Scalar Gauge Fields, C and CP violation, Global Scalar QED and Axions

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DOI: http://dx.doi.org/10.3204/DESY-PROC-2014-03/guendelman_eduardo

We study the implications of using the gradient of a scalar field instead of a standard gauge field at various points: a) in minimal coupling and b) in coupling electromagnetism to a global charge. The implications in a) can be surprising, leading to possible mixing of this gauge field with Goldstone bosons, leading to low mass weakly coupled particles, C and CP violation and in case b) we arrive to a new scalar QED that reproduces features of axion photon physics.

1 Generalizing minimal coupling using scalar gauge fields

We introduce [1] a complex scalar field ϕ and consider the local phase symmetry of ϕ by introducing a real, scalar $B(x_\mu)$ in addition to a normal vector gauge field and two types of covariant derivatives as

$$D^A_\mu = \partial_\mu + ieA_\mu \quad ; \quad D^B_\mu = \partial_\mu + ie\partial_\mu B \ . \tag{1}$$

The gauge transformation of the complex scalar, vector gauge field and scalar gauge field have the following gauge transformation

$$\phi \to e^{ie\Lambda}\phi \quad ; \quad A_\mu \to A_\mu - \partial_\mu \Lambda \quad ; \quad B \to B - \Lambda \ . \tag{2}$$

It is easy to see that terms like $D^A_\mu \phi$ and $D^B_\mu \phi$, as well as their complex conjugates will be covariant under (2). Thus one can generate kinetic energy type terms like $(D^A_\mu\phi)(D^{A\mu}\phi)^*$, $(D^B_\mu\phi)(D^{B\mu}\phi)^*$, $(D^A_\mu\phi)(D^{B\mu}\phi)^*$, and $(D^B_\mu\phi)(D^{A\mu}\phi)^*$. Unlike A_μ where one can add a gauge invariant kinetic term involving only A_μ (i.e. $F_{\mu\nu}F^{\mu\nu}$) this is apparently not possible to do for the scalar gauge field B. However note that the term $A_\mu - \partial_\mu B$ is invariant under the gauge field transformation alone (i.e. $A_\mu \to A_\mu - \partial_\mu\Lambda$ and $B \to B - \Lambda$). Thus one can add a term like $(A_\mu - \partial_\mu B)(A^\mu - \partial^\mu B)$ to the Lagrangian which is invariant with respect to the gauge field part only of the gauge transformation in (2). This gauge invariant term will lead to both mass-like terms for the vector gauge field and kinetic energy-like terms for the scalar gauge field. In total a general Lagrangian which respects the new gauge transformation, has the form

$$\begin{aligned}\mathcal{L} &= c_1 D^A_\mu\phi(D^{A\mu}\phi)^* + c_2 D^B_\mu\phi(D^{B\mu}\phi)^* + c_3 D^A_\mu\phi(D^{B\mu}\phi)^* + c_4 D^B_\mu\phi(D^{A\mu}\phi)^* - V(\phi) \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_5(A_\mu - \partial_\mu B)(A^\mu - \partial^\mu B)\ ,\end{aligned} \tag{3}$$

where c_i's are constants. The Maxwell field strength tensor appears in the equation above and is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{4}$$

One can see that $F_{\mu\nu}$ is invariant under $A_\mu \to A_\mu - \partial_\mu \Lambda$.Later we will show that it is possible to add non-derivative , polynomial interaction terms for the B field. An interesting point to remark upon is that the last term in (3) will have a term of the form $A_\mu A^\mu$ which is a mass term for the vector gauge field. This term (contrary to the usual gauge transformation given in the introduction) does not violate the expanded gauge symmetry of (2).

2 Particle content, the generalized unitary gauge, C and CP violation

We devote this section to the discussion of the physical reality of the newly introduced scalar gauge field $B(x)$ and to discuss the particle content of the theory when we have spontaneous symmetry breaking i.e. when the scalar field ϕ develops a vacuum expectation value due to the form of the potential $V(\phi)$ in (3). At first glance one might conclude that $B(x)$ is not a physical field – it appears that one could "gauge" it away by taking $\Lambda = B(x)$ in (2). However one must be careful since this would imply that the gauge transformation of the field ϕ would be of the form $\phi \to e^{ieB}\phi$ i.e. the phase factor would be fixed by the gauge transformation of $B(x)$. In this situation one would no longer to able to use the usual unitary gauge transformation to eliminate the Goldstone boson in the case when one has spontaneous symmetry breaking.

The unitary gauge is the standard procedure to find the particle content of a spontaneously broken theory. Let us recall how the unitary gauge works: One writes the complex scalar field as an amplitude and phase – $\phi(x) = \rho(x)e^{i\theta(x)}$. The two fields $\rho(x)$ and $\theta(x)$ represent the initial fields of the system. If $\phi(x)$ develops a VEV due to the form of the potential, $V(\phi)$, then one can transform to the unitary gauge $\phi \to e^{ie\Lambda(x)}\phi(x)$ with $\Lambda = -\theta(x)/e$. In this way one removes the field $\theta(x)$ (which is "eaten" by the gauge boson) and is left with only the $\rho(x)$ field. With the introduction of the scalar gauge field, $B(x)$, one no longer can gauge away *both* $\theta(x)$ and $B(x)$, and in the end one is left with some real, physical field which is some combination of the original $B(x)$ and $\theta(x)$.

We now fix, as far as possible, the character of the c_i's in (3). First c_1, c_2 and c_5 must be real since $D^A_\mu\phi(D^{A\mu}\phi)^*$, $D^B_\mu\phi(D^{B\mu}\phi)^*$ and $(A_\mu - \partial_\mu B)(A^\mu - \partial^\mu B)$ are real. Next c_3 and c_4 must be complex conjugates (i.e. $c_3 = c_4^*$) in order that the combination of the two crossed covariant derivative terms in (3) (i.e. the terms $D^A_\mu\phi(D^{B\mu}\phi)^*$ and $D^B_\mu\phi(D^{A\mu}\phi)^*$) be real. Finally we require that $(c_1 + c_2 + c_3 + c_4) = (c_1 + c_2 + Re[c_3 + c_4]) = 1$. This condition ensures that the kinetic energy term for the scalar field ϕ has the standard form $\partial_\mu\phi\partial^\mu\phi^*$. One could accomplish this as well by rescaling ϕ, but here we chose to accomplish this by placing conditions on the c_i's. Taking into account these conditions (and in particular writing out c_3 and c_4 in terms of their real and imaginary parts $c_3 = a + ib$ and $c_4 = a - ib$) the Lagrangian in (3) becomes

$$\begin{aligned} \mathcal{L} &= \partial_\mu\phi\partial^\mu\phi^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_5 A_\mu A^\mu + c_5\partial_\mu B\partial^\mu B - 2c_5 A_\mu\partial^\mu B \\ &+ ie[\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi]\left((c_1+a)A^\mu + (c_2+a)\partial_\mu B\right) \\ &+ e^2\phi\phi^*\left(c_1 A_\mu A^\mu + c_2\partial_\mu B\partial^\mu B + 2a\partial_\mu B A^\mu\right) - eb\partial_\mu(\phi^*\phi)(A^\mu - \partial^\mu B) \ . \end{aligned} \tag{5}$$

There are several interesting features of the Lagrangian in (5). First, the vector gauge field, A_μ, has a mass term (i.e. $c_5 A_\mu A^\mu$) which is allowed by the extended gauge symmetry (2). Thus in addition to the vector gauge field developing a mass through the term $e^2 c_1 \phi\phi^* A_\mu A^\mu$ if ϕ develops a vacuum expectation value (i.e. if $\langle \phi\phi^* \rangle = \rho_0^2$ with ρ_0 a constant), there is now an additional potential mass term for the vector gauge field, even in the absence of spontaneous symmetry breaking via ϕ. Second, the scalar gauge field appears to be a dynamical field through the presence of two possible kinetic energy terms. The term $c_5 \partial_\mu B \partial^\mu B$ is the standard kinetic energy term for a scalar field. Also, the term $c_2 e^2 \phi\phi^* \partial_\mu B \partial^\mu B$ takes the form of a kinetic energy term if ϕ develops a vacuum expectation value. Third, the term $-eb\partial_\mu(\phi^*\phi)(A^\mu - \partial^\mu B)$ will lead to C and CP violation. Let us now define exactly what will be the generalization of the unitary gauge appropriate to the situation here. In the presence of spontaneous symmetry breaking and where the field ϕ develops a VEV the unitary gauge eliminates cross terms like $A_\mu \partial^\mu \theta$ from the Lagrangian. In the present case the cross terms between the vector field A_μ and the scalars (in our case B and θ) are more involved. Explicitly the relevant cross terms that we wish to eliminate by a generalized unitary gauge are

$$\mathcal{L}_{cross} = -2c_5 A_\mu \partial^\mu B + ie(c_1 + a)[\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi] A^\mu + 2ae^2 \partial_\mu B A^\mu \phi^* \phi \ . \tag{6}$$

It is obvious why the first and third terms in the above equation are denoted as cross terms since they have the form $A_\mu \partial^\mu B$. To see why the second term above is considered a cross term between A_μ and θ in the presence of SSB (i.e. the scalar field develops a VEV $\langle \phi\phi^* \rangle = \rho_0^2$ where ρ_0 is a constant) we begin by approximating the scalar field as $\phi(x) \approx \rho_0 e^{i\theta(x)}$. With this the scalar current becomes $[\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi] \approx 2\rho_0^2 \partial_\mu \theta$. We have used the assumption that the amplitude of the scalar field is approximately constant – $\rho(x) \approx \rho_0$. Putting all this together show that the second term in (6) is a cross term between A_μ and θ of the form $A_\mu \partial^\mu \theta$. Thus (6) becomes

$$\mathcal{L}_{cross} = 2A_\mu \partial^\mu \left(-c_5 B + ec_1 \rho_0^2 \theta + ae\rho_0^2 \theta + ae^2 \rho_0^2 B \right) \ . \tag{7}$$

It is this more complex cross term that we want to eliminate via some generalized unitary gauge. Defining $F(x) = -c_5 B + c_1 e \rho_0^2 \theta + ae\rho_0^2 \theta + ae^2 \rho_0^2 B$, one can see that the cross term in (7) takes the form $\propto A_\mu \partial^\mu F$ which is similar to the more common form $\propto A_\mu \partial^\mu \theta$. By means of a gauge transformation (i.e. $\theta \to \theta + e\Lambda$, $B \to B - \Lambda$) we can take some initial non-zero value $F = F_0$, and always arrive at a gauge $F = 0$. From (7) one can check this is possible by choosing the gauge function as $\Lambda = -F_0/(c_5 + c_1 e^2 \rho_0^2)$. In this physical gauge, with $F = 0$, we can solve the θ field in terms of the B field as

$$\theta = \frac{c_5 - ae^2 \rho_0^2}{e\rho_0^2 (c_1 + a)} B \ . \tag{8}$$

What (8) shows is that θ and B are not independent fields – one is fixed in terms of the other. There is therefore only one physical scalar field in this generalized unitary gauge which one can call either θ or B. The above is different from the normal gauge procedure in the presence of symmetry breaking where the $\theta(x)$ field completely disappears. Here there is some left over hint of the Goldstone boson which we may call $B(x)$ (as we do here) or $\theta(x)$. At this stage the mixed θ/B field is massless and thus could be thought of as a true, massless Goldstone boson. However it is possible to add to the Lagrangian from (5), non-derivative potential terms for the B field. These terms will include a mass term and power law interaction terms. one can write down the following general interacting potential for ϕ and B

$$V(e^{ieB}\phi) = -m^2\phi\phi^* + \lambda(\phi\phi^*)^2 + \lambda_1 e^{ieB}\phi + \lambda_1^* e^{-ieB}\phi^* + \lambda_2 e^{i2eB}\phi^2 + \lambda_2^* e^{-i2eB}(\phi^*)^2 \\ +\lambda_3 e^{i3eB}\phi^3 + \lambda_3^* e^{-i3eB}(\phi^*)^3 + \lambda_4 e^{i4eB}\phi^4 + \lambda_4^* e^{-i4eB}(\phi^*)^4 \; . \tag{9}$$

If the λ_i 's, have an imaginary part, then those terms will violate charge conjugation symmetry. Defining a canonically normalized field, rescaling the B field, expressing the interactions and effective potential in terms of this field, one can see that the interactions of the new B-particles can be very weak if certain expectation values are big and they can be massive [1], a possible candidate for dark matter?.

3 Global scalar QED and axion photon dynamics

We work here with the following lagrangian density [2]

$$\mathcal{L} = g^{\mu\nu}\frac{\partial\psi^*}{\partial x^\mu}\frac{\partial\psi}{\partial x^\nu} - U(\psi^*\psi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j_\mu(A^\mu + \partial^\mu B) \tag{10}$$

where

$$j_\mu = ie(\psi^*\frac{\partial\psi}{\partial x^\mu} - \psi\frac{\partial\psi^*}{\partial x^\mu}) \tag{11}$$

and where we have also allowed an arbitrary potential $U(\psi^*\psi)$ to allow for the possibility of spontaneous breaking of symmetry. The model is separately invariant under local gauge transformations

$$A^\mu \rightarrow A^\mu + \partial^\mu\Lambda; \quad B \rightarrow B - \Lambda \tag{12}$$

and the independent global phase transformations

$$\psi \rightarrow exp(i\chi)\psi \tag{13}$$

A model of this type allows to couple electromagnetism to a global charge, it leads to a different version of scalar electrodynamics, without sea gull diagrams. Here also the correspondence between a scalar charged particle in an external electric field with the axion photon dynamics [3] is exact in this version of scalar electrodynamics as opposed to the standard scalar QED, where it is valid only to first order in the coupling constant.

4 Bibliography

References

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