finds $\Gamma_S/\Gamma = 19\,\%$.

2.5 Scalar contribution to the FB asymmetry A_{FB} of the lepton pair

An interesting observation concerns the scalar contribution to the Forward-Backward (FB) asymmetry of the lepton pair in the cm frame of the lepton pair or, put differently, in the W^* rest frame where its momentum is $(\sqrt{q^2}, \vec{0}\,)$. The notation $\vec{0}$ is rather symbolic and stands for the boost direction which brings the W^* to rest. The observation is that there are parity-conserving contributions to the FB asymmetry arising from scalar-vector interference effects.

Consider the FB asymmetry

$$A_{FB} = \frac{\Gamma_F - \Gamma_B}{\Gamma_F + \Gamma_B} \tag{5}$$



If $A_{FB} \neq 0$ one speaks of a parity-odd effect. Consider the J^P content of the currents coupling to the W^*: $V^\mu(1^-, 0^+)$ and $A^\mu(1^+, 0^-)$. There are two sources of parity-odd effects leading to $A_{FB} \neq 0$ given by

1. parity-violating interaction from $V(1^-)A(1^+)$, $V(0^+)A(0^-)$ interference
2. parity-conserving interaction from $V(0^+)V(1^-)$, $A(0^-)A(1^+)$ interference

Take, for example, the semileptonic decay $\Lambda_b \to \Lambda_c + \ell^- \bar{\nu}_\ell$. The numerator of Eq. (5) is given by (see Ref. [11])

$$\frac{d\Gamma_F}{dq^2} - \frac{d\Gamma_B}{dq^2} = \frac{G^2}{(2\pi)^3}|V_{us}|^2 \frac{(q^2 - m_\ell^2)^2 p}{8M_1^2 q^2} \left[-H^V_{\frac{1}{2}1} H^A_{\frac{1}{2}1} - 2\frac{m_\ell^2}{2q^2}(H^V_{\frac{1}{2}t} H^V_{\frac{1}{2}0} + H^A_{\frac{1}{2}t} H^A_{\frac{1}{2}0}) \right]. \tag{6}$$

The amplitudes $H_{\lambda_{\Lambda_c}\lambda_W}$ in Eq. (6) denote the helicity amplitudes in the transitions $\Lambda_b(\lambda_{\Lambda_b}) \to \Lambda_c(\lambda_{\Lambda_c}) + W^{-*}(\lambda_W)$. The first term in Eq. (6) arises from a truly parity-violating contribution while the remaining two contributions are parity-odd contributions arising from parity conserving interactions. The second contribution is negligible for the e^- and μ^- modes due to the helicity flip factor m_ℓ^2/q^2, but can be sizeable for the τ^- mode. In fact, for the τ mode $\Lambda_b \to \Lambda_c + \tau^- \bar{\nu}_\tau$ the FB asymmetry is dominated by the helicity flip contribution in Eq. (6) leading to a sign change in A_{FB} when going from the e^-, μ^- modes to the τ^- mode (see the corresponding quark-level calculation in Ref. [9]).

3 Helicity amplitudes and invariant amplitudes

The results of a dynamical calculation are usually obtained in terms of invariant amplitudes. The helicity amplitudes can be expressed as a linear superposition of the invariant amplitudes. In this section we show how to calculate the coefficient of this linear expansion for the process $B_1 \to B_2 + j_{\rm eff}$. In order to calculate the coefficients of the linear expansion one has to choose a definite frame.

3.1 System 1: Parent baryon B_1 at rest

Consider the decay $B_1(M_1) \to B_2(M_2) + j_{\rm eff}$ in the rest system of B_1. The effective current $j_{\rm eff}$ with momentum q^μ moves in the positive z direction while B_2 moves in the negative z direction.

$$p_1 = (M_1; 0, 0, 0) \qquad q^\mu = (q_0; 0, 0, |\vec{q}\,|) \qquad p_2^\mu = (E_2; 0, 0, -|\vec{q}\,|)$$

We do not explicitly annote the helicity of the parent baryon B_1 in the helicity amplitudes since, in system 1, λ_1 is fixed by the relation $\lambda_1 = -\lambda_2 + \lambda_j$.

Possible helicity configurations are

λ_1	λ_2	λ_j
1/2	-1/2	0 (t)
-1/2	1/2	0 (t)
1/2	1/2	1
-1/2	-1/2	-1

Convenient relations in system 1 are ($Q_{\pm} = (M_1 \pm M_2)^2 - q^2$)

$$2M_1(E_2 + M_2) = Q_+, \qquad 2M_1|\vec{q}\,| = \sqrt{Q_+ Q_-}. \tag{7}$$

The helicity spinors are given by

$$\bar{u}_2(\pm\frac{1}{2}, p_2) = \sqrt{E_2 + M_2}\left(\chi^{\dagger}_{\mp}, \frac{\mp|\vec{q}\,|}{E_2 + M_2}\chi^{\dagger}_{\mp}\right), \quad u_1(\pm\frac{1}{2}, p_1) = \sqrt{2M_1}\left(\begin{array}{c}\chi_{\pm}\\ 0\end{array}\right), \tag{8}$$

where $\chi_+ = \left(\begin{array}{c}1\\0\end{array}\right)$ and $\chi_- = \left(\begin{array}{c}0\\1\end{array}\right)$ are the usual Pauli two-spinors.

The helicity spinors satisfy the relations

$$\begin{aligned}\frac{1}{2}(1 + \gamma_5 \not{s}_{\pm})u(\pm\frac{1}{2}, p) &= u(\pm\frac{1}{2}, p), \\ \frac{1}{2}(1 + \gamma_5 \not{s}_{\mp})u(\pm\frac{1}{2}, p) &= 0,\end{aligned} \tag{9}$$

where $s_{\mu\,\pm} = \pm(|\vec{p}\,|/M; 0, 0, E/M)$ is the spin four-vector of the fermion with helicity $\pm 1/2$.

For the four polarization four-vectors of the effective current we have

$$\varepsilon^{\mu}(t) = \frac{1}{\sqrt{q^2}}\left(q_0; 0, 0, |\vec{q}\,|\right), \quad \varepsilon^{\mu}(\pm 1) = \frac{1}{\sqrt{2}}\left(0; \mp 1, -i, 0\right), \quad \varepsilon^{\mu}(0) = \frac{1}{\sqrt{q^2}}\left(|\vec{q}\,|; 0, 0, q_0\right). \tag{10}$$

They can be obtained by boosting the corresponding rest frame polarization vectors $\varepsilon^{\mu}(t; q = 0) = (1; 0, 0, 0)$ and $\varepsilon^{\mu}(0; q = 0) = (0; 0, 0, 1)$ by a boost with the boost matrix given by $M_{tt} = M_{00} = q_0/\sqrt{q^2}$ and $M_{t0} = M_{0t} = |\vec{q}\,|/\sqrt{q^2}$ (the transverse polarization vectors are boost invariant).

One defines helicity amplitudes through

$$H^{V,A}_{\lambda_2 \lambda_W} = M^{V,A}_{\mu}(\lambda_2)\epsilon^{*\mu}(\lambda_j). \tag{11}$$

The current matrix elements can be expanded in terms of a complete set of invariants

$$\begin{aligned}M^V_{\mu} &= \langle B_2|J^V_{\mu}|B_1\rangle = \bar{u}_2(p_2)\left[F^V_1(q^2)\gamma_{\mu} - \frac{F^V_2(q^2)}{M_1}i\sigma_{\mu\nu}q^{\nu} + \frac{F^V_3(q^2)}{M_1}q_{\mu}\right]u_1(p_1), \\ M^A_{\mu} &= \langle B_2|J^A_{\mu}|B_1\rangle = \bar{u}_2(p_2)\left[F^A_1(q^2)\gamma_{\mu} - \frac{F^A_2(q^2)}{M_1}i\sigma_{\mu\nu}q^{\nu} + \frac{F^A_3(q^2)}{M_1}q_{\mu}\right]\gamma_5 u_1(p_1)\end{aligned} \tag{12}$$

(we define $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})$). Using the definitions (11, 12), the helicity spinors (8) and polarization vectors (10), the helicity amplitudes can be calculated to be

$$\begin{aligned}H^{V/A}_{-\frac{1}{2}t} &= \frac{\sqrt{Q_{\pm}}}{\sqrt{q^2}}\Big((M_1 \mp M_2)F^{V/A}_1 \pm q^2/M_1 F^{V/A}_3\Big), \\ H^{V/A}_{\frac{1}{2}1} &= \sqrt{2Q_{\mp}}\Big(F^{V/A}_1 \pm (M_1 \pm M_2)/M_1 F^{V/A}_2\Big), \\ H^{V/A}_{\frac{1}{2}0} &= \frac{\sqrt{Q_{\mp}}}{\sqrt{q^2}}\Big((M_1 \pm M_2)F^{V/A}_1 \pm q^2/M_1 F^{V/A}_2\Big).\end{aligned} \tag{13}$$

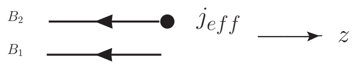
From parity or from an explicit calculation one has

$$\begin{aligned} H^V_{-\lambda_2,-\lambda_j} &= H^V_{\lambda_2,\lambda_j}, \\ H^A_{-\lambda_2,-\lambda_j} &= -H^A_{\lambda_2,\lambda_j}. \end{aligned}$$

For a general linear combination $H_{\lambda_2,\lambda_j} = aH^V_{\lambda_2,\lambda_j} + bH^A_{\lambda_2,\lambda_j}$ it is advantageous to make use of the linear superpositions $H_{\lambda_2,\lambda_j} \pm H_{-\lambda_2,-\lambda_j}$ which have definite transformation properties under parity.

3.2 System 2: The effective current is at rest

The effective current $j_{\rm eff}$ is at rest, or put differently, in system 2 we work in the cm frame of the lepton pair in the decay $j_{\rm eff} \to \ell\bar{\ell}$. Both B_1 and B_2 move in the negative z direction. One now has $\lambda_1 = \lambda_2 - \lambda_j$.



$$p_1^\mu = (E_1'; 0, 0, -|\vec{p}\,'|) \qquad q^\mu = (\sqrt{q^2}; 0, 0, 0) \qquad p_2^\mu = (E_2'; 0, 0, -|\vec{p}\,'|)$$

Convenient relations in system 2 are

$$|\vec{p}\,'| = \frac{\sqrt{Q_+Q_-}}{2\sqrt{q^2}}, \quad (E_1' + M_1)(E_2' + M_2) = \frac{Q_+}{4q^2}\,(M_1 - M_2 + \sqrt{q^2})^2. \tag{14}$$

The relevant spinors can be obtained from the rest frame spinor in Eq. (8) by a boost according to $2M_1 u(p_1) = (\not{p}_1 + M_1)u(p_1 = 0)$ and $2M_1\bar{u}(p_1) = \bar{u}(p_1 = 0)(\not{p}_1 + M_1)$. The spinors in system 2 are thus given by

$$\bar{u}_2(\pm\frac{1}{2}, p_2) = \sqrt{E_2' + M_2}\left(\chi^\dagger_\mp, \frac{\mp|\vec{p}\,'|}{E_2' + M_2}\chi^\dagger_\mp\right), \quad u_1(\pm\frac{1}{2}, p_1) = \sqrt{E_1' + M_1}\begin{pmatrix} \chi_\mp \\ \frac{\pm|\vec{p}\,'|}{E_1'+M_1}\chi_\mp \end{pmatrix}$$

For the four polarization four-vectors of the effective current we now have $\varepsilon^\mu(t) = (1; 0, 0, 0)$ and $\varepsilon^\mu(0) = (0; 0, 0, 1)$ while the transverse polarization vectors $\varepsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}}(0; \mp 1, -i, 0)$ remain unchanged.

With a little bit of work one can show that

$$H^{V,A}_{\lambda_2,\lambda_j}(\text{system 2})\ (F_i^{V,A}) = H^{V,A}_{\lambda_2,\lambda_j}(\text{system 1})\ (F_i^{V,A}),$$

i.e. Eq. (13) holds for both systems 1 and 2. One has recovered a general property of the linear coefficients relating the helicity amplitudes to invariant amplitudes: the coefficients of this linear relation are boost invariant. In this sense the helicity amplitudes are boost invariant. I have gone through this exercise in some detail to convince the reader that e.g. the expression $\sum_{\lambda_2} |H_{\lambda_2,\lambda_j}|^2$ is nothing but the (unnormalized) density matrix of the off-shell gauge boson in its own rest frame regardless of the system in which the helicity amplitudes are evaluated (as long as the systems are connected by a boost). We mention that corresponding relations between helicity amplitudes and invariant amplitudes for the cases $(1/2^+; 3/2^+) \to (1/2^+; 3/2^+)$ have been given in Ref. [18].

3.3 Helicity amplitudes and (LS) amplitudes

Looking at Eq. (13) one notes that at threshold $q^2 = (M_1 - M_2)^2$ there are only two independent nonvanishing helicity amplitudes, namely $H^V_{1/2,t}$ and $H^A_{1/2,1} = \sqrt{2}H^A_{1/2,0}$. This is no accident and can be understood by performing an (LS) amplitude analysis in terms of the (LS) amplitudes $A^{V,A}_{LS}$. For the vector component with J^P content $(1^-;0^+)$ one has the (LS) amplitudes $(A^V_{1,1/2}, A^V_{1,3/2}; A^V_{0,1/2})$, and for the axial component with J^P content $(1^+;0^-)$ one has the (LS) amplitudes $(A^A_{0,1/2}, A^A_{2,3/2}; A^A_{1,1/2})$. At threshold only the two S-wave amplitudes survive, namely $A^V_{0,1/2}$ and $A^A_{0,1/2}$. In fact, there is a linear relation between the set of helicity and (LS) amplitudes which reads $(J = 0,1)$

$$H_{\lambda_1\lambda_2}(J) = \sum_{LS} \left(\frac{2L+1}{2J+1}\right)^{1/2} \langle LS0\mu|J\lambda\rangle\langle s_1 s_2 - \lambda_2\lambda_1|S\mu\rangle\, A_{LS}, \tag{15}$$

where $\lambda = \lambda_1 - \lambda_2$. Eq. (15) can be inverted, and upon setting $A^A_{2,3/2} = 0$ at threshold one recovers the above threshold relation $H^A_{1/2,1} = \sqrt{2}H^A_{1/2,0}$. We emphasize that the set of (LS) amplitudes is completely equivalent to the set of helicity amplitudes and the definition of both sets of amplitudes is based on fully relativistic concepts. Some examples of threshold and near threshold relations have recently been discussed in Refs. [19, 20, 21].

4 Rotation of density matrices

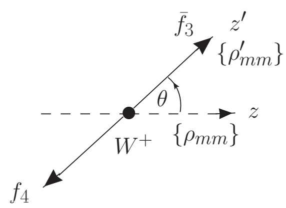
For concreteness we discus the decay of an on-shell W^+ into a fermion pair, i.e. $W^+ \to \bar{f}_3 f_4$ (as e.g. $W^+ \to \mu^+\nu_\mu$) described by the helicity amplitudes $h_{\lambda_3\lambda_4}$ $(\lambda_3, \lambda_4 = \pm 1/2)$. First consider a frame where W^+ is at rest and where the antifermion $\bar{f}_3$ moves in the positive z' direction.

- Consider first the decay of an unpolarized W^+ into a fermion pair. The decay rate in the z' frame is given by

$$\Gamma \quad \sim \quad \sum_{\text{helicities}} |h_{\lambda_3\lambda_4}|^2.$$

- Consider next the decay of a polarized W^+ into a fermion pair. The polarization of the W^+ is given in terms of the spin density matrix ρ'_{mm} with $m = \lambda_3 - \lambda_4$. One then has

$$\Gamma \quad \sim \quad \sum_{\text{helicities}} \rho'_{mm}|h_{\lambda_3\lambda_4}|^2.$$



- Now assume that the W^+ was polarized in a production process characterized by a z axis as e.g. in the decay $t \to b + W^+$ discussed before. In this case the z axis is defined by the momentum direction of the W^+ in the top quark rest system. The spin density matrix of the W^+ is given in terms of the helicity amplitudes for the decay $t \to b + W^+$, i.e. by $\sum_{\lambda_2} H_{\lambda_2 \lambda_W}$. In the present case (no azimuthal correlations) one only needs the diagonal terms of the density matrix of the W^+. For the unnormalized density matrix elements of the W^+ one has
$$\rho_{m=\lambda_W,\, m=\lambda_W} = \sum_{\lambda_2} |H_{\lambda_2 \lambda_W}|^2.$$
Then "rotate" the density matrix. Rotation is from (x, y, z) to (x', y, z') by the angle θ around the y axis. The differential $\cos\theta$ rate reads
$$\frac{d\Gamma(\theta)}{d\cos\theta} \sim \sum_{\text{helicities}} |h_{\lambda_3\lambda_4}|^2 \underbrace{d^1_{\lambda_W,\lambda_3-\lambda_4}(\theta)\rho_{\lambda_W,\lambda_W} d^1_{\lambda_W,\lambda_3-\lambda_4}(\theta)}_{\text{rotated density matrix } \rho'}$$

4.1 General polarized two-body decay

- Take the two particle decay $a \to b + c$ of a spin-J_a particle where the polarization of particle a in the frame (x, y, z) is given by $\rho_{\lambda_a \lambda'_a}$. Since we are also considering possible effects from azimuthal correlations one has to take into account the nondiagonal density matrix elements $\rho_{\lambda_a \lambda'_a}$ with $\lambda_a \neq \lambda'_a$.
- Consider a second frame (x', y', z') obtained from (x, y, z) by the rotation $R(\theta, \phi, 0)$ and whose z axis is defined by particle b. The polarization density matrix ρ' in the frame (x', y', z') is obtained by a "rotation" of the density matrix ρ from the frame (x, y, z) to the frame (x', y', z').
- The rate for $a \to b + c$ is then given by the sum of the decay probabilities $|h_{\lambda_b\lambda_c}|^2$ (with $\lambda_a = \lambda_b - \lambda_c$) weighted by the diagonal terms of the density matrix ρ' of particle a in the frame (x', y', z'). One has
$$\frac{d\Gamma_{a\to b+c}}{d\cos\theta\, d\phi} \sim \sum_{\lambda_a,\lambda'_a,\lambda_b,\lambda_c} |h_{\lambda_b\lambda_c}|^2 \underbrace{D^{J*}_{\lambda_a,\lambda_b-\lambda_c}(\theta,\phi)\ \rho_{\lambda_a,\lambda'_a}\ D^{J}_{\lambda'_a,\lambda_b-\lambda_c}(\theta,\phi)}_{\text{rotated density matrix } \rho'} \qquad (16)$$
where
$$D^J_{m,m'}(\theta,\phi) = e^{-im\phi} d^J_{m\,m'}(\theta).$$
- All master formulas discussed in this lecture can be obtained by a repeated application of the basic two-body formula.

5 T-odd contributions

Take again the cascade decay $\Xi^0 \to \Sigma^+(\to p\pi^0) + W^{-*}(\to \ell^-\nu_\ell)$ as an example. Using the master formula Eq. (1) one obtains, among others, contributions from the two helicity configurations [11]

$$(\lambda_\Sigma = 1/2, \lambda_W = 1; \lambda'_\Sigma = -1/2, \lambda'_W = 0) \quad \text{and} \quad (\lambda_\Sigma = -1/2, \lambda_W = 0; \lambda'_\Sigma = 1/2, \lambda'_W = 1).$$

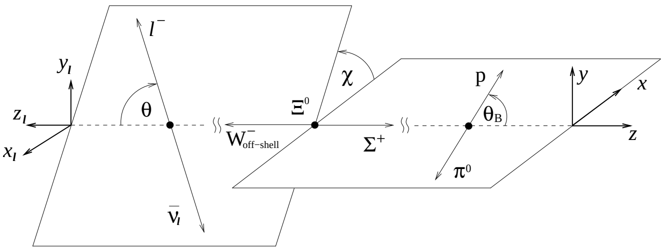


Figure 1: Definition of the polar angles θ and θ_B, and the azimuthal angle χ in the joint angular decay distribution of an unpolarized Ξ^0 in the cascade decay $\Xi^0 \to \Sigma^+(\to p+\pi^0) + \ell^- + \bar{\nu}_l \ell$. The coordinate system (x_ℓ, y_ℓ, z_ℓ) is obtained from the coordinate system (x,y,z) by a 180° rotation around the y axis.

These will lead to the bilinear combinations

$$\begin{aligned} & H_{\frac{1}{2}1}H^*_{-\frac{1}{2}0}\, e^{i(\pi-\chi)} + H_{-\frac{1}{2}0}H^*_{\frac{1}{2}1}\, e^{-i(\pi-\chi)} \\ = \quad & -2\cos\chi\,\mathrm{Re}\, H_{\frac{1}{2}1}H^*_{-\frac{1}{2}0} - 2\sin\chi\,\mathrm{Im}\, H_{\frac{1}{2}1}H^*_{-\frac{1}{2}0}\,. \end{aligned}$$

Take the imaginary part contributions and put in the remaining θ- and θ_B-dependent trigonometric factors. One has two terms proportional to $\sin\chi$,

$$\sin\theta\sin\chi\sin\theta_B\,\mathrm{Im}\, H_{\frac{1}{2}1}H^*_{-\frac{1}{2}0} \qquad \text{and} \qquad \cos\theta\sin\theta\sin\chi\sin\theta_B\,\mathrm{Im}\, H_{\frac{1}{2}1}H^*_{-\frac{1}{2}0}. \tag{17}$$

Rewrite the product of angular factors in terms of scalar and pseudoscalar products using the momentum representations in the (x,y,z) system. The normalized three-momenta are given by (see Fig. 1)

$$\begin{aligned} \hat{p}_{\ell^-} &= (\sin\theta\cos\chi, \sin\theta\sin\chi, -\cos\theta), \qquad \hat{p}_W = (0,0,-1), \\ \hat{p}_{\Sigma^+} &= (0,0,1), \qquad \hat{p}_p = (\sin\theta_B, 0, \cos\theta_B), \end{aligned}$$

where the three-momenta have unit length indicated by the hat notation.

The two angular factors (17) can be rewritten as

$$\sin\theta\sin\chi\sin\theta_B = \hat{p}_W\cdot(\hat{p}_{\ell^-}\times\hat{p}_p)\,, \tag{18}$$

$$\cos\theta\sin\theta\sin\chi\sin\theta_B = (\hat{p}_{\ell^-}\cdot\hat{p}_W)\,[\,\hat{p}_W\cdot(\,\hat{p}_{\ell^-}\times\hat{p}_p)] \tag{19}$$

Under time reversal ($t\to -t$) one has ($p\to -p$). The above two invariants (18) and (19) involve an odd number of momenta, i.e. they change sign under time reversal. This has led to the notion of the so-called T-odd observables: Observables that multiply T-odd momentum invariants are called T-odd observables.

In the same vein we rewrite the angular factors multiplying $\cos\chi$. One finds

$$\begin{aligned} \sin\theta\cos\chi\sin\theta_B &= \hat{p}_W\cdot\hat{p}_p + (\hat{p}_W\cdot\hat{p}_p)\,(\hat{p}_W\cdot\hat{p}_{\ell^-})\,, \\ \cos\theta\sin\theta\cos\chi\sin\theta_B &= (\hat{p}_{\ell^-}\cdot\hat{p}_W)\,\left(\hat{p}_W\cdot\hat{p}_p + (\hat{p}_W\cdot\hat{p}_p)\,(\hat{p}_W\cdot\hat{p}_{\ell^-})\right). \end{aligned}$$

There is an even number of momentum factors in the angular correlations involving $\cos\chi$, i.e. the momentum invariants correspond to T-even angular correlations.

The T-odd contributions can arise from two different sources. They can be contributed to by true CP-violating effects or by final state interaction effects (imaginary parts of loop contributions). One can distinguish between the two sources of T-odd effects by comparing with the corresponding antihyperon decays. Phases from CP-violating effects change sign whereas phases from final state interaction effects do not change sign when going from hyperon to antihyperon decays.

6 Two examples of polar angle decay distributions

6.1 The top quark decay $t \to b + W^+(\to \ell^+ + \nu_\ell)$

We are finally ready to derive the polar angle distribution $W(\theta) \sim L_{\mu\nu}H^{\mu\nu}$ in the decay $t \to b + W^+(\to \ell^+ + \nu_\ell)$ using helicity methods. We take the W^+ to be on-shell, i.e. the W^+ has three spin degrees of freedom with corresponding helicities $\lambda_W = \pm 1, 0$. Heeding Eq. (16) one has

$$L_{\mu\nu}H^{\mu\nu} = \frac{1}{8} \sum_{\lambda_b,\lambda_W,\lambda_\ell} |H^{V-A}_{\lambda_b\,\lambda_W}|^2 \, d^1_{\lambda_W,\lambda_\ell+\frac{1}{2}}(\theta)\, d^1_{\lambda_W,\lambda_\ell+\frac{1}{2}}(\theta)\, |h^{V-A}_{\lambda_\ell,-\frac{1}{2}}|^2. \tag{20}$$

At the scale of the process one can put the lepton-side helicity flip amplitude to zero, i.e. $|h^{V-A}_{-\frac{1}{2},-\frac{1}{2}}|^2 = 0$. The helicity nonflip amplitude is given by $|h^{V-A}_{\frac{1}{2},-\frac{1}{2}}|^2 = 8m_W^2$. One obtains

$$L_{\mu\nu}H^{\mu\nu} = \frac{m_W^2}{4}\Big(|H^{V-A}_{\frac{1}{2}1}|^2(1+\cos\theta)^2 + 2(|H^{V-A}_{\frac{1}{2}0}|^2 + |H^{V-A}_{-\frac{1}{2}0}|^2)\sin^2\theta + |H^{V-A}_{-\frac{1}{2}1}|^2(1-\cos\theta)^2\Big).$$

The corresponding three-fold angular decay distribution of polarized top decay $t(\uparrow) \to b + W^+(\to \ell^+ + \nu_\ell)$ [1, 2] can be derived with similar ease.

As emphasized in Sec. 3.2 the bilinear forms $\sum_{\lambda_b} |H^{V-A}_{\lambda_b\,\lambda_W}|^2$ ($\lambda_j = 1, 0, -1$) are the (unnormalized) density matrix elements of the on-shell W^+ in the W^+ rest frame. In their normalized form the density matrix elements $\sum_{\lambda_b} |\widehat{H}^{V-A}_{\lambda_b\,\lambda_j}|^2$ are usually referred to as the helicity fractions of the W^+ labelled by $\mathcal{H}_+$, $\mathcal{H}_0$ and $\mathcal{H}_-$. At the Born term level and for $m_b = 0$ one has ($y^2 = m_W^2/m_t^2$)

$$\mathcal{H}_+ : \mathcal{H}_0 : \mathcal{H}_- = 0 : \frac{1}{1+2y^2} : \frac{2y^2}{1+2y^2} = 0 : 0.70 : 0.30, \tag{21}$$

where we have used $m_t = 173.5\,\mathrm{GeV}$. NLO and NNLO QCD corrections to the helicity fractions have been calculated in Refs. [1, 2] and in Ref. [22], respectively. Results on the NLO elctroweak corrections to the helicity fractions have been given in Ref. [13].

6.2 The decay $\Lambda_b(\uparrow) \to \Lambda + J/\psi(\to \ell^+\ell^-)$

There has been a longstanding interest to measure the polarization of hadronically produced hyperons, and charm and bottom baryons [24, 25]. Recently the LHCb Collaboration has measured the polarization of hadronically produced Λ_b's [26]. At the same time they measured ratios of squared helicity amplitudes in the decay $\Lambda_b(\uparrow) \to \Lambda + J/\psi$ through an analysis of polar correlations in the cascade decay process. Consider the three polar angles θ, θ_1 and θ_2 that characterize the cascade decay $\Lambda_b(\uparrow) \to \Lambda(\to p + \pi^-) + J/\psi(\to \ell^+\ell^-)$ (see Fig. 2)

By now we know how to write down the master formula for this three-fold polar angle distribution which could also be obtained by azimuthal integration of Eq. (1). Since I also want

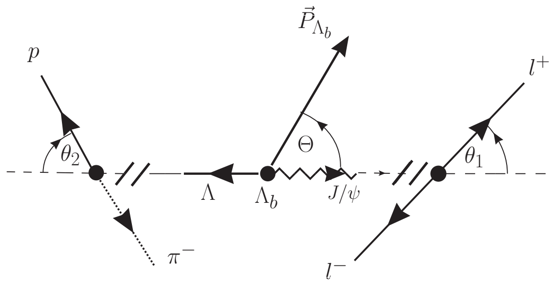


Figure 2: Definition of three polar angles in the decay $\Lambda_b(\uparrow) \to \Lambda(\to p + \pi^-) + J/\psi(\to \ell^+\ell^-)$

to discuss the decay $\Lambda_b(\uparrow) \to \Lambda(\to p + \pi^-) + \psi(2S)(\to \ell^+\ell^-)$ I use the generic notation V for the $J^{PC} = 1^{--}$ vector resonances J/ψ and $\psi(2S)$. In the $\psi(2S)$ mode one also has access to the decay $\psi(2S) \to \tau^+\tau^-$ which necessitates the incorporation of lepton mass effects in the decay distribution. One has

$$\begin{aligned} W(\theta,\theta_1,\theta_2) \;\; &\propto \;\; \frac{1}{2}\sum_{helicities} |h^V_{\lambda_1\lambda_2}|^2 \left[d^1_{\lambda_V,\lambda_1-\lambda_2}(\theta_2)\right]^2 \rho_{\lambda_b,\lambda_b}(\theta) \\ &\times\; \delta_{\lambda_b,\lambda_V-\lambda_\Lambda} |H_{\lambda_\Lambda\lambda_V}|^2 \left[d^{1/2}_{\lambda_\Lambda\lambda_p}(\theta_1)\right]^2 |h^B_{\lambda_p,0}|^2, \end{aligned}$$

where λ_V is the helicity of the J/ψ or $\psi(2S)$. The lepton non-flip (n.f.) and flip (h.f.) helicity amplitudes for the parity conserving decays $V \to \ell^+\ell^-$ are given by

$$\text{n.f.}: \quad h^V_{-\frac{1}{2},-\frac{1}{2}} = h^V_{+\frac{1}{2},+\frac{1}{2}} = 2m_l, \qquad \text{h.f.}: \quad h^V_{-\frac{1}{2},+\frac{1}{2}} = h^V_{+\frac{1}{2},-\frac{1}{2}} = \sqrt{2}m_V.$$

We also know how to rotate the density matrix of the Λ_b from its production direction (perpendicular to the production plane)

$$\rho_{\lambda_b\lambda_b'}(\theta_P) = \frac{1}{2}\begin{pmatrix} 1 + P_b\cos\theta_P & P_b\sin\theta_P \\ P_b\sin\theta_P & 1 - P_b\cos\theta_P \end{pmatrix}$$

Since I am not considering azimuthal correlations in this application only the diagonal density matrix elements $\rho_{\lambda_b\lambda_b}$ are needed.

Next introduce the linear combinations of normalized helicity bispinors $|\widehat{H}_{\lambda_\Lambda\lambda_V}|^2$ (where $|\widehat{H}_{\lambda_\Lambda\lambda_V}|^2 = |H_{\lambda_\Lambda\lambda_V}|^2/\sum_{\lambda_\Lambda,\lambda_V}|H_{\lambda_\Lambda\lambda_V}|^2$)

$$\begin{aligned} \alpha_b &= |\widehat{H}_{+\frac{1}{2}0}|^2 - |\widehat{H}_{-\frac{1}{2}0}|^2 + |\widehat{H}_{-\frac{1}{2}-1}|^2 - |\widehat{H}_{+\frac{1}{2}+1}|^2\,, \\ r_0 &= |\widehat{H}_{+\frac{1}{2}0}|^2 + |\widehat{H}_{-\frac{1}{2}0}|^2\,, \\ r_1 &= |\widehat{H}_{+\frac{1}{2}0}|^2 - |\widehat{H}_{-\frac{1}{2}0}|^2\,. \end{aligned}$$

We define $\varepsilon = m_\ell^2/m_V^2$ such that the velocity of the lepton is given by $v = (1-4\varepsilon)^{1/2}$.

The polar angle distribution can be written as

$$\widetilde{W}(\theta,\theta_1,\theta_2) = \sum_{i=0}^{7} f_i(\alpha_b,r_0,r_1)\; g_i(P_b,\alpha_\Lambda)\; h_i(\cos\theta,\cos\theta_1,\cos\theta_2)\; \ell_i(\varepsilon). \tag{22}$$

The functions f_i, g_i, h_i and ℓ_i are listed in the following table.

i	$f_i(\alpha_b, r_0, r_1)$	$g_i(P_b, \alpha_\Lambda)$	$h_i(\cos\theta, \cos\theta_1, \cos\theta_2)$	$\ell_i(\varepsilon)$
0	1	1	1	$v \cdot (1 + 2\varepsilon)$
1	α_b	P_b	$\cos\theta$	$v \cdot (1 + 2\varepsilon)$
2	$2r_1 - \alpha_b$	α_Λ	$\cos\theta_1$	$v \cdot (1 + 2\varepsilon)$
3	$2r_0 - 1$	$P_b\alpha_\Lambda$	$\cos\theta\cos\theta_1$	$v \cdot (1 + 2\varepsilon)$
4	$\frac{1}{2}(1 - 3r_0)$	1	$\frac{1}{2}(3\cos^2\theta_2 - 1)$	$v \cdot v^2$
5	$\frac{1}{2}(\alpha_b - 3r_1)$	P_b	$\frac{1}{2}(3\cos^2\theta_2 - 1)\cos\theta$	$v \cdot v^2$
6	$-\frac{1}{2}(\alpha_b + r_1)$	α_Λ	$\frac{1}{2}(3\cos^2\theta_2 - 1)\cos\theta_1$	$v \cdot v^2$
7	$-\frac{1}{2}(1 + r_0)$	$P_b\alpha_\Lambda$	$\frac{1}{2}(3\cos^2\theta_2 - 1)\cos\theta\cos\theta_1$	$v \cdot v^2$

The symbols in the table are

P_b	:	polarization of Λ_b
α_b	:	asymmetry parameter in the decay $\Lambda \to p + \pi^-$

The overall factor v in the fifth column is the phase space factor for $V \to \ell^+\ell^-$. The factors $(1 + 2\epsilon)$ (S-wave dominance) and v^2 (($S - D$)-wave interference) were calculated by us for the first time. The LHCb Collaboration finds a very small polarization of the Λ_b [26]

$$P_b = 0.05 \pm 0.07 \pm 0.02.$$

Our results on helicity amplitudes for the transitions $\Lambda_b \to \Lambda$ [10] agree with the experimental results [26]. Our calculation is based on the confined covariant quark model developed by us (see e.g. Refs. [3, 10, 27, 28, 29]).

6.3 The confined covariant quark model in a nutshell

The confined covariant quark model provides a field theoretic frame work for the constituent quark model (see e.g. Refs. [3, 10, 27, 28, 29]). Its main features can be summarized as follows.

Particle transitions are calculated from Feynman diagrams involving quark loops. For example, the $\Lambda_b \to \Lambda$ transition is described by a two-loop diagram requiring a genuine two-loop calculation. The high energy behaviour of quark loops is tempered by nonlocal Gaussian-type vertex functions with a Gaussian-type fall-off behaviour. The particle-quark vertices have interpolating current structure. Use free local quark propagators $(m - \not{p})^{-1}$ in the Feynman diagrams. The normalization of the particle-quark vertices is provided by the compositeness condition which embodies the correct charge normalization of the respective hadron. The compositeness condition can be viewed as the field theoretic equivalent of the normalization of the wave function of a quantum mechanical state. A universal infrared cut-off provides for a effective confinement of quarks. There are no free quark poles in the Feynman diagrams.

HQET relations are recovered by using a static propagator for the heavy quark (k_1 is a loop momentum)

$$\frac{1}{m_b - \not{k}_1 - \not{p}_1} \quad \to \quad \frac{1 + \not{v}_1}{-2k_1v_1 - 2\bar{\Lambda}}.$$

7 Summary

The helicity method provides an easy and simple access to angular decay distributions in sequential cascade decays. Polarization and mass effects are readily incorporated. The corresponding techniques should belong to the basic tool kit of every experimentalist and theorist working in particle physics phenomenology.

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