Light and Heavy Hadrons in AdS/QCD

Valery E. Lyubovitskij[1]*, *Thomas Gutsche*[1], *Ivan Schmidt*[2], *Alfredo Vega*[3]

[1]Institut für Theoretische Physik, Universität Tübingen,
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D–72076 Tübingen, Germany
[2]Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal),
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
[3]Departamento de Física y Astronomía, Universidad de Valparaíso,
Avenida Gran Bretaña 1111, Valparaíso, Chile

We discuss light and heavy hadrons in a holographic soft-wall AdS/QCD model. This approach is based on an action which describes hadron structure with broken conformal and chiral invariance and incorporates confinement through the presence of a background dilaton field. According to the gauge/gravity duality the five-dimensional boson and fermion fields propagating in AdS space are dual to four-dimensional fields leaving on the surface of AdS sphere, which correspond to hadrons. In this picture hadronic wave functions — basics blocks of hadronic properties — are dual to the profiles of AdS fields in the fifth (holographic) dimension, which is identified with scale variable. As applications we consider properties of light and heavy hadrons from unified point of view: mass spectrum, form factors, decay rates and parton distributions.

Based on the gauge/gravity duality [1], a class of AdS/QCD approaches which model QCD by using methods of extra-dimensional field theories formulated in anti-de Sitter (AdS) space, was recently successfully developed for describing the phenomenology of hadronic properties (for a recent review see e.g. [2]). One of the popular formalisms of this kind is the "soft-wall" model [3]-[6] which uses a soft infrared (IR) cutoff in the fifth dimension. This procedure can be introduced in the following ways: i) as a background field (dilaton) in the overall exponential of the action ("dilaton" soft-wall model), ii) in the warping factor of the AdS metric ("metric" soft-wall model), iii) in the effective potential of the action. In Ref. [5] we showed that these three ways of proceeding are equivalent to each other via a redefinition of the bulk fields and by inclusion of extra effective potentials in the action. In our opinion, the "dilaton" form of the soft-wall model is more convenient in performing the calculations.

In this paper we consider such type of soft-wall AdS/QCD approach. We report the applications of our approach to the properties of light and heavy hadrons. In particular, we present results for hadronic mass spectra, coupling constants and form factors [4]-[6].

*On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia

1 Approach

Here we briefly review our approach. First, we specify the five-dimensional AdS metric:

$$
\begin{aligned}
ds^2 = g_{MN} dx^M dx^N &= \eta_{ab}\, e^{2A(z)}\, dx^a dx^b = e^{2A(z)}\, (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)\,, \\
\eta_{\mu\nu} &= \mathrm{diag}(1,-1,-1,-1,-1)\,, \qquad (1)
\end{aligned}
$$

where M and $N = 0, 1, \cdots, 4$ are the space-time (base manifold) indices, $a = (\mu, z)$ and $b = (\nu, z)$ are the local Lorentz (tangent) indices, and g_{MN} and η_{ab} are curved and flat metric tensors, respectively, which are related by the vielbein $\epsilon_M^a(z) = e^{A(z)}\, \delta_M^a$ as $g_{MN} = \epsilon_M^a \epsilon_N^b \eta_{ab}$. Here z is the holographic coordinate, R is the AdS radius, and $g = |\mathrm{det} g_{MN}|$. In the following we restrict ourselves to a conformal-invariant metric with $A(z) = \log(R/z)$.

The relevant AdS/QCD actions for the boson and fermion field of spin J are [4]-[6]

$$
\begin{aligned}
S_B &= \int d^4x dz\, \sqrt{g}\, e^{-\varphi(z)} \biggl[\mathcal{D}_M \Phi_{M_1\cdots M_J}(x,z) \mathcal{D}^M \Phi^{M_1\cdots M_J}(x,z) \\
&- \Bigl((\mu_J^B)^2 + U_J^B(z)\Bigr)\, \Phi_{M_1\cdots M_J}(x,z) \Phi^{M_1\cdots M_J}(x,z)\biggr]\,, \qquad (2) \\
S_F &= S_F^+ + S_F^-\,, \quad S_F^\pm = \int d^4x dz\, \sqrt{g}\, e^{-\varphi(z)} \sum_{i=+,-} \biggl[\bar\Psi^\pm_{M_1\cdots M_J}(x,z) i \mathcal{D}_M^\pm \Psi^{\pm\, M_1\cdots M_J}(x,z) \\
&\mp \bar\Psi^\pm_{M_1\cdots M_J}(x,z) \Bigl((\mu_J^F)^2 + U_J^F(z)\Bigr) \Psi^{\pm\, M_1\cdots M_J}(x,z) \qquad (3)
\end{aligned}
$$

where $\mathcal{D}_M$ and $\mathcal{D}_M^\pm$ are the covariant derivative (including external vector and axial fields) acting on boson $\Phi_{M_1\cdots M_J}$ and fermion $\Psi^\pm_{M_1\cdots M_J}$ fields, respectively. $\Psi^\pm_{M_1\cdots M_J}$ is the pair of bulk fermion fields, which are the holographic analogues of the left- and right-chirality fermion operators in the 4D theory. $\varphi(z) = \kappa^2 z^2$ is the dilaton field with κ being a free scale parameter. The quantities μ_J^B and μ_J^F are the bulk boson and fermion masses related to the conformal dimensions (Δ_J^B, Δ_J^F) of the spin-J AdS boson and fermion fields, respectively

$$
(\mu_J^B R)^2 = \Delta_J^B (\Delta_J^B - 4)\,, \quad \mu_J^F R = \Delta_J^F - 2 \qquad (4)
$$

As was shown in Refs. [7] and [5] the field dimensions Δ_J^B and Δ_J^F are related to twist-dimension $\tau_{B/F}$ of hadronic operators as

$$
\Delta_J^B = \tau_B = 2 + L\,, \quad \Delta_J^F = \tau_F + \frac{1}{2} = \frac{7}{2} + L\,. \qquad (5)
$$

where $L = \max|L_z|$ is the maximal value of the z component of the quark orbital angular momentum in hadron [7]: $U_J^B(z) = 4\varphi(z)(J-1)/R^2$ and $U_J^F(z) = \varphi(z)/R$ are the effective dilaton potentials. Note the choice of quadratic dilaton profile and potentials $U_J^B(z)$ and $U_J^F(z)$ is necessary in order to guarantee correct Regge behavior of hadronic mass spectra and asymptotic power scaling of hadronic factors at large momenta transfer in agreement with quark counting rules [4]-[6].

Notice that the fermion masses and the effective potentials corresponding to the fields Ψ^+ and Ψ^- have opposite signs according to the P-parity transformation. The absolute sign of the fermion mass is related to the chirality of the boundary operator. According to our conventions the QCD operators $\mathcal{O}_R$ and $\mathcal{O}_L$ have positive and negative chirality, and therefore the mass terms of the bulk fields Ψ^+ and Ψ^- have absolute signs "plus" and "minus", respectively.

One of the main advantages of the soft-wall AdS/QCD model is that the most of the calculations can be done analytically. In a first step, we show how in this approach the hadron wave functions and spectrum are generated. We follow the procedure pursued in Refs. [4]-[6]. We drop the external vector and axial fields in covariant derivatives, turn to the tangent space with Lorentz signature, where the AdS fields are rescaled as

$$\Phi_{\mu_1\cdots\mu_J} = e^{\varphi(z)/2+A(z)J}\phi_{\mu_1\cdots\mu_J}\,, \quad \Psi^{\pm}_{\mu_1\cdots\mu_J} = e^{\varphi(z)/2+A(z)(J-1/2)}\psi^{\pm}_{\mu_1\cdots\mu_J}\,. \tag{6}$$

Next we split the fermion field into left- and right-chirality components

$$\psi^{\pm}_{\mu_1\cdots\mu_J}(x,z) = \psi^{\pm\,L}_{\mu_1\cdots\mu_J}(x,z) + \psi^{\pm\,R}_{\mu_1\cdots\mu_J}(x,z) \tag{7}$$

and perform Kaluza-Klein (KK) expansion for $\phi_{\mu_1\cdots\mu_J}(x,z)$ and $\psi^{\pm\,L/R}_{\mu_1\cdots\mu_J}(x,z)$

$$\begin{aligned}\phi_{\mu_1\cdots\mu_J}(x,z) &= \sum_n \phi_{n\,\mu_1\cdots\mu_J}(x)\,F_{n\tau}(z)\,,\\ \psi^{\pm\,L/R}_{\mu_1\cdots\mu_J}(x,z) &= \frac{1}{\sqrt{2}}\sum_n \psi^{L/R}_{n\,\mu_1\cdots\mu_J}(x)\,G^{\pm\,L/R}_{n\tau}(z)\,,\end{aligned} \tag{8}$$

where the tower of the KK fields $\phi_{n\,\mu_1\cdots\mu_J}(x)$ is dual to four-dimensional fields describing mesons with spin J, while KK fields $\psi^{L/R}_{n\,\mu_1\cdots\mu_J}(x)$ are dual left/right-chirality fermion fields describing baryons with spin J. The number n corresponds to the radial quantum number. The set of functions $F_{n\tau}(z)$ are the profiles of boson AdS fields in holographic direction, which are dual to the mesonic wave functions with twist τ and radial quantum number n. In case of baryon we have four sets of such profiles dual to baryonic wave functions, which satisfy to the following relation (due P- and C-invariance)

$$G^{\pm\,R}_{n\tau}(z) = \mp G^{\mp\,L}_{n\tau}(z)\,. \tag{9}$$

Then it is convenient to rescale the boson and fermion profiles as

$$F_{n\tau}(z) = e^{-3/2A(z)}\,f_{n\tau}(z)\,, \quad G^{\pm\,R/L}_{n\tau}(z) = e^{-2A(z)}\,g^{\pm\,R/L}_{n\tau}(z) \tag{10}$$

in order derive the Schrödinger-type equation of motions (EOMs) for the wave functions $f_{n\tau}$ and $g^{\pm\,L/R}_{n\tau}(z)$

$$\Big[-\partial_z^2 + \frac{4L^2-1}{4z^2} + \kappa^4 z^2 + 2\kappa^2(J-1)\Big]\,f_{n\tau}(z) \;=\; M^2_{B,n\tau J}\,f_{n\tau}(z) \tag{11}$$

and

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\Big(m \mp \frac{1}{2}\Big) + \frac{m(m\pm 1)}{z^2}\right]\,g^{L/R}_{n\tau}(z) \;=\; M^2_{F,n\tau}\,g^{L/R}_{n\tau}(z)\,, \tag{12}$$

where $m = \tau - 3/2$; $M_{B,n\tau J}$ and $M_{F,n\tau}$ are the masses of bosons and fermions dual to corresponding hadrons (mesons and baryons) with specific values of quantum numbers.

Above EOMs have analytical solutions for both wave functions

$$\begin{aligned} f_{n\tau}(z) &= \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\, \kappa^{\tau-1}\, z^{\tau-3/2}\, e^{-\kappa^2 z^2/2}\, L_n^{\tau-2}(\kappa^2 z^2)\,, \\ g^L_{n\tau}(z) &= \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}}\, \kappa^{\tau}\, z^{\tau-1/2}\, e^{-\kappa^2 z^2/2}\, L_n^{\tau-1}(\kappa^2 z^2)\,, \\ g^R_{n\tau}(z) &= \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\, \kappa^{\tau-1}\, z^{\tau-3/2}\, e^{-\kappa^2 z^2/2}\, L_n^{\tau-2}(\kappa^2 z^2) \end{aligned} \tag{13}$$

and mass spectrum

$$M^2_{B,n\tau J} = 4\kappa^2\Big(n + \frac{\tau+J}{2} - 1\Big)\,, \qquad M^2_{F,n\tau} = 4\kappa^2\Big(n+\tau-1\Big)\,. \tag{14}$$

Therefore, our main idea is to find the solutions for the bulk profiles of the AdS field in the z–direction, and then calculate the physical properties of hadrons in terms of the bulk profiles of AdS fields dual to hadronic wave functions. In this way both mass spectrum and dynamical hadronic properties like form factors and parton distributions will be calculated from a unified point of view based on the solutions of the Schrödinger-type EOMs (13). One can see that the bulk profiles of AdS fields have the correct scaling behavior for small z, which leads to correct power behavior of calculated hadronic form factors at large Q^2. Another important property of the bulk profiles is that they vanish at large z (confinement). Up to now we discussed the solutions of EOMs for the bulk profiles on its mass shell $p^2 = M^2$. In case when we go beyond mass shell, we can calculate so-called bulk-to-boundary propagators describing the behavior of bulk profiles at arbitrary p^2, which are necessary for calculation of momentum dependence of matrix elements in our approach. In particular, the bulk-to-boundary propagator for the vector AdS field dual to electromagnetic field is given in analytical form in terms of the Gamma $\Gamma(n)$ and Tricomi $U(a,b,z)$ functions:

$$V(Q,z) = \Gamma\bigg(1 + \frac{Q^2}{4\kappa^2}\bigg) U\bigg(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\bigg)\,. \tag{15}$$

The bulk-to-boundary propagator $V(Q,z)$ obeys the normalization condition $V(0,z) = 1$ consistent with gauge invariance and fulfils the following ultraviolet (UV) and infrared (IR) boundary conditions: $V(Q,0) = 1\,, \quad V(Q,\infty) = 0$. The UV boundary condition corresponds to the local (structureless) coupling of the electromagnetic field to matter fields, while the IR boundary condition implies that the vector field vanishes at $z = \infty$. E.g. a generic expression for the meson form factor is given in the form integral over z variable of the product of $V(Q,z)$ and bulk profiles corresponding to the wave functions of initial (in) and final (fin) meson

$$F_M(Q^2) = \int\limits_0^\infty dz V(Q,z) f_{\rm in}(z) f_{\rm fin}(z)\,. \tag{16}$$

Another advantage of our approach is a possibility to constraint the form of light-front wave functions (see detailed discussion in Refs. [4]-[6]) from matching of matrix elements of physical processes in AdS/QCD and Light-Front QCD. The idea of such matching was proposed in Ref. [7]. Next step is inclusion of effects of quark masses in agreement with constraints imposed by chiral symmetry and heavy quark effective theory.

2 Applications

2.1 Meson mass spectrum and leptonic decay constants

We consider applications of our approach to mass spectrum, decay constants, form factors and parton distributions. First we present the results for the mass spectrum and decay constants of mesons: light, heavy-light and heavy quarkonia (see Tables I-V).

Table I. Masses of light mesons.

Meson	n	L	S	Mass [MeV]			
π	0,1,2,3	0	0	140	1010	1421	1738
K	0	0,1,2,3	0	495	1116	1498	1801
η	0,1,2,3	0	0	566	11494	1523	1822
$f_0[\bar{n}n]$	0,1,2,3	1	1	721	1233	1587	1876
$f_0[\bar{s}s]$	0,1,2,3	1	1	985	1404	1723	1993
$\rho(770)$	0,1,2,3	0	1	721	1233	1587	1876
$\omega(782)$	0,1,2,3	0	1	721	1233	1587	1876
$\phi(1020)$	0,1,2,3	0	1	985	1404	1723	1993
$a_1(1260)$	0,1,2,3	1	1	1010	1421	1738	2005

Table II. Masses of heavy-light mesons.

Meson	$J^{\rm P}$	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1870	2000	2121	2235
$D^*(2010)$	1^-	0	0,1,2,3	1	2000	2121	2235	2345
$D_s(1969)$	0^-	0	0,1,2,3	0	1970	2093	2209	2320
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2093	2209	2320	2425
$B(5279)$	0^-	0	0,1,2,3	0	5280	5327	5374	5420
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5374	5420	5466
$B_s(5366)$	0^-	0	0,1,2,3	0	5370	5416	5462	5508
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5462	5508	5553

Table III. Masses of heavy quarkonia.

Meson	$J^{\rm P}$	n	L	S	Mass [MeV]			
$\eta_c(2980)$	0^-	0,1,2,3	0	0	2975	3477	3729	3938
$\psi(3097)$	1^-	0,1,2,3	0	1	3097	3583	3828	4032
$\chi_{c0}(3415)$	0^+	0,1,2,3	1	1	3369	3628	3843	4038
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3477	3729	3938	4129
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3583	3828	4032	4219
$\eta_b(9390)$	0^-	0,1,2,3	0	0	9337	9931	10224	10471
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10048	10338	10581
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	9813	10110	10359	10591
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	9931	10224	10471	10700
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10048	10338	10581	10808
$B_c(6277)$	0^-	0,1,2,3	0	0	6277	6719	6892	7025

Table IV. Decay constants f_P (MeV) of pseudoscalar mesons.

Meson	Data	Our
π^-	$130.4 \pm 0.03 \pm 0.2$	153
K^-	$156.1 \pm 0.2 \pm 0.8$	153
D^+	206.7 ± 8.9	207
D_s^+	257.5 ± 6.1	224
B^-	193 ± 11	163
B_s^0	$253 \pm 8 \pm 7$	170
B_c	$489 \pm 5 \pm 3$	489

Table V. Decay constants f_V (MeV) of vector mesons.

Meson	Data	Our	Meson	Data	Our
ρ^+	210.5 ± 0.6	216	ρ^0	154.7 ± 0.7	153
D^*	$245 \pm 20^{+3}_{-2}$	207	ω	45.8 ± 0.8	51
D_s^*	$272 \pm 16^{+3}_{-20}$	224	ϕ	76 ± 1.2	72
B^*	$196 \pm 24^{+39}_{-2}$	170	J/ψ	277.6 ± 4	223
B_s^*	$229 \pm 20^{+41}_{-16}$	170	$\Upsilon(1s)$	238.5 ± 5.5	170

One should stress that our analytical results for the masses of light pseudoscalar mesons are consistent with chiral symmetry: $M_\pi^2, M_K^2, M_\eta^2 \to 0$ at $m_{u,d}, m_s \to 0$. The masses and leptonic decay constant of heavy-light mesons are consistent with constraints imposed by heavy quark mass limit. In particular, the heavy quark mass expansion of heavy-light mesons masses reads $M_{Qq} = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q)$ and their leptonic decay constants scale as $f_{Qq} \sim 1/\sqrt{m_Q}$.

2.2 Electromagnetic structure of nucleon

Here from unified point of view we describe nucleon form factors and the electroproduction of the $N(1440)$ Roper resonance. The Roper resonance is identified as the first radially excited state of the nucleon. The obtained results for helicity amplitudes of the Roper electroproduction are in good agreement with the recent results of the CLAS Collaboration at JLab. In Table VI we present our results for the nucleon properties: mass, magnetic moments, electromagnetic and axial charge radii. In Figs. 1-2 we present selected results for the electromagnetic form factors of nucleon.

Table VI. Mass and electromagnetic properties of nucleons.

Quantity	Our results	Data [9]
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
g_A	1.270	1.2701
r_E^p (fm)	0.840	0.8768 ± 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 ± 0.0022
r_M^p (fm)	0.785	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.792	$0.862^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67 ± 0.01

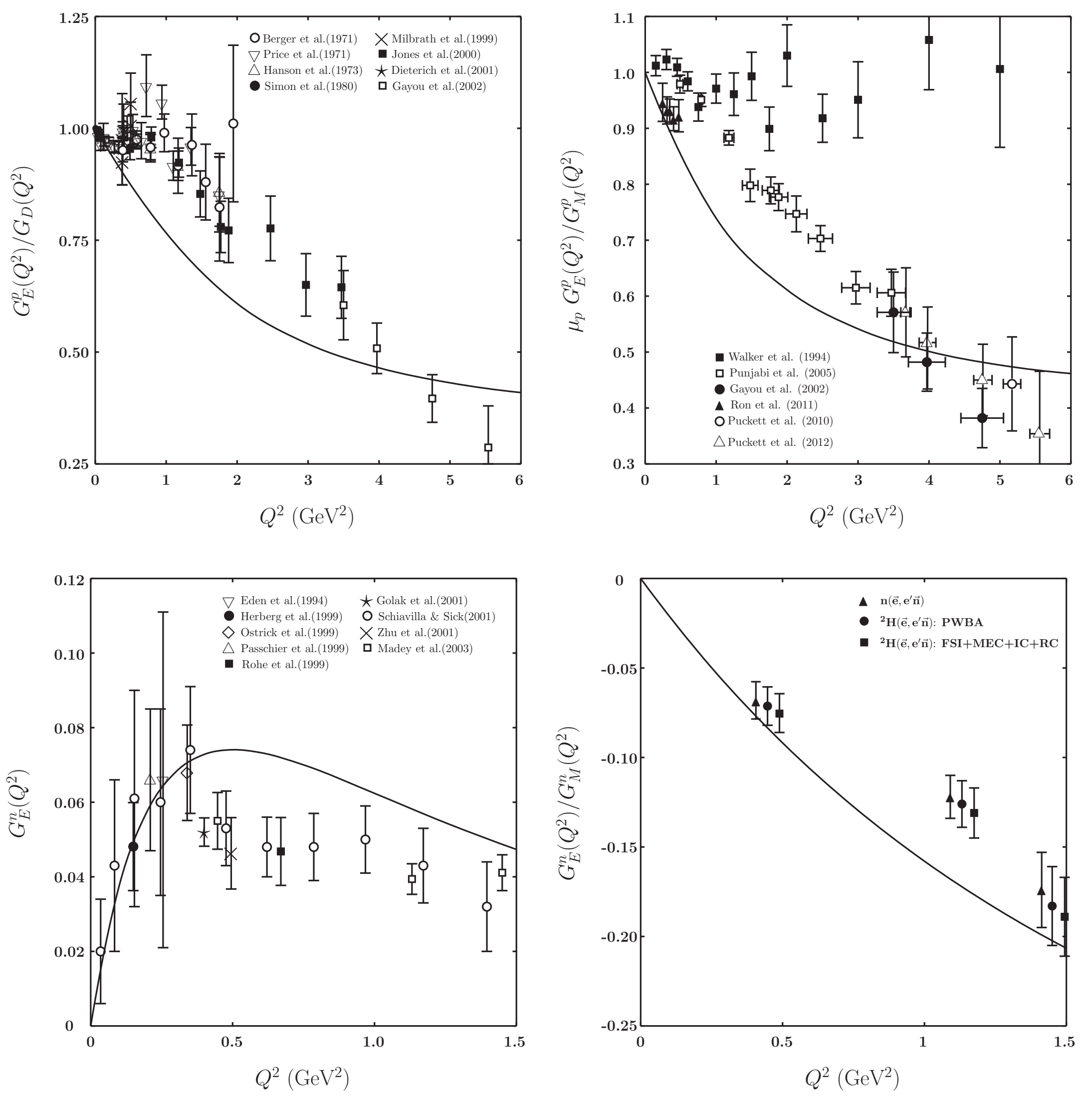

Figure 1: The ratios $G_E^p(Q^2)/G_D(Q^2)$, $G_E^p(Q^2)/G_M^p(Q^2)$, $G_E^n(Q^2)/G_M^n(Q^2)$ and charge neutron form factor $G_E^n(Q^2)$.

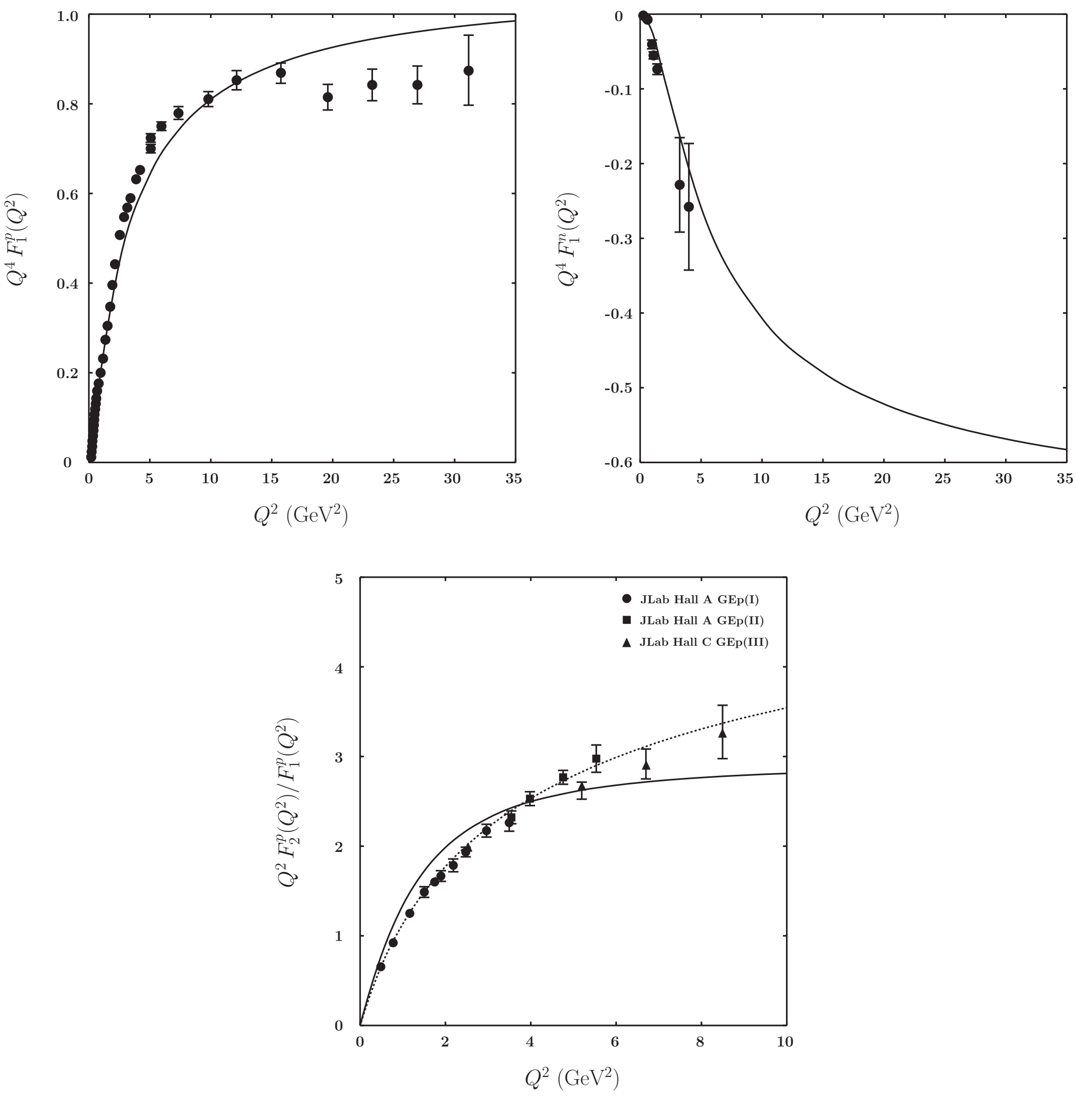

Figure 2: Proton and neutron Dirac form factor multiplied with Q^4, ratios $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$ (the dashed line is the approximation of data suggested in Ref. [8]) and $G_A(Q^2)/G_A^D(Q^2)$.

Table VII. Helicity amplitudes $A^N_{1/2}(0)$ and $S^N_{1/2}(0)$, $N = p, n$.

Quantity	Our results	Data [9]
$A^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.065 (-0.065)	-0.065 $\pm$ 0.004
$A^n_{1/2}(0)$ (GeV$^{-1/2}$)	0.040 (0.040)	0.040 $\pm$ 0.010
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	0.047 (0.048)	
$S^n_{1/2}(0)$ (GeV$^{-1/2}$)	-0.044 (-0.045)	

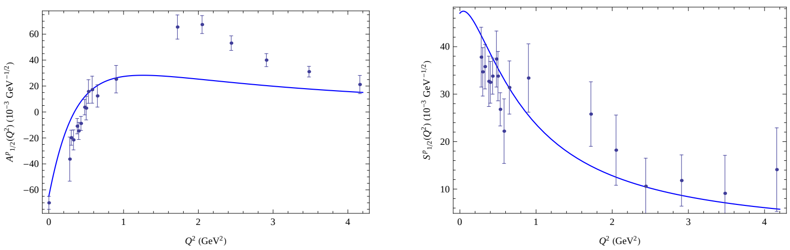

Figure 3: Helicity amplitudes $A^p_{1/2}(Q^2)$ and $S^p_{1/2}(Q^2)$ up to $Q^2 = 4$ GeV^2.

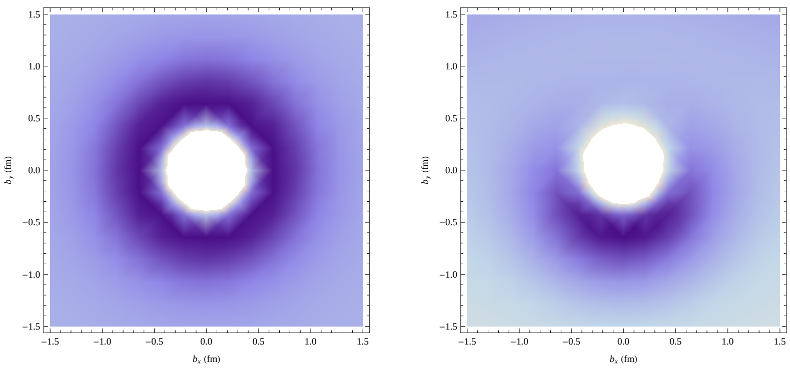

Figure 4: Transition charge densities for unpolarized and transversely polarized nucleon and Roper.

In Table VII we present our results for the helicity amplitudes $A^N_{1/2}(0)$ and $S^N_{1/2}(0)$, $N = p, n$ at $Q^2 = 0$. In Fig. 3 we present our predictions for the Q^2 dependence of helicity amplitudes. Results for the transition charge densities for unpolarized and transversely polarized nucleon and Roper in the transverse impact parameter plane $\mathbf{b}_\perp = (b_x, b_y)$ are shown in Fig. 4.

From Fig.3 it should be evident that our results for the helicity amplitudes in the proton case have qualitative agreement with the present data of the CLAS Collaboration [10]. Within the current approach it is difficult to reproduce the maximum of data for $A^p_{1/2}$ at about 2 GeV^2. Further data for the helicity amplitudes in the region from 1.6 to 4 GeV^2 could be accumulated at the upgraded facilities of JLab and certainly help to clarify the theoretical understanding.

Acknowledgements

This work was supported by the DFG under Contract No. LY 114/2-1, by FONDECYT (Chile) under Grant No. 1100287 and by CONICYT (Chile) under Grant No. 7912010025. The work is done partially under the project 2.3684.2011 of Tomsk State University. V. E. L. would like to thank Heisenberg-Landau Program for financial support.

References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[2] Y. Kim and D. Yi, Adv. High Energy Phys. **2011**, 1 (2011).

[3] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, "Linear Confinement and AdS/QCD," Phys. Rev. D **74**, 015005 (2006); S. J. Brodsky and G. F. de Teramond, "Hadronic spectra and light-front wavefunctions in holographic QCD," Phys. Rev. Lett. **96**, 201601 (2006); O. Andreev, "$1/q^2$ corrections and gauge / string duality," Phys. Rev. D **73**, 107901 (2006)

[4] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, "Light and heavy mesons in a soft-wall holographic approach," Phys. Rev. D **82**, 074022 (2010); A. Vega, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, "Meson wave function from holographic models," Phys. Rev. D **80**, 055014 (2009); T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, "Chiral Symmetry Breaking and Meson Wave Functions in Soft-Wall AdS/QCD," Phys. Rev. D **87**, 056001 (2013).

[5] T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, "Dilaton in a soft-wall holographic approach to mesons and baryons," Phys. Rev. D **85**, 076003 (2012).

[6] T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, "Nucleon structure including high Fock states in AdS/QCD," Phys. Rev. D **86**, 036007 (2012); A. Vega, I. Schmidt, T. Gutsche, V. E. Lyubovitskij, "Generalized parton distributions in AdS/QCD," Phys. Rev. D **83**, 036001 (2011); T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, "Nucleon resonances in AdS/QCD," Phys. Rev. D **87**, 016017 (2013).

[7] S. J. Brodsky, G. F. de Teramond, "Hadronic spectra and light-front wavefunctions in holographic QCD," Phys. Rev. Lett. **96**, 201601 (2006); S. J. Brodsky and G. F. de Teramond, "Light-Front Dynamics and AdS/QCD Correspondence: The Pion Form Factor in the Space- and Time-Like Regions," Phys. Rev. D **77**, 056007 (2008).

[8] S. J. Brodsky, "Perspectives on exclusive processes in QCD," arXiv:hep-ph/0208158 (2002).

[9] J. Beringer *et al.* (Particle Data Group), "Review of Particle Physics (RPP)," Phys. Rev. D **86**, 010001 (2012).

[10] V. I. Mokeev *et al.* (CLAS Collaboration), "Experimental Study of the $P_{11}(1440)$ and $D_{13}(1520)$ resonances from CLAS data on $ep \to e'\pi^+\pi^-p'$," Phys. Rev. C **86**, 035203 (2012).