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I will discuss the significance of precise knowledge of the top quark and Higgs boson masses for physics beyond the Standard Model and cosmology.

1 Introduction

After the discovery of the Higgs boson at the LHC by ATLAS [1] and CMS [2], the last missing particle of the Standard Model (SM) has been found. At present, the message from the LHC can be formulated as follows: the SM is a self-consistent, weakly coupled effective field theory all the way up to the Planck scale. First, no significant deviations from the SM predictions are seen and no convincing signal in favour of existence of new physics beyond the SM is observed. Second, the mass of the Higgs boson M_H is smaller than $M_H^{\text{max}} = 175$ GeV. If this were not the case, the Landau pole in the Higgs scalar self-coupling would be below the Planck quantum gravity scale $M_P = 2.44 \times 10^{18}$ GeV (see, e.g. [3]), calling for an extension of the SM at some energies between Fermi and Planck scales. Finally, the mass of the Higgs is sufficiently large, $M_H > 111$ GeV, meaning that our vacuum is stable or metastable with a lifetime greatly exceeding the Universe age [4]. The schematic behaviour of the Higgs boson self-coupling λ as the function of energy and the lifetime of the Universe as a function of the Higgs boson and top quark masses are shown in Fig. 1.

At the same time, the mass of the Higgs boson, found experimentally, $(M_H = 125.5 \pm 0.2_{stat} \stackrel{+0.5}{-0.6_{syst}}$ GeV, ATLAS [1] $M_H = 125.7 \pm 0.3_{stat} \pm 0.3_{syst}$ GeV, CMS [2]) is very close to the "critical Higgs mass" M_{crit} , which appeared in the literature well before the Higgs discovery in different contexts. The value of M_{crit} is the stability bound on the Higgs mass $M_H > M_{crit}$, see Fig. 2 (the "multiple point principle", put forward in [5], leads to prediction $M_H = M_{crit}$), to the lower bound on the Higgs mass coming from requirement of the Higgs inflation [6, 7], and to the prediction of the Higgs mass coming from asymptotic safety scenario for the SM [8]. The value of M_{crit} depends strongly on the mass of the top quark, calling for its precise measurement.

In this talk, based on the paper we written together with Fedor Bezrukov, Mikhail Kalmykov and Bernd A. Kniehl [9] and on the contribution to Proceedings of the European Physical Society Conference on High Energy Physics (2013) [10], I will discuss the significance of the top quark and Higgs boson masses for physics beyond the Standard Model and cosmology. The paper is organised as follows. In Section 2 we will discuss the absolute stability bound on the Higgs mass, Section 3 provides a short overview of the asymptotic safety scenario for the Standard Model, in Section 4 we will discuss the amazing relationship between the Planck



Figure 1: Left panel: Different patterns of the behaviour of the Higgs self coupling with energy. For $M_H > M_H^{\text{max}}$ the Landau pole appears at energies below the Planck scale. If $M_H < M_{\text{crit}}$ the scalar constant becomes negative at energies below the Planck mass, and electroweak vacuum becomes metastable. Right panel (courtesy of F. Bezrukov): The lifetime of the electroweak vacuum as a function of top quark and Higgs boson masses. Ellipses correspond to 1 and 2 σ contours in M_H and m_t , t_U is the age of the Universe. Along the straight lines the lifetime of the vacuum is given by the number in the plot. The light green region in the upper left corner corresponds to the stable vacuum.



Figure 2: The form of the effective potential for the Higgs field ϕ which corresponds to the stable (left), critical (middle) and metastable (right) electroweak vacuum. The form of the effective potential is tightly related to the energy dependence of the Higgs self-coupling constant $\lambda(\mu)$: the potential is negative almost in the same domain where $\lambda(\phi) < 0$.

and Fermi scales, in Section 5 we discuss a lower bound on the Higgs mass coming from Higgs inflation, and in Section 6 we conclude.

2 Top and Higgs: absolute stability bound

To find the numerical value of $M_{\rm crit}$, one should compute the effective potential for the Higgs field $V(\phi)$ and determine the parameters at which it has two degenerate minima:

$$V(\phi_{SM}) = V(\phi_1), \quad V'(\phi_{SM}) = V'(\phi_1) = 0,$$
(1)

The renormalisation group improved potential has the form

$$V(\phi) \propto \lambda(\phi)\phi^4 \left[1 + O\left(\frac{\alpha}{4\pi}\log(M_i/M_j)\right)\right],$$
 (2)

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Figure 3: A very small change in the top quark mass converts the monotonic behaviour of the effective potential for the Higgs field to that with an extra minimum at large values of the Higgs field. Horizontal axis: ϕ in GeV; vertical axis : $V(\phi)$ in GeV⁴

where α is the common name for the SM coupling constants, and M_i are the masses of different particles in the background of the Higgs field. So, instead of computing the effective potential, one can solve the "criticality equations":

$$\lambda(\mu_0) = 0, \qquad \beta_\lambda^{\rm SM}(\mu_0) = 0 \ . \tag{3}$$

This simplified procedure works with accuracy $\simeq 0.15\,{\rm GeV}$ for the masses of the Higgs and of the top.

The contribution of the top quark to the effective potential is very important, as it has the largest Yukawa coupling to the Higgs boson. Moreover, it comes with the minus sign and is responsible for appearance of the extra minimum of the effective potential at large values of the Higgs field, see Fig. 3.

The most recent result for $M_{\rm crit}$ is convenient to write in the form¹

$$M_{\rm crit} = \left[129.3 + \frac{y_t(\mu_t) - 0.9361}{0.0058} \times 2.0 - \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.5\right] \,\text{GeV} \ . \tag{4}$$

Here $y_t(\mu_t)$ is the top Yukawa coupling in the $\overline{\text{MS}}$ renormalisation scheme taken at $\mu_t = 173.2$ GeV, and $\alpha_s(M_Z)$ is the QCD coupling at the Z-boson mass. The computation consists of matching of $\overline{\text{MS}}$ parameters of the SM to the physical parameters such as the masses of different particles and the renormalisation group running of coupling constants to high energy scale,

¹Note that this form is different from the original works, as well as the uniform estimates of the theoretical errors, which are the sole responsibility of the speaker.

supplemented by the computation of the effective potential for the Higgs field. All recent works [9, 11, 12] used 3-loop running of the coupling constants found in [13]-[18]; Ref. [9] accounted for $\mathcal{O}(\alpha \alpha_s)$ corrections to the matching procedure, getting 129.4 GeV for the central value of $M_{\rm crit}$ with the theoretical error 1.0 GeV, Ref. [11] got 129.6 GeV with smaller error 0.7 GeV, accounting for $\mathcal{O}(\alpha \alpha_s, y_t^2 \alpha_s, \lambda^2, \lambda \alpha_s)$ terms in the matching, while the complete analysis of 2-loop corrections in [12] gives 129.3 GeV for the central value with very small theoretical error 0.07 GeV.

At present, we do not know whether our vacuum is stable or metastable. Fig. 4 shows the behaviour of the scalar self-coupling within experimental and theoretical uncertainties, together with confronting the value of $M_{\rm crit}$ from Eq. (4) with the data. For making these plots, the pole top mass was taken from the Tevatron [19], $m_t = 173.2 \pm 0.51_{stat} \pm 0.71_{sys}$ GeV (the combined ATLAS and CMS value is $m_t = 173.4 \pm 0.4 \pm 0.9$ GeV [20]), and the value of $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [21].

To determine the relation between $M_{\rm crit}$ and M_H , the precision measurements of m_H, y_t and α_s are needed. The main uncertainty is in the value of the top Yukawa coupling, y_t . In general, an x GeV experimental error in m_t leads to $\simeq 2 \times x$ GeV error in $M_{\rm crit}$. The difficulties in extraction of y_t from experiments at the LHC or Tevatron are discussed in [22]. Here we just mention that the non-perturbative QCD effects, $\delta m_t \simeq \pm \Lambda_{QCD} \simeq \pm 300$ MeV lead to $\delta M_{\rm crit} \simeq \pm 0.6$ GeV. The similar in amplitude effect comes from (unknown) $\mathcal{O}(\alpha_s^4)$ corrections to the relation between the pole and $\overline{\rm MS}$ top quark masses. According to [23], this correction can be as large as $\delta y_t/y_t \simeq -750(\alpha_s/\pi)^4 \simeq -0.0015$, leading to $\delta M_{\rm crit} \simeq -0.5$ GeV.

What do the (meta) stability of our vacuum and the agreement of the Standard Model with the LHC experiments mean for cosmology? We can consider two different possibilities.

(i) The Higgs mass is smaller than $M_{\rm crit}$, so that the scalar self coupling crosses zero at energy scale $M_{\lambda} \ll M_P$, where M_{λ} can be as "small" as 10⁸ GeV, within the experimental and theoretical error-bars, see Fig. 4.

(ii) The Higgs mass is larger or equal to $M_{\rm crit}$, and the Higgs self coupling never crosses zero (or does so close to the Planck scale, where gravity effects must be taken into account), see Fig. 4.

If (i) is realised, there are at least two ways to deal with the metastability of our vacuum. The first one is cosmological: it is sufficient that the Universe after inflation finds itself in our vacuum with reheating temperature below M_{λ} . Then this guarantees that we will stay in it for a very long time. This happens, for example, in R^2 inflation [24]. The other possibility is related to possible existence of new physics at M_{λ} scale, which makes our vacuum unique (see, e.g. [25]).

If (ii) is realised, then no new physics is needed between the Fermi and Planck scales.

It is very interesting that the values of M_t and M_H are amazingly close to the critical values, determined from (3). Though this could be a pure coincidence, the discussion below indicates that this may be a very important message about the structure of high energy theory.

3 Top and Higgs: asymptotically safe SM+gravity

The asymptotic safety of the SM [8], associated with the asymptotic safety of gravity [26], is strongly related to the value of the Higgs boson and top quark masses. Though General Relativity is non-renormalizable by perturbative methods, it may exist as a field theory non-perturbatively, exhibiting a non-trivial ultraviolet fixed point (for a review see [27]). If true, all



Figure 4: Both panels: The shaded regions account for 1 and 2 σ experimental uncertainties in α_s and the pole top quark mass m_t , and theoretical errors in extraction of y_t from experiment. Left panel: Running of the scalar self coupling constant with energy. Dashed and dotted lines correspond to varying in addition the mass of the Higgs boson within 1 and 2 σ experimental errors. Right panel: The blue line gives the relation between α_s and the pole top mass following from Eq. (4) if M_H is identified with $M_{\rm crit}$. The shaded regions around it correspond to 1 and 2 σ experimental errors in the Higgs mass. Red ellipses correspond to the accuracy achievable at e^+e^- collider [22].

other couplings of the SM (including the Higgs self-interaction) should exhibit an asymptotically safe behavior with the gravity contribution to the renormalisation group running included.

The prediction of the Higgs boson mass from the requirement of asymptotic safety of the SM is found as follows [8]. Consider the SM running of the coupling constants and add to the β -functions extra terms coming from gravity, deriving their structure from dimensional analysis:

$$\beta_h^{\text{grav}} = \frac{a_h}{8\pi} \frac{\mu^2}{M_P^2(\mu)} h,\tag{5}$$

where a_1 , a_2 , a_3 , a_y , and a_λ are some constants (anomalous dimensions) corresponding to the gauge couplings of the SM g, g', g_s , the top Yukawa coupling y_t , and the Higgs self-coupling λ . In addition,

$$M_P^2(\mu) \simeq M_P^2 + 2\xi_0 \mu^2$$
 (6)

is the running Planck mass with $\xi_0 \approx 0.024$ following from numerical solutions of functional RG equations [28, 29, 30]. Now, require that the solution for all coupling constants is finite for all μ and that λ is always positive. The SM can only be asymptotically safe if a_1, a_2, a_3, a_y are all negative, leading to asymptotically safe behavior of the gauge and Yukawa couplings. For $a_{\lambda} < 0$ we are getting the interval of admissible Higgs boson masses, $M_{\min}^{\text{safety}} < M_H < M_{\max}^{\text{safety}} \simeq 175$ GeV. However, if $a_{\lambda} > 0$, as follows from computations of [29, 30], only one value of the Higgs boson mass $M_H = M_{\min}^{\text{safety}}$ leads to asymptotically safe behavior of λ . As is explained in [8], this behavior is only possible provided $\lambda(M_P) \approx 0$ and $\beta_{\lambda}(\lambda(M_P)) \approx 0$. And, due to miraculous coincidence of μ_0 and M_P , the difference $\Delta m^{\text{safety}} \equiv M_{\min}^{\text{safety}} - M_{\min}$ is extremely small, of the order 0.1 GeV. The evolution of the Higgs self-coupling for the case of $a_h < 0$ is shown in Fig. 5, and for the case $a_h > 0$ in Fig. 6.

In fact, in the discussion of the asymptotic safety of the SM one can consider a more general situation, replacing the Planck mass in Eq. (6) by some cutoff scale $\Lambda = \kappa M_P$. Indeed, if the Higgs field has non-minimal coupling with gravity (see below), the behavior of the SM coupling may start to change at energies smaller than M_P by a factor $1/\xi$, leading to an expectation for



Figure 5: Schematic depiction of the behavior of the scalar self-coupling if $a_h < 0$ for $M_{\min}^{\text{safety}} < M_H < M_{\max}^{\text{safety}}$ (left) and $M_H < M_{\min}^{\text{safety}}$ (right). In both cases gravity leads to asymptotically free behavior of the scalar self-coupling. Negative λ lead to instability and are thus excluded.

the range of κ as $1/\xi \lesssim \kappa \lesssim 1$. Still, the difference between M_{\min} and M_{\min}^{safety} remains small even for $\kappa \sim 10^{-4}$, where $M_{\min}^{\text{safety}} \simeq 128.4 \,\text{GeV}$, making the prediction $M_H \simeq M_{\min}$ sufficiently stable against specific details of Planck physics within the asymptotic safety scenario.



Figure 6: Schematic depiction of the behavior of the scalar self-coupling if $a_h > 0$ for $M_H > M_{\min}^{\text{safety}}$, leading to Landau-pole behavior (left), $M_H < M_{\min}^{\text{safety}}$, leading to instability (right) and $M_H = M_{\min}^{\text{safety}}$, asymptotically safe behavior (middle). Only this choice is admissible.

4 New physics between the Fermi and Planck scales?

If we fix mass of the top quark, then Eq. 3 determines also the value of the scale μ_0 at which the scalar self-coupling and its β -function vanish simultaneously. The central value for μ_0 is 2.9×10^{18} GeV and is quite stable if m_t and α_s are varied in their confidence intervals (see Fig. 7). One can see that there is a remarkable coincidence between μ_0 and the (reduced) Planck scale $M_P = 2.44 \times 10^{18}$ GeV. The physics input in the computation of μ_0 includes the parameters of the SM only, while the result gives the gravity scale. A possible explanation may

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be related to the asymptotic safety of the SM, as discussed above.

It remains to be seen if this is just the random play of the numbers or a profound indication that the electroweak symmetry breaking is related to Planck physics. If real, this coincidence indicates that there should be no new energy scales between the Planck and Fermi scales, as they would remove the equality of μ_0 and M_P unless some conspiracy is taking place.



Figure 7: The scale μ_0 depending on the top mass M_t . The dashed lines correspond to 1σ uncertainty in α_s . The yellow shaded region corresponds to adding the α_s experimental error and the theoretical uncertainty in the matching of the top Yukawa y_t and top pole mass.

5 Top and Higgs: cosmological inflation

It is well known that for inflation we better have some bosonic field, which drives it (for a review see e.g. [31]). At last, the Higgs boson has been discovered. Can it make the Universe flat, homogeneous, and isotropic, and produce the necessary spectrum of fluctuations for structure formation? The answer to this question is affirmative [32].

The main idea of Higgs inflation is related to a non-minimal coupling of the Higgs field to gravity, described by the action

$$S_G = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2}R - \frac{\xi |\phi|^2}{2}R \right\} \,. \tag{7}$$

Here R is the scalar curvature, the first term is the standard Hilbert-Einstein action, ϕ is the Higgs field, and ξ is a new coupling constant, fixing the strength of "non-minimal" interaction. This constant cannot be fixed by a theoretical computation, but its presence is actually required for consistency of the SM in curved space-time (see, e.g. [33]).

Consider now large Higgs fields, typical for chaotic inflation [34]. Then the gravity strength, given by the effective Planck mass in the Higgs background, is changed as $M_P^{\text{eff}} = \sqrt{M_P^2 + \xi |\phi|^2} \propto |\phi|$. In addition, all particle masses are also proportional to the Higgs field. This means that for $|\phi| \gg \frac{M_P}{\sqrt{\xi}}$ the physics does not depend on the value of the Higgs field, as all dimensionless

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Figure 8: Schematic depiction of the effective potential V for the Higgs field in the Higgsinflationary theory in the Einstein frame for $M_H > M_{crit}$ (left) and $M_H < M_{crit}$ (right). In the second case successful inflation cannot take place.

ratios are $|\phi|$ independent. This leads to an existence of the flat direction for a canonically normalized scalar field χ , related to the Higgs field by conformal transformation. After inflation with $N \simeq 58$ e-foldings the energy of the Higgs field is transferred to other particles of the SM, reheating the Universe up to the temperature $T_{reh} \sim 10^{13-14}$ GeV [35, 36].

For the Higgs inflation to work, the scalar self-coupling constant λ must be positive up to the scale of inflation $\mu_{infl} = M_P/\sqrt{\xi}$. Numerically, this leads to the constraint $M_H > M_{\rm crit}$ with extra theoretical uncertainty of $\delta M_H \sim 1$ GeV [37], see Fig. 8. Though the theory in the electroweak vacuum enters into strong coupling regime at energies smaller than the Planck scale by a factor ξ [38, 39], the analysis of higher dimensional operators and radiative corrections at large Higgs background, necessary for inflation, shows that the Higgs inflation occurs in the weak coupling regime and is self-consistent [37].

The cosmological predictions of the Higgs inflation can be compared with observations performed by the Planck satellite. The Higgs-inflaton potential depends on one unknown parameter, ξ . It can be fixed by the amplitude of the CMB temperature fluctuations $\delta T/T$ at the WMAP normalization scale ~ 500 Mpc, with the use of precise knowledge of the top quark and Higgs masses, and α_s . In general, $\xi > 600$ [6]. Since the Higgs mass lies near $M_{\rm crit}$, the actual value of ξ may be close to the lower bound.

Also, the value of spectral index n_s of scalar density perturbations

$$\left\langle \frac{\delta T(x)}{T} \frac{\delta T(y)}{T} \right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n_s-1}$$
(8)

and the amplitude of tensor perturbations $r = \frac{\delta \rho_s}{\delta \rho_t}$ can be determined. The predictions, together with the Planck results, are presented in Fig. 9, and are well inside the 1 sigma experimental contour. Moreover, as for any single field inflationary model, the perturbations are Gaussian, in complete agreement with Planck [40].

6 Conclusions

For experimental values of Higgs boson and top quark masses there is no necessity for a new energy scale between the Fermi and Planck scales. The EW theory remains in a weakly coupled



Figure 9: Left panel: The predictions of Higgs inflation versus observations. Right panel: Predictions of different inflationary models contrasted with the Planck results (from Ref. [40]).

region all the way up to M_P , whereas the SM vacuum state lives longer than the age of the Universe. If the Higgs and top masses would satisfy the "criticality equation" (4), this would put a strong argument in favor of the *absence* of such a scale and indicate that the electroweak symmetry breaking may be associated with the physics at the Planck scale.

The experimental precision in the Higgs boson mass measurements at the LHC can eventually reach 200 MeV. So, the largest uncertainty will remain in the measurement of the mass of the top quark. It does not look likely that the LHC will substantially reduce the error in the top quark mass determination. Therefore, to clarify the relation between the Fermi and Planck scales a construction of an electron-positron or muon collider with a center-of-mass energy of $\sim 200 + 200 \text{ GeV}$ (Higgs and t-quark factory) would be needed. This would be decisive for setting up the question about the necessity for a new energy scale besides the two ones already known—the Fermi and the Planck scales. In addition, this will allow to study in detail the properties of the two heaviest particles of the Standard Model, potentially most sensitive to any types on new physics.

Surely, even if the SM is a valid effective field theory all the way up the the Planck scale, it cannot be complete as it contradicts a number of observations. In fact, all the confirmed observational signals in favor of physics beyond the Standard Model which were not discussed in this talk (neutrino masses and oscillations, dark matter and baryon asymmetry of the Universe) can be associated with new physics *below* the electroweak scale, for reviews see [41, 42] and references therein. The minimal model explaining all these phenomena, ν MSM, contains, in addition to the SM particles, three relatively light singlet Majorana fermions. The ν MSM predicts that the LHC will continue to confirm the Standard Model and see no deviations from it. At the same time, new experiments at the high-intensity frontier, discussed in [43], may be needed to uncover the new physics below the Fermi scale. In addition, new observations in astrophysics, discussed in [42], may shed light on the nature of Dark Matter.

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References

- [1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1, [arXiv:1207.7214 [hep-ex]].
- [2] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30, arXiv:1207.7235 [hep-ex].
- [3] J. Ellis, J. R. Espinosa, G. F. Giudice, A. Hoecker and A. Riotto, Phys. Lett. B 679 (2009) 369, arXiv:0906.0954 [hep-ph].
- [4] J. R. Espinosa, G. F. Giudice and A. Riotto, JCAP 0805 (2008) 002, arXiv:0710.2484 [hep-ph]].
- [5] C. D. Froggatt and H. B. Nielsen, Phys. Lett. B 368 (1996) 96, hep-ph/9511371.
- [6] F. Bezrukov and M. Shaposhnikov, JHEP 0907 (2009) 089, arXiv:0904.1537 [hep-ph].
- [7] A. De Simone, M. P. Hertzberg and F. Wilczek, Phys. Lett. B 678 (2009) 1, arXiv:0812.4946 [hep-ph].
- [8] M. Shaposhnikov and C. Wetterich, Phys. Lett. B 683 (2010) 196, arXiv:0912.0208 [hep-th].
- [9] F. Bezrukov et al., JHEP 1210 (2012) 140, arXiv:1205.2893 [hep-ph].
- [10] M. Shaposhnikov, arXiv:1311.4979 [hep-ph].
- [11] G. Degrassi et al., JHEP 1208 (2012) 098, arXiv:1205.6497 [hep-ph].
- [12] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, JHEP 1312 (2013) 089, arXiv:1307.3536.
- [13] L. N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. Lett. 108 (2012) 151602, arXiv:1201.5868 [hep-ph].
- [14] L. N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. D 86 (2012) 096008, arXiv:1208.3357 [hep-ph].
- [15] K. G. Chetyrkin and M. F. Zoller, JHEP 1206 (2012) 033, arXiv:1205.2892 [hep-ph].
- [16] K. G. Chetyrkin and M. F. Zoller, JHEP 1304 (2013) 091 [Erratum-ibid. 1309 (2013) 155], arXiv:1303.2890 [hep-ph].
- [17] A. V. Bednyakov, A. F. Pikelner and V. N. Velizhanin, Phys. Lett. B 722 (2013) 336, arXiv:1212.6829.
- [18] A. V. Bednyakov, A. F. Pikelner and V. N. Velizhanin, Nucl. Phys. B 875 (2013) 552, arXiv:1303.4364.
- [19] M. Muether et al. [Tevatron Electroweak Working Group and CDF and D0 Collaborations], arXiv:1305.3929 [hep-ex].
- [20] C. Battilana [for the ATLAS and The CMS Collaborations], arXiv:1309.5307 [hep-ex].
- [21] J. Beringer et al. [Particle Data Group], Phys. Rev. D 86 (2012) 010001.
- [22] S. Alekhin, A. Djouadi and S. Moch, Phys. Lett. B 716 (2012) 214, arXiv:1207.0980 [hep-ph].
- [23] A. L. Kataev and V. T. Kim, Phys. Part. Nucl. 41 (2010) 946, arXiv:1001.4207 [hep-ph].
- [24] F. L. Bezrukov and D. S. Gorbunov, Phys. Lett. B 713 (2012) 365, arXiv:1111.4397 [hep-ph].
- [25] J. Elias-Miro et al., JHEP 1206 (2012) 031, arXiv:1203.0237 [hep-ph].
- [26] S. Weinberg, in General Relativity: An Einstein centenary survey, edited by S. W.Hawking and W.Israel (Cambridge University Press, 1979), chapter 16, p. 790.
- $[27]\,$ M. Niedermaier and M. Reuter, Living Rev. Rel. 9 (2006) 5.
- $[28]\,$ M. Reuter, Phys. Rev. D ${\bf 57}$ (1998) 971, hep-th/9605030.
- [29] R. Percacci and D. Perini, Phys. Rev. D 68 (2003) 044018, hep-th/0304222.
- [30] G. Narain and R. Percacci, arXiv:0911.0386 [hep-th].
- [31] A. D. Linde, Lect. Notes Phys. 738 (2008) 1, arXiv:0705.0164 [hep-th].
- [32] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659 (2008) 703, arXiv:0710.3755 [hep-th].
- [33] R. P. Feynman, F. B. Morinigo, W. G. Wagner and B. Hatfield, "Feynman lectures on gravitation," Reading, USA: Addison-Wesley (1995) 232 p.
- [34] A. D. Linde, Phys. Lett. B **129** (1983) 177.
- [35] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, JCAP 0906 (2009) 029, arXiv:0812.3622 [hep-ph].
- [36] J. Garcia-Bellido, D. G. Figueroa and J. Rubio, Phys. Rev. D 79 (2009) 063531, arXiv:0812.4624 [hep-ph].

- [37] F. Bezrukov et al., JHEP **1101** (2011) 016, arXiv:1008.5157 [hep-ph].
- [38] J. L. F. Barbon and J. R. Espinosa, Phys. Rev. D 79 (2009) 081302, arXiv:0903.0355 [hep-ph].
- [39] C. P. Burgess, H. M. Lee and M. Trott, JHEP 1007 (2010) 007, arXiv:1002.2730 [hep-ph].
- [40] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].
- [41] M. Shaposhnikov, arXiv:0708.3550 [hep-th].
- [42] A. Boyarsky, O. Ruchayskiy and M. Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191, arXiv:0901.0011 [hep-ph].
- [43] W. Bonivento, A. Boyarsky, H. Dijkstra, U. Egede, M. Ferro-Luzzi, B. Goddard, A. Golutvin and D. Gorbunov et al., arXiv:1310.1762 [hep-ex].

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