The Pressure of Misalignment Axions: a Difference from WIMPs in Galaxy Formation?

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Two populations of axions can contribute to cold dark matter: the classical field produced via the misalignment mechanism, and the modes produced in the decay of strings. The classical field has extra pressure, as compared to WIMPs, which could have observable consequences in non-linear galaxy formation.

1 Introduction

It is interesting to study whether axion dark matter could be distinguished from WIMP dark matter, using Large Scale Structure (LSS) data. It is well known that axion dark matter can be composed of two components[1]: the misalignment axions and the non-relativistic modes radiated by strings (here taken to be cold particles). Both redshift like CDM, and grow linear density inhomogeneities like WIMPs. However, as pointed out by Sikivie [2], the misalignment axions have a different pressure from WIMPs, which could be relevant during non-linear stucture formation. The consequences of this additional pressure could be reliably addressed by the numerical galaxy formation community. The aim of this proceedings, which is based on [3], is to clarify the relevant variables and equations for studying non-linear stucture formation with axions.

There has been considerable confusion in the literature about whether axion dark matter is a Bose Einstein condensate. In a scenario proposed by Sikivie, the dark matter axions "thermalise" via their gravitational interactions, and therefore form a Bose Einstein condensate due to the high occupation number of the low momentum modes. Then, Sikivie and collaborators hypothesize that a galactic halo made of condensate could form vortices, which could be observed as caustics in the dark matter distribution. This interesting scenario, which proposes an observable signature for axion dark matter in LSS data, has been studied by many people: Saikawa and collaborators[4] confirmed in Q uantum Field Theory and General Relativity, the gravitational interaction rate estimated by Sikivie and collaborators. However, with Martin Elmer[5], we could not confirm that the interaction rate was a thermalisation rate (= generated entropy). Rindler-Daller and Shapiro[6] studied rotating halos of non-relativistic scalar field, and found that vortices were energetically favoured for much lighter bosons than the QCD axion, or for scalars with repulsive self-interactions (opposite to axions, whose self-interactions are attractive). This proceedings will argue that the notion of Bose Einstein condensation is an unneccessary confusion, somewhat akin to trying to describe a classical electromagnetic field in terms of photons. The misalignment axions are a classical scalar field, as such they have a different pressure from WIMPs, and this could allow them to leave distinctive features in LSS data. First, I review the formalism and results of [3], and discuss the relation to Bose Einstein condensation after eqn (1).

Structure formation within the horizon is a classical gravity problem, so at first sight, a Quantum Field Theory treatment seems unneccessary. However, the classical field and classical particle limits of a QFT are different, and dark matter axions are composed of both. So to obtain a self-consistent and well-defined formalism, where approximations can be catalogued, and particles and fields can be included simultaneously, this proceedings starts from the path integral. A very beautiful earlier treatment of axion dark matter in field theory is [7]. We will find that it is straightforward to describe axion dark matter as a classical field (the misalignment axions) plus cold axion particles (produced by strings). The leading order gravitational interactions of the axion field and particles are simply found by computing their contributions to the stress-energy tensor. And while cold axion particles, in the limit of negligeable velocity, have the stress-energy as WIMPs, the classical axion field has extra contributions to the pressure, which could be relevant during non-linear stucture formation.

Finally, in section 3, I discuss the rate at which gravity can move axions between the field and the cold particle bath.

2 Formalism

From a theoretical perspective, it should be true that "the path integral knows everything": our world is usually at a saddle-point of the effective action. This perspective is rarely useful, because the path integral cannot be solved. However, I imagine to follow it here for two reasons: first, it gives an formal framework which can describe all aspects of axion behaviour, and second, the axion is so feebly coupled that the path integral could be computed pertubatively, which organises the various corrections to the classical= leading order solutions.

I imagine to write the path integral for the axion field, and evalulate it in a closed-time-path (CTP) formalism, as a perturbative expansion in Newton's constant G_N and the axion self-interaction parameter m_a^2/f_{PQ}^2. The path integral offers to decribe axions via n-point functions. I am particularily interested in the one-point function, which is the classical axion field, and the two-point function, which in CTP formalism includes the number distribution f of axion particles. Higher point functions can be neglected because the axion is so feebly coupled.

Next, equations of motion are required. Since we are interested in the gravitational interactions of axions, the leading order (= classical) equations are Einstein's Equations, with contributions to the stress-energy tensor from the axion field and particle density. These contributions will be obtained in second-quantised Field Theory in flat space-time, for simplicity. The details of the calculation can be found in [3]. The order G_N^2 effects will be discussed in section 3.

For cold axion particles, self-interactions are $\sim \lambda^2 \sim (m_a/f_{PQ})^4$ which is negligeable, so $T_{\mu\nu}$ has the form expected for non-relativistic non-interacting particles (the same as for WIMPs):

$$T_{\mu\nu} = \rho U_\mu U_\nu = \left[\begin{array}{cc} \rho & -\rho\vec{v} \\ -\rho\vec{v} & \rho v^i v^j \end{array} \right] \qquad \text{where} \qquad \rho(x) = \int \frac{d^3q}{(2\pi)^3} m_a f(x,q) \quad ,$$

which is the expected classical particle result, with matter four-velocity $U^\mu = (1, \vec{v})$.

For the non-relativistic classical field $\phi = \eta e^{-i(mt+S)}$, $T_{\mu\nu}$ is of the form

$$T_{\mu\nu} = \left[\begin{array}{cc} \rho & -\rho\vec{v} \\ & \\ -\rho\vec{v} & \rho v^i v^j + \Delta T_{ij} \\ & \end{array} \right] \qquad \begin{array}{rcl} \Delta T_{ij} & = & 2\partial_i\eta\partial_j\eta - \delta_{ij}\nabla\eta\cdot\nabla\eta \\ & & -\delta_{ij}\left(\rho|\vec{v}|^2 - 2m\eta^2\partial_t S + \lambda\eta^4\right) \end{array} \tag{1}$$

where $\rho \simeq 2m^2\eta^2$, $v_j \simeq \partial_j S/m$, and ΔT_{ij} is the extra pressure intrinsic to the classical scalar field. There is a part related to the field gradients, sometimes refered to as "quantum pressure"[8], and a part due to self-interactions, which is linear in the four-axion coupling $\lambda = -m_a^2/(12f_{PQ}^2)$. Since the self-interaction pressure of axions is *negative*, it causes axions to clump (like gravity). Rindler-Daller and Shapiro [6] argue that this negative pressure discourages vortices in axion halos.

Notice that the pressure excess ΔT_{ij} arises because the misalignment axions are a classical field. There is no additional requirement of "bose einstein condensation". In calculations, Bose Einstein condensates are described at leading order as non-relativistic classical fields[8]; since this is what the misalignment axions are, one could also say the misalignment axions are a bose einstein condensate. However, in my opinion, this comes with baggage of un-useful [1] intuitions from classical equilibrium statistical mechanics, and from well-known strongly interacting Bose Einstein condensates such as ^4He. So its simpler and clearer to refer to misalignment axions as a classical scalar field.

The pressure excess ΔT_{ij} confirms Sikivie's expectation that axions could differ from WIMPs during non-linear structure formation. This can most easily be seen from the "continuity" and "Euler" equations obtained from $T^{\nu\mu}_{\ ;\mu} = 0$:

$$\begin{array}{rclc} \partial_t\rho + \nabla\cdot(\rho\vec{v}) & = & 0 & \text{continuity} \\ \rho\partial_t\vec{v} + \rho(\vec{v}\cdot\nabla)\vec{v} & = & -\rho\nabla\psi - \rho\nabla Q - \nabla P_{SI} & \text{Euler} \end{array} ,$$

where ψ is the Newtonian gravitational potential, $Q = -\frac{1}{2m\eta}\nabla^2\eta$ describes the "quantum kinetic energy" related to the "quantum pressure" of the classical field, and $P_{SI} = 2\lambda\eta^4$ is proportional to the pressure arising from the Self Interactions of the field. The point is that the extra pressures generate extra forces on the dark matter distribution. It would be interesting to numerically simulate galaxy formation using fluid equations for the dark matter, rather than N-body. This could allow to identify differences between galaxies made of classical axion field versus cold particles.

3 Gravitational evaporation of the field?

An important question remains, before studying galaxy formation in the the presence of the classical axion field: can gravity move axions between the field and the particle bath? One could imagine that during violent moments of galaxy formation, the field gets stirred up, and evaporates into particles. Or maybe the particles condense to the field, due to their high occupation numbers?

This process of "decay" of the field into particles should arise somewhere in the path integral formalism. For instance, in the case of a $\lambda\phi^4$ interaction, this "decay" of the field will arise as an

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[1] Statistical mechanics is un-useful because its about the classical particle limit of QFT, and ^4He is a poor analogy because axions are feebly coupled.

imaginary part of loop corrections to the potential, in a 2 Particle Irreducible Closed-Time-Path evaluation of the path integral. A more simple-minded estimate was performed in [3].

First, notice that classical gravity, at $\mathcal{O}(G_N)$, does not move the particles between the field and the bath, because the field and particles contribute independently to the stress-energy tensor:

$$T_{\mu\nu} = T^{(\phi_c)}_{\mu\nu} + T^{(part)}_{\mu\nu} \quad .$$

Then, one can estimate the $\mathcal{O}(G_N^2)$ cross-section for axion scattering via graviton exchange, $aa \to aa$. If one of the incident axions is in the classical field, and both final state axions are free particles, this would correspond to field "evaporation. The cross-section is infra-red divergent, so the choice of infra-red cut-off is crucial. I claim that a reasonable IR cutoff is the inverse axion momentum, because on much larger distances, the graviton will not see individual axions, but rather will interact coherently with a large number of axions. Therefore the the two-axion scattering process, $aa \to aa$, only occurs when the momentum exchange is of order the axion momentum. In this case, the rate for the field to evaporate to particles via gravitational interactions is found to be negligeably small, because it is suppressed by $(m_a/m_{pl})^3$.

4 Summary

The axion misalignment field has additional contributions to the pressure, as compared to a bath of cold particles. This is automatic, no dynamical process of Bose Einstein condensation is required. As suggested by Sikivie, this extra pressure could give observational signatures in the structure of galaxies. It would be interesting to numerically simulate galaxy formation with a fluid code which allowed the dark matter to have pressure, to discover whether galaxies made of axion dark matter look different.

References

[1] See the proceedings of K. Saikawa at this conference.

[2] P. Sikivie and Q. Yang, "Bose-Einstein Condensation of Dark Matter Axions," Phys. Rev. Lett. **103** (2009) 111301 [arXiv:0901.1106 [hep-ph]].

[3] S. Davidson, "Axions: Bose Einstein Condensate or Classical Field?," arXiv:1405.1139 [hep-ph].

[4] K. 'i. Saikawa and M. Yamaguchi, "Evolution and thermalization of dark matter axions in the condensed regime," Phys. Rev. D **87** (2013) 085010 [arXiv:1210.7080 [hep-ph]]. T. Noumi, K. 'i. Saikawa, R. Sato and M. Yamaguchi, "Effective gravitational interactions of dark matter axions," arXiv:1310.0167 [hep-ph].

[5] S. Davidson and M. Elmer, "Bose Einstein condensation of the classical axion field in cosmology?," JCAP **1312** (2013) 034 [arXiv:1307.8024].

[6] T. Rindler-Daller and P. R. Shapiro, "Finding new signature effects on galactic dynamics to constrain Bose-Einstein-condensed cold dark matter," arXiv:1209.1835 [astro-ph.CO]. *ibid.* "Angular Momentum and Vortex Formation in Bose-Einstein-Condensed Cold Dark Matter Haloes," arXiv:1106.1256 [astro-ph.CO]. *ibid.* "Vortices and Angular Momentum in Bose-Einstein-Condensed Cold Dark Matter Halos," arXiv:0912.2897 [astro-ph.CO].

[7] Y. Nambu and M. Sasaki, "Quantum Treatment Of Cosmological Axion Perturbations," Phys. Rev. D **42** (1990) 3918.

[8] F Dalfovo, S Giorgini, L.P. Pitaevskii, S. Stingari, "Theory of Bose-Einstein condensation in trapped gases", Rev. Mod Phyis. **71** (1999), 463.