From the LHC to a future lepton collider

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I discuss the complementarity of top quark physics that can be studied at the LHC and at a future lepton collider.

1 Introduction

The goal of this talk is to discuss a synergy between top quark studies at the LHC and at a future lepton collider. I rely here on the results obtained by the Top Quark Working Group [1] in the context of the Snowmass community planning exercise, an important event that occurred in the first half of 2013 in the United States.

I think everybody would agree that a top quark is a "known unknown". Indeed, on one hand, we understand very well how top quarks are embedded into the Standard Model and, as the result, we can precisely describe what to expect in any process that involves top quarks. On the other hand, we believe that top quarks should have unusual features since they appear to couple to the Higgs boson so strongly. As a consequence, it is then logical to assume that top quarks play some role in the resolution of the Naturalness problem and to search for clues to the solutions to this problem in physics of top quarks.

The majority of solutions to the Naturalness problem suggested so far are based on one of three distinct mechanisms that stabilize the Higgs mass or the scale of electroweak symmetry breaking: spin-zero partners of top quarks (e.g. supersymmetric models), spin one-half partners of top quarks (e.g. little Higgs models) and composite models of Higgs bosons and top quarks. When we think about the future of the top quark physics and, in particular, discuss an interplay of lepton and hadron colliders, it is useful to have these three mechanisms in mind.

Top quark studies at hadron colliders (for a review see Refs. [2, 3]) have given us measurements of the top quark mass, the top quark charge, the top quark spin and the top quark couplings to gluons and W-bosons, as well as cross-sections and kinematic distributions in various top quark production channels with reasonable precision. Except for an intriguing problem with the forward-backward asymmetry at the Tevatron [4], results of all existing measurements are consistent with the Standard Model. However, one has to keep in mind that after almost twenty years of top quark discovery, some top quark properties are still poorly known experimentally. A striking example is provided by the top quark couplings to neutral electroweak gauge bosons. As we discuss below, probing these couplings may turn out to be very important; hopefully, first interesting results on $t\bar{t}Z/\gamma$ will be obtained at the LHC and then substantiated at a lepton collider.

It is well-known that hadron and lepton colliders are complimentary. The LHC is a broadband discovery machine with huge rates and large backgrounds where precision measurements are difficult but not impossible. Lepton colliders are precision tools with low backgrounds, small event rates and high luminosity. These are great machines for accurate measurements of top quark properties, especially those that involve electroweak interactions. When we talk about lepton colliders it is important to emphasize that we do not talk about a definite machine at the moment. Indeed, energy, layout and the geographic location of a future lepton collider are not yet fixed so that one can consider different scenarios including the Higgs factory with the center-of-mass energy of 250 GeV, as well as 500 and 1000 GeV machines. It is still being discussed if the next collider should be linear such as the ILC [5] or circular, such as TLEP [6]. For the purposes of this talk, I will consider a generic lepton collider where energy is a continuous parameter and the luminosity is not unreasonably high. The important difference between circular and linear colliders is the beam polarization option that is available at a linear collider. As we will see shortly, access to beam polarization makes significant differences in physics reach in some cases.

Similar to other aspects of collider physics, top quark physics involves measurements and searches. It is obviously important to have a balanced combination of the two. To some extent, the desire to have this balance explains why we always talk about hadron and lepton colliders since, almost by design and certainly in the popular culture, hadron colliders are associated with searches and lepton colliders with precision measurements. It is important to understand, however, that times have changed and that there is a great track record of precision measurements at hadron colliders. Quite often, lepton colliders improve on precision obtained at a hadron collider but they can hardly contribute to searches. It is therefore important to understand in which cases added precision provided by a lepton collider relative to what the LHC can do justifies building a new machine, at least from the perspective of the top quark physics. This is a difficult question but it is central for the discussion of the complementarity of hadron and lepton colliders. To illustrate possible answers to this question, I will to discuss 1) the top quark mass measurements; 2) studies of top quark production at threshold at a lepton collider; 3) top quark couplings to electroweak gauge bosons; 4) top quark flavor-violating decays; 5) physics beyond the Standard Model. I conclude in Section 7.

2 The top quark mass

The discussion of the top quark mass provides a great illustration of all issues related to the complementarity of lepton and hadron colliders. It is well-known that the top quark mass can be measured much more precisely at a lepton collider than at a hadron collider. However, it is important to understand why the increased precision in the top quark mass measurements is relevant. We will consider a few examples to illustrate that.

First, recall that intrinsic consistency of the Standard Model can be checked through precision electroweak fits that express, e.g. m_W through m_t, m_Z, etc. The rule of thumb [7] is that a 6 MeV precision on m_W corresponds to 900 MeV precision on m_t. Pushing only one observable to higher precision does not help much with the interpretation of precision electroweak fits. Currently, we know m_W to about 15 MeV and it is expected that both the LHC and the ILC will probably reduce the error on M_W to 6-7 MeV but not much beyond that [7]. Hence, from the point of view of precision electroweak fits, measurement of the top quark mass to $300-500$ MeV is entirely sufficient, even in the post-LHC era.

Another place where precise knowledge of the top quark mass appears to be necessary is the issue of the vacuum stability in the Standard Model. Given the uncertainty on the top quark

	Ref.[11]	Projections		
CM Energy	7 TeV	14 TeV		
Luminosity	$5fb^{-1}$	$100fb^{-1}$	$300fb^{-1}$	$3000fb^{-1}$
Syst. (GeV)	1.8	1.0	0.7	0.5
Stat. (GeV)	0.90	0.10	0.05	0.02
Total	**2.0**	**1.0**	**0.7**	**0.5**

Table 1: Projections [1, 10] for the uncertainty in m_t determined using the CMS end-point method [11]. Extrapolations are based on the published CMS analysis.

mass of about 500 MeV, one determines the scale where the Higgs self-coupling turns negative to within a factor of five. This is sufficient to understand if we live in an unstable, stable or metastable Universe, so that unless the precise lifetime of the Universe becomes important, the 500 MeV uncertainty on m_t does not prevent us from drawing physics conclusions.

Taking $\delta m_t = 500$ MeV as a reasonable goal, we should ask whether or not the LHC can achieve it. Interestingly, we are already not too far from it. Indeed, the current uncertainty from combined Tevatron measurements is close to 900 MeV [8], while precision of current measurements at the LHC is close to one GeV [9]. One can expect that precision of traditional methods, such as the matrix element method and the template method, can be pushed further to approximately $\delta m_t \sim 600$ MeV by collecting more data. Unfortunately, improvements in precision do not continue past 300 fb^{-1} integrated LHC luminosity, due to increased pileup and related difficulties with the top quark reconstruction [10].

While $\delta m_t \sim 600$ MeV is definitely in the right ballpark, the standard criticism of traditional methods is that, due to biases in parton shower event generators, they provide us with a top mass parameter that is neither the pole mass nor the $\overline{\rm MS}$ mass. Even if we know this parameter to high precision, its deviation from the $m_t^{\rm pole}$ or $m_t^{\overline{\rm MS}}$ can be much larger than the quoted error, leading to a significant confusion. While this might be a valid concern as a matter of principle, in my opinion, the whole issue is not important at the $\mathcal{O}(500)$ MeV level of precision. For example, there should be little doubt that pole quark masses are used in perturbative parts of event generators such as PYTHIA and HERWIG but, of course, one has to be sure that non-perturbative effects are not large for observables from which the top quark mass is extracted.

The good news is that a number of new techniques for measuring the top quark mass, such as the CMS end-point method [11], were recently suggested. These methods appear to be more clean theoretically because it is easier to understand what mass parameter is measured and why potential contamination of top quark pair production by BSM physics does not affect the extracted value of m_t. The new methods also show better behavior when extrapolated to higher luminosities and pileups. As can be seen from Table 1, a precision of about 500 MeV using the end-point method can be reached at 3000 fb^{-1}. It can be expected that after combining a few of such clean measurements with each other, the precision $\delta m_t \sim 300 - 400$ MeV can be reached – that is better than what is required for precision electroweak fits both during the LHC era and beyond.

The need for additional improvements in m_t measurements may be justified if there are significant breakthroughs in m_W measurements that push its precision to a few MeV range *or* if the "vacuum (in)stability scenario", i.e. no new physics all the way to the Planck scale, becomes a leading paradigm in BSM physics. In the latter case, on can argue that the scale μ_Q where the Higgs quartic coupling turns negative is related to underlying fundamental physics, making precise determination of μ_Q an important task. In the SM, μ_Q depends on m_H and m_t. The Higgs mass can be measured to about 100 MeV at the LHC. The error on the scale where the quartic coupling turns negative is equally shared between δm_t and δm_H [12]. Therefore, from this perspective, measuring m_t to 100 MeV precision is valuable.

Measuring m_t to 100 MeV can only be done at a lepton collider [13], either from the threshold scan or from the measurement of the invariant mass distribution of top quark decay products at higher energies. In case of the threshold mass extraction, the error on m_t is very small, close to 40 MeV, but it gets amplified by almost a factor of two because the transition from the pole mass to the $\overline{\mathrm{MS}}$ mass is not known sufficiently well. This, however, is a theoretical issue which will be definitely worked out if the need arises.

3 Top quark threshold production at a lepton collider

One of the attractive points of a lepton collider is that it can study production of top quarks at threshold, with $E_{\mathrm{CM}} \sim 2m_t$. Independent of any utilitarian goal, top quark threshold production gives us an opportunity to a study beautiful and unique physics system of almost free but nevertheless strongly interacting quarks. Indeed, the lifetime of a top quark is short enough to prevent it from hadronizing into t-hadrons, yet it is long enough to allow produced t and $\bar{t}$ to get sufficiently far from each other to experience long-range Coulomb-like QCD interaction. These Coulomb-like interactions are enhanced by the inverse relative velocity of top quarks and, at threshold, require a resummation to all-orders in perturbation theory. The top quark width plays an important conceptual role by screening the non-perturbative effects and providing an opportunity to compute the entire line-shape for threshold top quark production. Let me also note that an accurate approximation for the cross-section has been worked out theoretically [14] and an even better approximation is in the works [15].

What are the interesting measurements that one can do at the top threshold? To understand this, let us imagine that we measure the top production cross-section as a function of the center-of-mass energy, and the top quark momentum distribution in the threshold region. Both of these quantities are affected by a number of things including the top quark mass, the top quark width, the strong coupling constant at relatively low energy scales and even the top-Higgs Yukawa coupling. Going back to measurements of interesting quantities and focusing on threshold line-shape and momentum distributions, it is easy to see that the cross-section at the peak, the position of the peak and the average momentum of produced top quarks are determined by three interesting parameters – the top quark mass, the top quark width and the strong coupling constant [16]

$$\sigma_{\mathrm{peak}} \sim \frac{\alpha_s^3}{m_t \Gamma_t}, \quad E_0 \sim -m_t \alpha_s^2, \quad \langle p_t \rangle \sim m_t \alpha_s. \tag{1}$$

Hence, by measuring the three quantities experimentally, we obtain the mass, the width and the strong coupling constant and, as was shown in Ref. [16], this can be done to very high precision. I want to emphasize, in particular, that a few percent precision with which the top quark width can be measured at a lepton collider is about fifty times higher than what one can do at the LHC. The importance of measuring the width with high precision is discussed at the end of this talk.

4 Couplings of top quarks to electroweak bosons

Couplings of top quarks to W and Z bosons and photons are well-known in the Standard Model, because gauge charges of left-handed and right-handed top quarks are fixed and the Standard Model interactions are restricted to a renormalizable subset. Of course, in a more general case that also includes radiative corrections in the Standard Model, these restrictions do not apply and one can write more general couplings that involve non-renormalizable interactions and arbitrary mixtures of left- and right-handed currents. The simplest extension of the tbW interaction is an addition of the right-handed current and the dipole dimension-five operator

$$\mathcal{L} = -\frac{g}{\sqrt{2}}\bar{b}\gamma^{\mu}\left(V_L P_L + V_R P_R\right) t W^{-}_{\mu} - g\bar{b}\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_W}\left[g_L P_L + g_R P_R\right] t W^{-}_{\mu} + \text{h.c.} \tag{2}$$

The strength of the left-handed current V_L is arbitrary; in the Standard Model, it is related to the CKM matrix element V_{tb}. The strength of the left-handed current is measured in the t-channel single top production. Current LHC measurements [17, 18, 19, 20] give $V_L \approx 1.13 \pm 0.13$, in agreement with the Standard Model at the ten percent level. One can expect that measurements of single top production cross-section at 3000 fb^{-1} LHC will provide a three percent measurement of $V_{\rm L}$.

The Lorentz structure of anomalous tWb coupling can be studied in top quark decays. There, a useful observable is the distribution of a relative angle between electron and W-boson momentum in the top quark rest frame. This distribution reads

$$\frac{{\rm d}\Gamma_t}{\Gamma_t {\rm d}\cos\theta} = \frac{3}{4}\sin^2\theta F_0 + \frac{3}{8}(1+\cos\theta)^2 F_R + \frac{3}{8}(1-\cos\theta)^2 F_L. \tag{3}$$

Coefficients F_0 and $F_{\pm}$ correspond to relative probabilities for the top quark to decay to a W-boson with longitudinal, left-handed and right-handed polarizations. Currently, theoretical [21, 22, 23] and experimental [24, 25, 26] results are in agreement, but theory predictions are about one order of magnitude more precise. One can also use these measurements to constrain g_L, g_R and V_R in Eq.(2) to be smaller than 0.1.

We have described how V_L can be obtained from the measurement of a single-top production cross-section. However, one can also measure V_L assuming that the top quark can only decay to the Wb final state and that the CKM matrix is unitary. If so, then the ratio of the number of b-jets and light-quark jets from top decays is fixed in terms of V_{tb} and, if this ratio is measured, V_{tb} can be extracted. In fact, this was recently done by the CMS collaboration [27]. The result is the measurement of V_{tb} with the precision of just two percent. It is interesting to note that if the top quark width at a lepton collider will indeed be measured to two percent precision, it will be immediately possible to put strong limits on invisible decays of top quarks or to check the consistency of the above-mentioned CMS measurement. Indeed, by measuring the top width and assuming that it is saturated by $t \to Wb$ decays, we can turn a 2% width measurement into a 1% measurement of V_{tb} which is very competitive with V_{tb} determinations from single-top production. Furthermore, given the possibility to fully reconstruct top quarks at a lepton collider in a low-background environment, it should be possible to explore anomalous couplings in the tWb vertex from angular distributions of top and W decay products with a much higher

Collider	LHC		ILC/CLIC
CM Energy [TeV]	14	14	0.5
Luminosity [fb^{-1}]	300	3000	500
SM Couplings			
photon, F_{1V}^{γ} (0.666)	0.042	0.014	0.002
Z boson, F_{1V}^{Z} (0.24)	0.50	0.17	0.003
Z boson, F_{1A}^{Z} (0.6)	0.058	–	0.005
Non-SM couplings			
photon, F_{1A}^{γ}	0.05	–	–
photon, F_{2V}^{γ}	0.037	0.025	0.003
photon, F_{2A}^{γ}	0.017	0.011	0.007
Z boson, F_{2V}^{Z}	0.25	0.17	0.006
Z boson, ReF_{2A}^{Z}	0.35	0.25	0.008
Z boson, ImF_{2A}^{Z}	0.035	0.025	0.015

Table 2: Table from Ref. [1]. Expected precision of the $t\bar{t}\gamma$ and $t\bar{t}Z$ coupling measurements at the LHC [31, 32] and the linear collider [33]. Expected magnitude of such couplings in the SM is shown in brackets. Note that the "non-standard model" couplings appear in the Standard Model through radiative corrections; their expected magnitude, therefore, is 10^{-2}.

precision than what is achievable at the LHC. This may be particularly important for studying possible CP-violation in top decays.

The situation becomes different if we consider interactions of electrically neutral electroweak gauge bosons with top quarks. The generic interaction vertex of top quarks with photons or Z bosons is parametrized as

$$\Gamma_\mu = ie\left\{ -\gamma_\mu \left(F_{1V} + F_{2V} + \gamma_5 F_{1A}\right) + \frac{(p-p')_\mu}{2m_t}\left(F_{2V} - i\gamma_5 F_{2A}\right)\right\}, \tag{4}$$

where form factors F_i are functions of the momentum transfer q^2. If the mass scale of BSM physics is high, the q^2-dependence can be neglected. Current experimental knowledge of $t\bar{t}Z$ and $t\bar{t}\gamma$ couplings is poor. There is a measurement of the top quark charge from the sum of charges of jets that originate from top decay [28], and evidence that $t\bar{t}Z$ and $t\bar{t}\gamma$ couplings are non-vanishing since cross-sections for $t\bar{t}Z$ and $t\bar{t}\gamma$ production in proton collisions are different from zero [29, 30].

Cross sections for producing top quarks in association with electroweak gauge bosons increase dramatically once higher energies become available. This leads to higher statistics and to the possibility to measure top quark couplings much more precisely, as can be seen from projections in Table 2. For both the photon and the Z-boson, couplings that do not vanish at tree level in the SM can be measured to $10 - 50$ percent. In fact, by measuring these couplings to ten percent we start probing an interesting region of parameter space since there are examples of physics beyond the Standard Model that lead to this level of deviations. In particular, $\mathcal{O}(10\%)$ deviations in $t\bar{t}Z$ are a smoking gun of composite or extra-dimensional models [34], while much smaller changes in $t\bar{t}Z$ vertex are expected in weakly-interacting extensions of the SM, e.g. the supersymmetry.

Collider	LHC		ILC	ILC	CLIC
CM Energy [TeV]	14	14	0.5	1.0	1.4
Luminosity [fb^{-1}]	300	3000	1000	1000	1500
Top Yukawa coupling κ_t	$(14-15)\%$	$(7-10)\%$	10%	4%	4%

Table 3: Table from Ref.[1]. Expected precision of the top quark Yukawa coupling measurement expected at the LHC and the linear collider [35]. The range for the LHC precision corresponds to an optimistic scenario where systematic uncertainties are scaled by a factor 0.5 and a conservative scenario where systematic uncertainties remain at the 2013 level [36, 37, 38]. The ILC [33, 39] and CLIC [40] projections assume polarized beams and nominal integrated luminosities.

Measurements of $t\bar{t}Z$ and $t\bar{t}\gamma$ couplings at a lepton collider will lead to extremely precise results that may be of interest to either explore deviations observed at the LHC or to search for even smaller indirect evidence for physics beyond the Standard Model. Since the precision of the couplings measurement improves by more than one order of magnitude at a lepton collider relative to what can be achieved at the LHC, the sensitivity to energy scales of BSM physics increases by almost a factor three. It should be also emphasized that new opportunities at a linear collider arise thanks to the beam polarization that allows us to disentangle different (c.f. Eq.(4)) anomalous couplings in $t\bar{t}Z$ and $t\bar{t}\gamma$ vertices.

Measurements of the top-Higgs Yukawa coupling are exceptionally important. This coupling holds clues to the Naturalness problem and it is the main ingredient in the discussion of the vacuum stability. A smaller top Yukawa coupling will ameliorate many concerns related to the above-mentioned problems while a larger Yukawa coupling will amplify them. Knowledge of the top Yukawa coupling allows us to predict the strength of $H\gamma\gamma$ and Hgg interactions in the Standard Model, so that by comparing our expectations to direct measurements of $H\gamma\gamma$ and Hgg rates, we can probe for additional contributions to these interactions.

The Higgs-top Yukawa coupling can be measured at the LHC in the process $pp \to t\bar{t}H$ by considering rare decays of the Higgs boson. In Table 3 we summarize prospects for measuring the top Yukawa coupling at the LHC; it follows from that table that one can probably reach the 10% precision on the top-Higgs Yukawa coupling at the high-luminosity LHC. At a lepton collider, the situation with measuring the top Higgs Yukawa coupling depends on the energy of the machine. This is so, because the primary process for measuring top Yukawa is $e^+e^- \to t\bar{t}H$ and it has a kinematic threshold at $2m_t + m_H \approx 470$ GeV, which is rather high. The maximum of $e^+e^- \to t\bar{t}H$ cross-section occurs at above 500 GeV; if it can be reached, the top Yukawa coupling can be measured to about 4 percent precision.

It is also possible that the top Yukawa coupling is of a more general type than what exists in the Standard Model. The most general $Ht\bar{t}$ coupling is $H\bar{\psi}\left(a + ib\gamma_5\right)\psi$. A non-vanishing b implies CP-violation. The possibility to study CP violation in top quark interactions with the Higgs boson at the ILC was discussed in Ref. [41] where further details can be found.

Process	Br Limit	Search	Dataset	Reference
$t \to Zq$	2.2×10^{-4}	ATLAS $t\bar{t} \to Wb + Zq \to \ell\nu b + \ell\ell q$	300 fb^{-1}, 14 TeV	[43]
$t \to Zq$	7×10^{-5}	ATLAS $t\bar{t} \to Wb + Zq \to \ell\nu b + \ell\ell q$	3000 fb^{-1}, 14 TeV	[43]
$t \to Zq$	$5\,(2) \times 10^{-4}$	ILC single top, γ_μ ($\sigma_{\mu\nu}$)	500 fb^{-1}, 250 GeV	Extrap.
$t \to Zq$	$1.5\,(1.1) \times 10^{-4\,(-5)}$	ILC single top, γ_μ ($\sigma_{\mu\nu}$)	500 fb^{-1}, 500 GeV	[44]
$t \to Zq$	$1.6\,(1.7) \times 10^{-3}$	ILC $t\bar{t}$, γ_μ ($\sigma_{\mu\nu}$)	500 fb^{-1}, 500 GeV	[44]
$t \to \gamma q$	8×10^{-5}	ATLAS $t\bar{t} \to Wb + \gamma q$	300 fb^{-1}, 14 TeV	[43]
$t \to \gamma q$	2.5×10^{-5}	ATLAS $t\bar{t} \to Wb + \gamma q$	3000 fb^{-1}, 14 TeV	[43]
$t \to \gamma q$	6×10^{-5}	ILC single top	500 fb^{-1}, 250 GeV	Extrap.
$t \to \gamma q$	6.4×10^{-6}	ILC single top	500 fb^{-1}, 500 GeV	[44]
$t \to \gamma q$	1.0×10^{-4}	ILC $t\bar{t}$	500 fb^{-1}, 500 GeV	[44]
$t \to gu$	4×10^{-6}	ATLAS $qg \to t \to Wb$	300 fb^{-1}, 14 TeV	Extrap.
$t \to gu$	1×10^{-6}	ATLAS $qg \to t \to Wb$	3000 fb^{-1}, 14 TeV	Extrap.
$t \to gc$	1×10^{-5}	ATLAS $qg \to t \to Wb$	300 fb^{-1}, 14 TeV	Extrap.
$t \to gc$	4×10^{-6}	ATLAS $qg \to t \to Wb$	3000 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	2×10^{-3}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \ell\ell q X$	300 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	5×10^{-4}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \ell\ell q X$	3000 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	5×10^{-4}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \gamma\gamma q$	300 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	2×10^{-4}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \gamma\gamma q$	3000 fb^{-1}, 14 TeV	Extrap.

Table 4: Projected limits [1, 42] on top FCNC at the LHC and ILC. “Extrap.” denotes estimates based on extrapolation as described in the text. For the ILC/CLIC, limits for various tensor couplings are shown in brackets.

5 Flavor changing decays of top quarks

Flavor-changing decays of top quarks $t \to u\gamma$, $t \to cZ$, $t \to Hu$ etc. are allowed in the Standard Model, but have tiny branching fractions thanks to CKM and loop suppression. Given the tiny SM contributions it is natural to expect that if there are other contributions to these decays they may become clearly observable. The branching fractions that can be expected in various extensions of the Standard Model are between 10^{-4} and 10^{-10}; further details can be found in Refs. [1, 42]. Current experimental measurements are becoming sensitive to top quark flavor-violating branching fractions as small as 10^{-3}, which means that the next step in the exploration of flavor-changing top decays may become very interesting.

Expectations for further improvements in measuring flavor-violating decays of top quarks are shown in Table 4. The entries in that table are collected from various studies, referenced there, or are obtained by extrapolation. It follows from that table that high-luminosity LHC will be able to probe top flavor-changing branching fractions down to the $10^{-4} - 10^{-5}$ level which is a very interesting range. Except for a few cases, the ILC will probably not be able to do significantly better. A truly new element that the ILC will provide is related to the possibility to use polarization information to disentangle decays facilitated either by current operators $q\gamma_\mu tZ^\mu$ or dipole operators $q\sigma_{\mu\nu}tZ^{\mu\nu}$. Such a separation does not appear possible at the LHC although it will be important for understanding the flavor-changing interactions of top quarks if a signal is discovered. Moreover, it is interesting to point out that one can study top quark flavor-violating processes at a 250 GeV machine by searching for single top production $e^+e^- \to Z/\gamma \to t\bar{q}$ that occurs through flavor-violating tqZ and $tq\gamma$ vertices The reach appears to be quite comparable to what can be expected at the LHC and at a higher-energy lepton collider, see Table 4.

6 Searches for physics beyond the Standard Model

We will now turn to the question of what a lepton collider can do to search for physics beyond the Standard Model related to top quarks. Admittedly, given bounds on such physics that have already been provided by the LHC, one should not expect many BSM discoveries from a lepton collider. Indeed, the existing limits are in the range of 1 TeV for generic stops, fermionic partners of top quarks, resonances that decay to $t\bar{t}$ pairs and so on. Clearly, all these particles are way too heavy to be produced at a lepton collider whose energy is below a TeV.

However, there are cases when generic mass bounds mentioned above do not apply. Such cases are typically connected with complicated mass patterns of BSM particles which significantly change the experimental acceptances assumed for setting direct limits. To give an example, suppose that a dominant decay mode of a stop is $\tilde{t} \to t + \chi_0$ and that masses of tops and stops are close, $m_{\tilde{t}} \approx m_t \gg m_\chi$. Then the stop signal is difficult to pick up over the $t\bar{t}$ background since the neutralino in the final state is soft and does not produce additional missing energy. One can use other observables – for example spin correlations – to disentangle tops from stops [45] but these techniques are difficult and not yet fully established. So, it is conceivable that regions where stops are relatively light will remain undiscovered at the LHC. However, a linear collider with sufficient energy can easily discover these particles and perform a detailed analysis of their properties through the threshold scan.

7 Conclusions

According to conventional wisdom, hadron and lepton colliders are complimentary: hadron colliders are designed for discoveries and lepton colliders – for precision measurements. While this premise is correct in general, it may not accurately reflect the actual situation now since the Tevatron and LHC experiments have demonstrated that impressive precision measurements can be performed at hadron colliders. It is expected that such measurements will successfully continue at a higher-energy and higher-luminosity LHC and that, in many cases, they will be probing interesting and important regions of parameter spaces which will further benefit our understanding of the top quark physics.

Significant improvements in precision at a lepton collider are possible in measurements of the top quark mass, the top quark width, the top quark couplings to electroweak gauge bosons and the Higgs boson, and in top quark flavor-violating decays. There are scenarios – in particular models of composite tops and Higgs – where such precision measurements will be crucial for understanding physics beyond the Standard Model. Further discussions of these issues can be found in Ref.[1]. In general, expected improvements in our understanding in top quark physics, Higgs physics and precision electroweak physics leave little doubt that a lepton collider with a reach that includes the top quark threshold and *beyond* is a fantastic tool for future research in particle physics.

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