

Alternative methods for top-quark mass determination at the Tevatron and LHC

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I am summarizing here the determination of the top-quark mass from the CDF and D0 Collaborations at the Tevatron and the ATLAS and CMS Collaborations at the LHC using non-conventional methods. I am concentrating on the extraction of the top-quark mass from the top-antitop cross-section, on the mass measurement using the so-called endpoint method and on the top-quark mass determination from the b -lifetime.

1 Motivations

The top-quark mass is now measured with a remarkable precision around 0.5 % both at the Tevatron and at the LHC using well-developed “standard” methods based on templates, matrix elements or ideogram. Despite this precision, some questions remain. Indeed since the top quark is a color object, it is non trivial to know which mass is really measured using these standard methods. In all standard methods, Monte Carlo (MC) is used to calibrate the measurements. This mass implemented in MC generators is different from a well-defined mass in theory. A way to get some hints about these points experimentally is to determine the top-quark mass using alternative methods. Such methods can use less inputs from MC or can have different sensitivity to systematic uncertainties than the standard analyses. In this article I will concentrate on the extraction of the top-quark mass from the top-antitop ($t\bar{t}$) cross-section, on the mass measurement using the so-called endpoint method and on the top-quark mass determination from the b -lifetime.

Before studying methods which rely differently on MC, it is interesting to look at the dependence of the measured top-quark mass using standard methods with the event kinematics and to compare the data measurements with the predictions from MC. This allows to test the description of the top-quark mass by MC in various phase space regions and to detect potential large deviations due to the pole mass definition problem described above. This has been looked at by the CMS Collaboration [1] in the ℓ +jets final state asking for two b -tag jets using 5 fb^{-1} of LHC at 7 TeV [5]. For these comparisons, the top-antitop final state is fully reconstructed and the top-quark mass is measured using the ideogram technique either solely or together with the jet energy scale. The measurements are compared to Madgraph [2] with different Pythia [3] tunes and to MC@NLO [4]. Differential measurements as a function of several variables have been performed [5] that are sensitive to different physics effects. For instance, the top-quark mass distribution as a function of the opening angles between the two light jets (see Figure 1) or as a function of the pseudo-rapidity (η) of the hadronic decaying top is sensitive to color

reconnection. The influence of initial and final state radiation can be investigated by looking at the top-quark mass as a function of the invariant mass of the $t\bar{t}$ pair or as a function of the transverse momentum (p_T) of the $t\bar{t}$ pair. To test the sensitivity to the b -quark kinematics, the top-quark mass is measured as a function of the transverse momentum or the pseudo-rapidity of the b -jet assigned to the hadronic decaying top quark. The mass distribution as a function of the distance between the b - and \bar{b} -jets ($\Delta R_{b\bar{b}} = \sqrt{\Delta\eta^2 + \Delta\phi^2}$) is also scrutinized (see Figure 1). Even if the statistical error on these differential measurements is still large, there is currently no indication of specific biases due to the choice of generators.

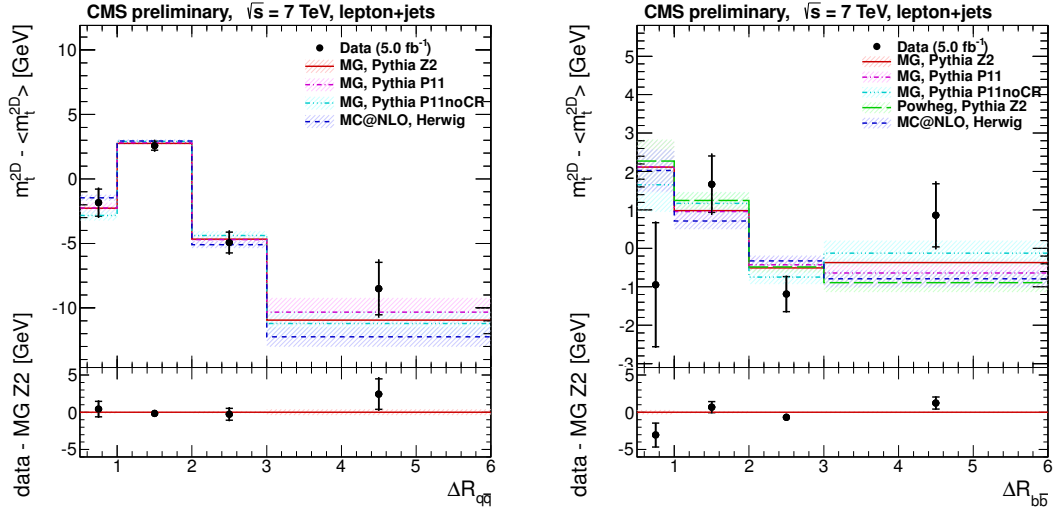


Figure 1: Differential top-quark mass measurements as a function of the separation of the light-quark jets (left) and of the b -quark jets (right) performed by CMS [5] compared to several MC predictions.

2 Mass extraction from the $t\bar{t}$ cross section

The principle for the mass extraction from the $t\bar{t}$ cross section is to compare the experimental measured $t\bar{t}$ cross section with the one computed theoretically. Both the experimental and theoretical cross sections depend on the top-quark mass but the dependence is different in the two cases. In the experimental case, the dependency comes from the acceptance cuts while in the theoretical case, it originates from the matrix element. The advantage of this alternative method lies in the fact that it allows to extract a top-quark mass in a well-defined renormalization scheme (the one that is used in the theory computation) in contrast to the one that is implemented in the MC generators. This method has however the drawback that it is less precise than direct measurements.

This determination of the top-quark mass has been performed by the D0 Collaboration using the $t\bar{t}$ cross section measured in the ℓ +jets channel using b -tagging requirement with 5.4 fb^{-1} . This measured cross section is the one that exhibits the weakest dependence on the top-quark mass. The variation of the measurement as a function of the MC mass (m_t^{MC}) is

parameterized using a third-order polynomial divided by the mass to the fourth power. As theory input cross section, the next-to-leading order (NLO), the NLO including next-to-leading log (NNL) resummation computations and some approximation of the next-to-next-to-leading order (NNLO) calculations. The mass is extracted from the maximum of a normalized likelihood distribution defined as:

$$L(m_t) = \int f_{\text{exp}}(\sigma|m_t) [f_{\text{scale}}(\sigma|m_t) \otimes f_{\text{PDF}}(\sigma|m_t)] d\sigma,$$

where f_{exp} comes from the experimental measurement which uncertainties are assumed to be Gaussian distributed, f_{scale} represents the theoretical scale uncertainty, taken to be flat and f_{PDF} represents the uncertainty of parton density functions (PDF) taken to be a Gaussian function. The mass determination is performed assuming that m_t^{MC} corresponds to the pole mass (m_t^{pole}) and assuming that m_t^{MC} corresponds to the $\overline{\text{MS}}$ mass ($m_t^{\overline{\text{MS}}}$). The experimental and theoretical $t\bar{t}$ cross sections used in the extraction are shown in Figure 2. With this technique, D0 measures the top-quark pole mass shown in Table 1 [6]. These values are compatible but slightly lower than the top-quark mass world average [7]. The $\overline{\text{MS}}$ mass is also extracted [6].

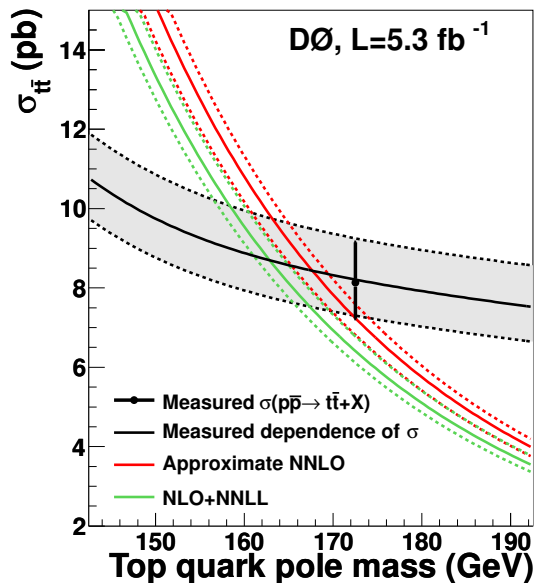


Figure 2: Experimental and theoretical $t\bar{t}$ cross sections used by D0 to extract the top-quark mass [6].

A similar method has been developed by CMS. In that analysis, CMS uses the $t\bar{t}$ cross section measured in the dilepton channel using 2.3 fb^{-1} at 7 TeV as experimental input. This cross section is the most precise one measured by CMS with a total uncertainty of 4.1 %. As for D0, it is parameterized using a third-order polynomial divided by the mass to the fourth power. The full NNLO prediction including next-to-next-to-leading log (NNLL) resummation is employed as theoretical input. The mass is extracted using a probability function similar to

Theoretical prediction	m_t^{pole} (GeV)	Δm_t^{pole} (GeV)
MC mass assumption	$m_t^{\text{MC}} = m_t^{\text{pole}}$	$m_t^{\text{MC}} = m_t^{\text{MS}}$
NLO	$164.8^{+5.7}_{-5.4}$	-3.0
NLO+NLL	$166.5^{+5.5}_{-4.8}$	-2.7
NLO+NNLL	$163.0^{+5.1}_{-4.6}$	-3.3
Approximate NNLO	$167.5^{+5.2}_{-4.7}$	-2.7

Table 1: Values of the pole top-quark mass m_t^{pole} , with their 68% C.L. uncertainties extracted for different theoretical predictions by D0 [6].

the D0 analysis. A 1 GeV addition uncertainty is added to the experimental result to cover the possible difference between m_t^{MC} and m_t^{pole} . CMS also studies the interplay of the mass extraction with the value of the strong coupling constant α_S (see [8] for more details).

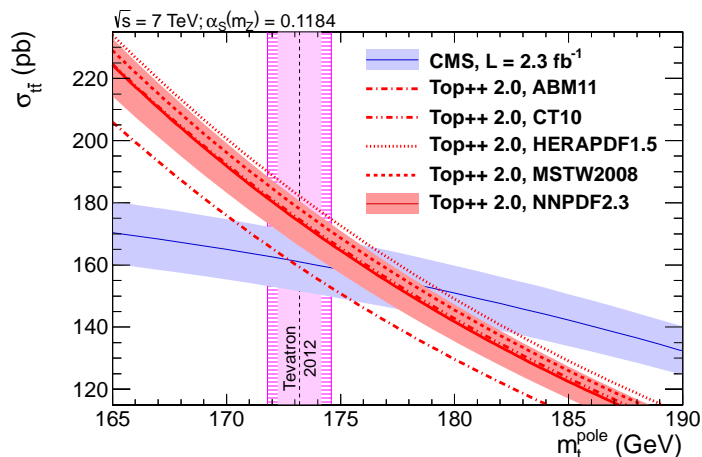


Figure 3: Experimental and theoretical $t\bar{t}$ cross sections used by CMS to extract the top-quark mass [9].

The experimental and theoretical $t\bar{t}$ cross sections used in the extraction are shown in Figure 3. The extracted top-quark pole mass by CMS are shown in Table 2 for different PDFs [9]. These values are compatible but slightly higher than the top-quark mass world average [7]. The same kind of extraction has been also performed by the ATLAS Collaboration [10] using the first 35 pb^{-1} of LHC data leading to a top-quark mass of $m_t^{\text{pole}} = 166^{+7.8}_{-7.3} \text{ GeV}$ [11].

To summarize, the top-quark pole mass has been extracted from the $t\bar{t}$ cross section by D0 leading to a precision of 3 % (where the input experimental cross section has a precision of 12 %, and the input theoretical cross section of 3 %), by ATLAS with a precision of 4.5 % (where the experimental input has an uncertainty of 13 % and the theory input of 5 %) and by CMS with a precision of 2 % (where the experimental input has an uncertainty of 4 % and the theory input of 4 %). Looking at the current theoretical uncertainty on the $t\bar{t}$ cross section and

	Most likely m_t^{pole} value (GeV)	Uncertainty (GeV)		
		Total	From $\delta\alpha_S$	From δE_{LHC}
ABM11	172.7	+3.8 -3.5	+1.0 -1.0	+0.8 -0.8
CT10	177.0	+4.3	+0.8	+0.9
HERAPDF1.5	179.5	+4.3	+1.2	+1.0
MSTW2008	177.9	+3.8 +4.0	-1.1 +0.9	-1.0 +0.9
NNPDF2.3	176.7	+3.6 -3.4	-0.9 -0.7	-0.9 -0.9

Table 2: Results obtained by CMS for m_t^{pole} by comparing the measured $t\bar{t}$ cross section to the NNLO+NNLL prediction with different NNLO PDF sets [9].

assuming no experimental errors, one can estimate the ultimate uncertainty on the top-quark mass achievable with this method to be around 3 GeV (1.7 %).

3 Mass measurement using the endpoint method

The endpoint method employed for the first time by CMS to measure the top-quark mass [12] was originally developed to measure masses of potentially pair produced new particles with two cascade decays each ending in an invisible particle, like neutralino. It is thus also applicable to the $t\bar{t}$ dilepton final state which contains two escaping neutrinos. This method relies on the end distribution of the variable named M_{T2} used as mass estimator. This M_{T2} is a generalization of the usual transverse mass and is defined as:

$$M_{T2} \equiv \min_{\vec{p}_T^a + \vec{p}_T^b = \vec{p}_T} \{ \max(M_T^a, M_T^b) \}.$$

This variable corresponds to the minimum parent mass consistent with the observed kinematics for the hypothetical \vec{p}_T^a and \vec{p}_T^b . To limit the sensitivity to the transverse momentum of the $t\bar{t}$ system ($p_T(t\bar{t})$), the variable $M_{T2\perp}$ is rather used. It is computed with the p_T components perpendicular to $p_T(t\bar{t})$.

Three variables are needed to solve the dilepton event kinematics. The chosen variables are $M_{T2\perp}$ computed at the lepton level ($\mu_{\ell\ell}$) after the W -boson decays, $M_{T2\perp}$ computed at the b -jet level (μ_{bb}), ignoring that leptons are in fact observed and the invariant mass between the b -jet and the lepton (M_{lb}) which is very correlated with $M_{T2\perp}$ constructed with the b -jet+lepton combined.

In the analysis, the physics background is estimated using MC while the background with mistag b -jets is evaluated using antitag events. The combinatoric background is suppressed using a dedicated selection algorithm [12]. The top-quark mass is extracted using a maximum likelihood fit of the endpoint of the three chosen variables taking the object resolution into account. Indeed in the limit of perfect object measurements, the maximum of the $\mu_{\ell\ell}$ distribution is equal to the W -boson mass (assuming zero neutrino mass), the maximum of the μ_{bb} distribution is equal to the top-quark mass while the maximum of M_{lb} can be expressed analytically using the energies and momenta of the daughter of $t \rightarrow Wb$ in the top-quark rest frame. The fitted distributions are shown in Figure 4. Using this technique, CMS measures [12]: $m_t = 173.9 \pm 0.9(\text{stat})_{-2.1}^{+1.7}(\text{syst})$ GeV. The precision of this result is comparable to the one from the standard measurement in the same channel. As can be seen in Table 3, the largest systematic uncertainty comes from the uncertainty on the jet energy scale.

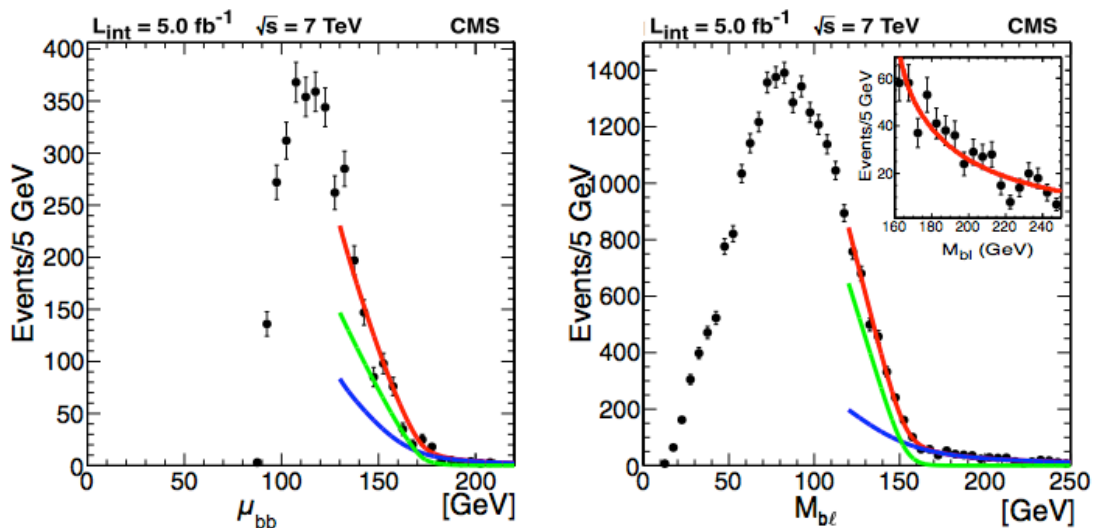


Figure 4: Results for the endpoint fit in CMS where the red line represents the full fit while the green and blue curves are for the signal and background shapes respectively [12].

4 Mass measurement using the B-hadron lifetime

The top-quark mass can also be measured using different observables. For instance the lifetime and decay length of the B-hadrons from the top-quark decay depends almost linearly on the top-quark mass as can be seen in Figure 5. Alternatively the lepton p_T from the decay of the W -boson from the top quark can also be used as a mass estimator. The advantage of such estimators is that they minimally rely on the calorimeter-based uncertainty like the jet energy scale uncertainty. However these methods can potentially be rather sensitive to the modeling of the top production kinematics or to the calibration of the b decay length or the b fragmentation model.

Source	δm_t (GeV)
Jet Energy Scale	+1.3 -1.8
Jet Energy Resolution	± 0.5
Lepton Energy Scale	+0.3 -0.4
Fit Range	± 0.6
Background Shape	± 0.5
Jet and Lepton Efficiencies	+0.1 -0.2
Pileup	< 0.1
QCD effects	± 0.6
Total	+1.7 -2.1

Table 3: Summary of the systematic uncertainties affecting the CMS measurement of the top-quark using the endpoint method [12].

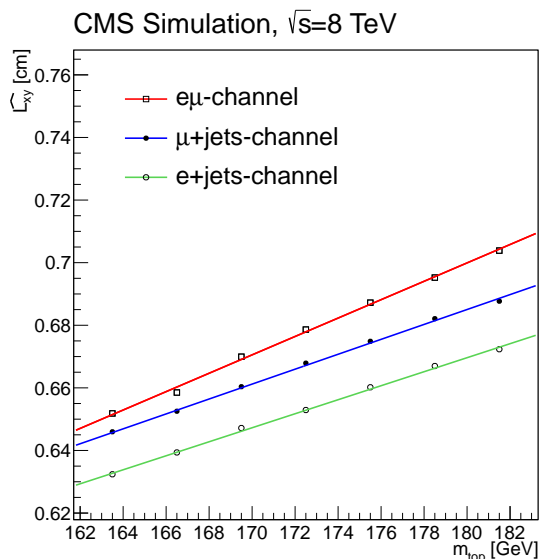


Figure 5: Median of the transverse b decay length distribution between the primary and the secondary vertex as a function of the simulated top-quark mass for three final states studied by CMS [13].

4.1 Measurement using the B-hadron lifetime at CDF

These alternative methods were first developed at CDF in the ℓ +jets channel with at least one b -tagged jet using 1.9 fb^{-1} [14]. The top-quark mass was simultaneously extracted from the B-hadron lifetime and from the lepton p_T . The main difficulty of this analysis appears to be the calibration of the transverse decay length. Indeed corrections for the inaccuracy of the fragmentation simulation in EVTGEN has been necessary as well as corrections for the tracker modeling in the simulation. These corrections are determined using a sample of $b\bar{b}$ events (with 95% purity) as a function the p_T of jets reconstructed only in the tracker. These track-based jets are previously calibrated using γ +jets events. The uncertainty on the calibration of the transverse decay length are the dominant systematic uncertainty on the final result. In the case of the measurement using the lepton p_T , the understanding of the lepton p_T scale is the largest systematic uncertainty. Constructing a combined likelihood shown in Figure 6 with the two observables, CDF measures [14]: $m_t = 170.7 \pm 6.3(\text{stat}) \pm 2.6(\text{syst}) \text{ GeV}$. Details on the systematic uncertainties limiting the measurements are presented in Table 4.

4.2 Measurement using the B-hadron lifetime at CMS

CMS has adapted CDF method using both the ℓ +jets and dilepton final state using 19 fb^{-1} of LHC data at 8 TeV. In this analysis, the chosen observable is the median of the distribution of secondary vertices with maximal transverse decay length (L_{xy}). The calibration for L_{xy} is cross-checked using dijet events with one muon-tagged jet, taken to be the tag jet, while the second jet is taken to be the probe. The distribution of the secondary vertex mass of this

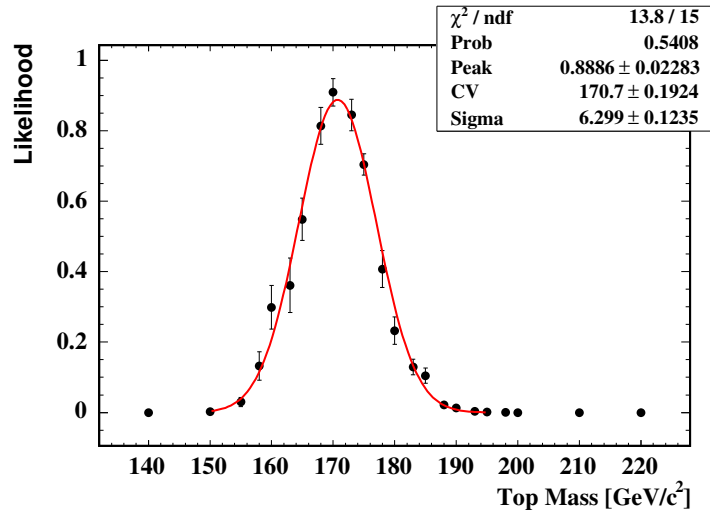


Figure 6: Likelihood constructed from 23 m_t test points using the transverse B-hadron decay length and the lepton p_T in the top-quark decay by CDF [14].

probe jet is then compared with the prediction after fitting the light, c and b -jets fractions. The agreement appears to be good as shown in Figure 7.

The top-quark mass extraction using the median values of L_{xy} after calibration leads to [13]: $m_t = 173.5 \pm 1.5(\text{stat}) \pm 1.3(\text{syst}) \pm 2.6(p_T(t))$ GeV. As can be seen in Table 5, the modeling of the top-quark p_T , which is mass dependent, has a huge influence on the result. A systematic uncertainty based on reweighting the simulation to the unfolded top-quark p_T spectrum from data is assigned. This is currently the limiting uncertainty. In the future, the possibility to use an invariant quantity like the lepton-vertex invariant mass could be studied since it would keep the information on the top-quark mass while being less dependent on the top-quark kinematics.

Systematic [GeV/ c^2]	Lxy	Lepton p_T	Simultaneous
Background Shape	1.0	2.3	1.7
QCD Radiation	0.5	1.2	0.7
PDF	0.3	0.6	0.5
Generator	0.7	0.9	0.3
Lepton p_T Scale	0	2.3	1.2
Lxy Calibration	2.5	0	1.1
Multiple Interactions	0.2	1.2	0.7
Calorimeter JES	0.4	0.4	0.3
Systematics Total	2.9	3.8	2.6

Table 4: Final systematic uncertainties for the transverse B-hadron decay length and the lepton p_T CDF measurement [14].

Source		Δm_t [GeV]		
		μ +jets	e +jets	$e\mu$
Statistical		1.0	1.0	2.0
Experimental	Jet energy scale	0.30 ± 0.01	0.30 ± 0.01	0.30 ± 0.01
	Multijet normalization (ℓ +jets)	0.50 ± 0.01	0.67 ± 0.01	-
	W+jets normalization (ℓ +jets)	1.42 ± 0.01	1.33 ± 0.01	-
	DY normalization ($\ell\ell$)	-	-	0.38 ± 0.06
	Other backgrounds normalization	0.05 ± 0.01	0.05 ± 0.01	0.15 ± 0.07
	W+jets background shapes (ℓ +jets)	0.40 ± 0.01	0.20 ± 0.01	-
	Single top background shapes	0.20 ± 0.01	0.20 ± 0.01	0.30 ± 0.06
	DY background shapes ($\ell\ell$)	-	-	0.04 ± 0.06
Calibration		0.42 ± 0.01	0.50 ± 0.01	0.21 ± 0.01
Theory	Q^2 -scale	0.47 ± 0.13	0.20 ± 0.03	0.11 ± 0.08
	ME-PS matching scale	0.73 ± 0.01	0.87 ± 0.03	0.44 ± 0.08
	PDF	0.26 ± 0.15	0.26 ± 0.15	0.26 ± 0.15
	Hadronization model	0.95 ± 0.13	0.95 ± 0.13	0.67 ± 0.10
	B hadron composition	0.39 ± 0.01	0.39 ± 0.01	0.39 ± 0.01
	B hadron lifetime	0.29 ± 0.18	0.29 ± 0.18	0.29 ± 0.18
	Top quark p_T modeling	3.27 ± 0.48	3.07 ± 0.45	2.36 ± 0.35
	Underlying event	0.27 ± 0.51	0.25 ± 0.48	0.19 ± 0.37
Color reconnection		0.36 ± 0.51	0.34 ± 0.48	0.26 ± 0.37

Table 5: Statistical, experimental and theoretical systematic uncertainties on the measured top-quark mass based on the median of the transverse B-hadron decay length distribution between the primary and the secondary vertex by CMS [13].

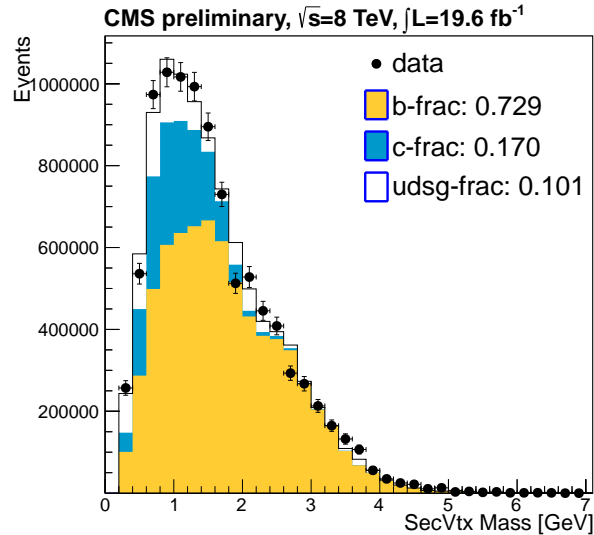


Figure 7: Inclusive fit to the flavor content of a dijet sample based on the secondary vertex mass distribution to check the calibration of L_{xy} in CMS B-hadron lifetime measurement [13].

5 Conclusion

Now that the precision on the direct top-quark mass measurements reaches 1 GeV, alternative methods that are less sensitive to MC (and so less sensitive to the top-quark mass scheme implemented in MC) or with different sensitivity to systematic uncertainties need to be developed. Some of these alternative approaches have been described here. For some of them the achieved precision is still modest. However with plenty of statistics foreseen, the LHC Run 2 will enable to improve them allowing in particular to study the systematic limitation using data.

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