Relativistic Corrections to Pair Charmonium Production at the LHC

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On the basis of perturbative QCD and relativistic quark model we calculate relativistic and bound state corrections to processes of a pair S-wave charmonium production at the LHC. The obtained result for J/ψ pair production at the energy $\sqrt{S} = 7$ TeV lies below the experimental value measured by LHCb collaboration. In the case of η_c pair the examined effects decrease total nonrelativistic cross section more than two times and on 20 percents in the rapidity region of LHCb detector.

1 Introduction

Relativistic corrections caused by relative motion of constituent quarks are known to bring an essential modifications to the values of pair charmonium production cross sections. For example, it was found in Ref. [1] that account of relativistic corrections in the NRQCD formalism increases the nonrelativistic result for $\sigma[e^+e^- \to J/\psi + \eta_c]$ by about 40%. Analogously, the large values of discussed corrections to cross sections of S- and P-wave charmonium production in e^+e^- annihilation were revealed in the framework of potential models and other approaches in Refs. [2] and [3]. In the current work we present the results of relativistic corrections calculation to the processes of pair J/ψ and η_c production in proton–proton collisions at the LHC relevant energies $\sqrt{S} = 7$ TeV and 14 TeV [4]. Within the quasipotential approach we consider two types of the relativistic corrections sources: quark bound state wave functions, which are described by means of the potential model based on the QCD generalization of the Breit potential, and expansions of quark and gluon propagators entering production amplitude. More detailed description of the used approach and the results obtained with it can be found in Refs. [3, 4].

2 General formalism

The differential cross section $d\sigma$ for the inclusive double charmonium production in proton–proton interaction can be presented in the form of the convolution of partonic cross section $d\sigma[g+g \to 2\,J/\psi(\eta_c)]$ with the parton distribution functions (PDF) in the initial protons [5, 6, 7]:

$$d\sigma[p+p \to 2\,J/\psi(\eta_c) + X] = \int dx_1 dx_2\, f_{g/p}(x_1,\mu) f_{g/p}(x_2,\mu)\, d\sigma[g+g \to 2\,J/\psi(\eta_c)],$$

where $f_{g/p}(x,\mu)$ is a partonic distribution function for the gluon in the proton, $x_{1,2}$ are longitudinal momentum fractions of gluons, μ is the factorization scale. Neglecting the proton mass

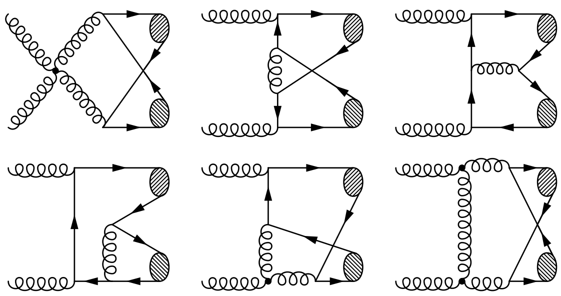

Figure 1: The typical LO diagrams contributing to the partonic process $g + g \to 2\,J/\psi(\eta_c)$. The others can be obtained by reversing the quark lines or interchanging the initial gluons.

and taking the c.m. reference frame of initial protons with the beam along the z-axis we can introduce the gluon on mass-shell momenta in the form $k_{1,2} = x_{1,2}\frac{\sqrt{S}}{2}(1,0,0,\pm1)$. $\sqrt{S}$ is the center-of-mass energy in proton–proton collision.

In the quasipotential approach the double charmonium production amplitude for the parton subprocess $g + g \to 2\,J/\psi(\eta_c)$ can be expressed as a convolution of perturbative production amplitude of two c-quark and $\bar{c}$-antiquark pairs $\mathcal{T}(p_1,p_2;q_1,q_2)$ and the quasipotential wave functions of the final mesons Ψ [3, 4]:

$$\mathcal{M}[g+g\to 2\,J/\psi(\eta_c)](k_1,k_2,P,Q) = \int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\,\bar{\Psi}(p,P)\bar{\Psi}(q,Q)\otimes\mathcal{T}(p_1,p_2;q_1,q_2), \quad (1)$$

where p_1 and p_2 are four-momenta of c-quark and $\bar{c}$-antiquark in the pair forming the first meson, and q_2 and q_1 are the appropriate four-momenta for quark and antiquark in the second meson. They are defined in subsequent transformations in terms of total momenta $P(Q)$ and relative momenta $p(q)$ as follows: $p_{1,2} = \frac{1}{2}P \pm p$, $(pP) = 0$; $q_{1,2} = \frac{1}{2}Q \pm q$, $(qQ) = 0$. Here $p = L_P(0,\mathbf{p})$ and $q = L_Q(0,\mathbf{q})$ are the relative four-momenta obtained by the Lorentz transformation of four-vectors $(0,\mathbf{p})$ and $(0,\mathbf{q})$ to the reference frames moving with the four-momenta P and Q.

At leading order of perturbation theory in strong coupling constant α_s there are 31 Feynman diagrams contributing to the amplitude of pair J/ψ production due to gluon fusion. The typical diagrams from this set are presented in Fig. 1. In the case of η_c pair there are also 8 additional diagrams shown in Fig. 2. The calculation of the diagrams and subsequent analytical transformations are performed by means of the package FeynArts [8] for the system Mathematica and FORM [9]. Then, the production amplitude (1) has the following structure:

$$\mathcal{M}[g+g\to 2\,J/\psi(\eta_c)](k_1,k_2,P,Q) = \frac{1}{9}M\pi^2\alpha_s^2\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\big[\mathrm{Tr}\,\mathfrak{M} + 3\,\tau\Delta\mathfrak{M}\big], \quad (2)$$

$$\mathfrak{M} = \mathcal{D}_1\gamma_\beta\bar{\Psi}_{q,Q}\Gamma_1^\beta\bar{\Psi}_{p,P}\,\hat{\varepsilon}_2\frac{m-\hat{k}_2+\hat{p}_1}{(k_2-p_1)^2-m^2} + \mathcal{D}_2\gamma_\beta\bar{\Psi}_{q,Q}\Gamma_2^\beta\bar{\Psi}_{p,P}\,\hat{\varepsilon}_1\frac{m-\hat{k}_1+\hat{p}_1}{(k_1-p_1)^2-m^2}+$$

$$\mathcal{D}_3\bar{\Psi}_{q,Q}\Gamma_3^\beta\bar{\Psi}_{p,P}\,\gamma_\beta + \mathcal{D}_4\bar{\Psi}_{p,P}\Gamma_4^\beta\bar{\Psi}_{q,Q}\,\gamma_\beta + \mathcal{D}_1\bar{\Psi}_{q,Q}\Gamma_5^\beta\bar{\Psi}_{p,P}\,\gamma_\beta\frac{m+\hat{k}_2-\hat{q}_1}{(k_2-q_1)^2-m^2}\hat{\varepsilon}_2+$$

$$\mathcal{D}_2\bar{\Psi}_{q,Q}\Gamma_6^\beta\bar{\Psi}_{p,P}\,\gamma_\beta\frac{m+\hat{k}_1-\hat{q}_1}{(k_1-q_1)^2-m^2}\hat{\varepsilon}_1,$$

where inverse denominators of gluon propagators are defined as $\mathcal{D}_{1,2}^{-1} = (k_2 - p_{1,2} - q_{1,2})^2$ and $\mathcal{D}_{3,4}^{-1} = (p_{1,2} + q_{1,2})^2$, $\varepsilon_{1,2}$ and $k_{1,2}$ are polarization vectors and four-momenta of the initial gluons, m is c-quark mass, M is observable $J/\psi(\eta_c)$ mass, and the hat symbol means a contraction of the four-vector with the Dirac gamma matrices. The amplitude (2) contains wave functions $\Psi_{p,P}$ and $\Psi_{q,Q}$ of the mesons taken in the reference frame moving with four momenta P and Q. The transformation law of the bound state wave function from the rest frame to the moving one was derived in the Bethe–Salpeter approach in Ref. [10] and in the quasipotential method in Ref. [11]. The $\Delta\mathfrak{M}$ integrand contribution corresponds to the 8 additional diagrams from Fig. 2, so the parameter τ in (2) equals zero in the case of J/ψ and $\tau = 1$ for η_c pair. Explicit expressions for $\Delta\mathfrak{M}$ and vertex functions Γ_i entering (2) can be found in Refs. [4].

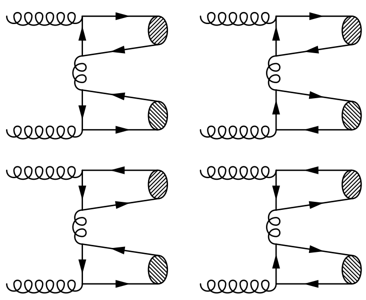

Figure 2: The additional LO diagrams contributing to the partonic process $g + g \to 2\,\eta_c$.

In order to calculate relativistic corrections to the amplitude (2) we expand the inverse denominators of gluon and quark propagators as series in relative quark momenta p and q:

$$\frac{1}{(p_1+q_1)^2} = \frac{4}{s} - \frac{16}{s^2}\left[(p+q)^2 + pQ + qP\right] + \cdots,$$

$$\frac{1}{(k_2-q_2)^2 - m^2} = \frac{2}{t-M^2} - \frac{4}{(t-M^2)^2}\left[q^2 + 2(k_2 q) + \frac{1}{4}M^2 - m^2\right] + \cdots,$$

where $s = (k_1+k_2)^2$ and $t = (P-k_1)^2$ are Mandelstam variables of the partonic subprocess. Preserving in the expanded amplitude terms up to the second order in the relative momenta p and q, we can perform angular integration and calculate the squared modulus of the amplitude summed over polarizations of the initial gluons and, if necessary, over polarizations of the final charmonium states. Then, we obtain the following result for the pair production cross sections:

$$\frac{d\sigma}{dt}[g+g \to 2\,J/\psi(\eta_c)](s,t) = \frac{\pi M^2 \alpha_s^4}{9216\,s^2}\,|\tilde{R}(0)|^4 \sum_{i=0}^{3} \omega_i F^{(i)}(s,t). \quad (3)$$

The auxiliary functions $F^{(i)}$ entering the cross sections (3) are written explicitly in Refs. [4]. Note that the function $F^{(0)}$ describes non-relativistic result, which coincides in the limit $M = 2m$ with the corresponding function obtained in Refs. [12, 13, 14] for the case of pair J/ψ production and in Ref. [13] for η_c pair production in the approach of NRQCD. Relativistic corrections in (3) are determined by a number of relativistic parameters ω_i:

$$\omega_0 = 1, \quad \omega_1 = \frac{I_1}{I_0}, \quad \omega_2 = \frac{I_2}{I_0}, \quad \omega_3 = \omega_1^2,$$

$$I_0 = \int_0^\infty \frac{m+\epsilon(p)}{2\epsilon(p)} R(p) p^2 dp, \quad I_{1,2} = \int_0^m \frac{m+\epsilon(p)}{2\epsilon(p)} \left(\frac{m-\epsilon(p)}{m+\epsilon(p)}\right)^{1,2} R(p) p^2 dp,$$

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{m+\epsilon(p)}{2\epsilon(p)} R(p) p^2 dp,$$

Table 1: The comparison of relativistic and nonrelativistic cross sections of a pair S-wave charmonium production in proton–proton collisions obtained for different sets of partonic distribution functions.

Energy $\sqrt{S}$	Meson pair, cross section type	σ(total), nb		$\sigma(2 < y_{P,Q} < 4.5)$, nb	
		CTEQ5L	CTEQ6L1	CTEQ5L	CTEQ6L1
$\sqrt{S} = 7$ TeV	$J/\psi\, J/\psi$, relativistic	9.6	7.4	1.6	1.2
	$J/\psi\, J/\psi$, nonrelativistic	23.0	17.7	3.8	2.9
	$\eta_c\, \eta_c$, relativistic	23.7	19.9	1.3	1.0
	$\eta_c\, \eta_c$, nonrelativistic	56.3	48.1	1.5	1.2
$\sqrt{S} = 14$ TeV	$J/\psi\, J/\psi$, relativistic	17.1	13.2	3.0	2.1
	$J/\psi\, J/\psi$, nonrelativistic	41.0	31.6	7.1	5.1
	$\eta_c\, \eta_c$, relativistic	47.8	39.3	2.4	1.7
	$\eta_c\, \eta_c$, nonrelativistic	116.5	94.7	2.8	2.0

where $\epsilon(p) = \sqrt{m^2 + \mathbf{p}^2}$ is quark energy and $R(p)$ is the radial charmonium wave function. All parameters, which contain the meson wave functions and describe the transition of $(c\bar{c})$ pairs to the bound state, are calculated in the framework of relativistic quark model. This model is based on the Schrödinger equation with the Breit Hamiltonian in QCD and the nonperturbative confinement terms. Using the program of numerical solution of the Schrödinger equation, we obtain relativistic wave functions and bound state energies of S-wave charmonia. The additional details on our relativistic quark model can be found in Refs. [3, 4].

3 Numerical results and discussion

The numerical results of our calculation of the pair S-wave charmonium production cross sections in the case of non-relativistic approximation as well as with the account of relativistic corrections of order v^2 are presented in Table 1. Along with total cross section values, we have also included there the cross section predictions corresponding to the rapidity interval $2 < y_{P,Q} < 4.5$ of the LHCb experiment [15] calculated with two different sets of linear PDFs: CTEQ5L [16] and CTEQ6L1 [17]. At the current moment, the only known experimental result for the cross section of pair S-wave charmonium production in proton–proton collisions is the result measured by LHCb collaboration for the pair J/ψ production at the energy $\sqrt{S} = 7$ TeV [15]: $\sigma^{exp}_{LHCb} = 5.1 \pm 1.0 \pm 1.1$ nb. The corresponding relativistic result obtained in our model with CTEQ5L partonic function is $\sigma^{theor}_{rel} = 1.6$ nb. It is evident that this results lies below the experimental value measured by LHCb collaboration. Nevertheless, despite the difference between σ^{theor}_{rel} and σ^{exp}_{LHCb}, we consider that at present it is difficult to state that there is the discrepancy between the theory and experiment in double charmonium production. Along with the possibility of large contribution from NLO α_s corrections (as in the case of e^+e^- production [7, 18]), there exists an additional mechanism through the double parton scattering, which gives the contribution comparable with the standard nonrelativistic result [19]: $\sigma_{DPS}[p + p \to 2\, J/\psi + X] = 2$ nb. Accounting for this result and our value of the cross section σ^{theor}_{rel}, we obtain the summary value $\sigma[p + p \to 2\, J/\psi + X] = 3.6$ nb. Then, taking into account the experimental error, the difference with the LHCb result does not look so significant. A new experimental data as well as theoretical exploration of NLO α_s corrections and other uncertainties are desirable to clarify the situation.

Recently, the NRQCD calculation of relativistic corrections to pair J/ψ production cross section was performed [20]. Contrary to our results, relativistic effects were found to be much smaller in that approach. The authors of [20] investigate only one part of relativistic corrections to the production amplitude, so direct comparison of our results with [20] is difficult. Moreover, the choice of numerical value $\langle 0|O^{J/\psi}(^3S_1^{[1]})|0\rangle$ in [20] is at variance with quark model calculations.

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