Modelling γ-ray-axion-like particle oscillations in turbulent magnetic fields: relevance for observations with Cherenkov telescopes

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Axion-like particles (ALPs) are a common prediction of certain theories beyond the Standard Model and couple to photons in the presence of external magnetic fields. As a consequence, photon-ALP conversion could lead to an enhancement of the flux of extragalactic γ-ray sources that is otherwise attenuated due to the interactions with background radiation fields. The magnetic fields traversed by the γ rays are often turbulent and frequently modelled with a simple domain-like structure. Given a maximum mixing between photons and ALPs, we show that in such models realisations of the fields exist for which the photon-ALP oscillation probability vanishes. This behaviour does not occur in more sophisticated magnetic-field models.

1   Introduction

Very high energy γ rays (VHE; energy $E \gtrsim 100\,\mathrm{GeV}$) originating from extragalactic objects interact with photons of the extragalactic background light (EBL) leading to an exponential attenuation of the γ-ray flux emitted by the source [8]. Direct detections of the EBL are difficult due to foreground emission [11] and thus the exact level of the EBL photon density remains unknown. Recent EBL models (e.g. Refs. [15, 6, 10]) predict densities close to lower limits deduced from galaxy number counts [16, 9]. In simple emission models of blazars[1], no spectral hardening is expected at VHE and consequently spectra corrected for EBL absorption should not show such features. Nevertheless, evidence for such signatures has been found [4, 3, 7, 13, 24]. An explanation might be the oscillations of γ-rays into axion-like particles (ALPs), spin-0 pseudo-Nambu-Goldstone bosons that arise in certain Standard Model extensions. These particles couple to photons in external magnetic fields (see Ref. [14] for a review) and could lead to a flux enhancement as ALPs do not interact with EBL photons.

Several turbulent magnetic-field environments have been studied in this respect including the intergalactic magnetic field [22, 5], the AGN host galaxy [27], intra-galaxy-cluster fields [12], magnetic fields in AGN lobes [26], and in the Milky Way [25]. The turbulent fields are commonly modelled with a simple domain-like structure: the path length is split up into N domains of coherence length L_{coh}. While the field strength remains constant over all cells, the orientation

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[1]Blazars are active galactic nuclei (AGN) with a jet closely aligned to the line of sight. It is the most common source class for extragalactic VHE γ-ray emitters, see e.g. http://tevcat.uchicago.edu/.

of the field is assumed to change randomly from one cell to the next. The random nature of the field makes it necessary to calculate the conversion probability for a large number of random realisations. As N grows, the photon-ALP oscillations can often only be calculated numerically. Here, we show analytically that in these simple models realisations exist for which the photon-ALP conversion probability vanishes.

2 Photon-ALP oscillations

The equations of motion of a monochromatic photon-ALP beam composed of the two photon polarisation states $A_{1,2}$ and the ALP field strength a, $\Psi = (A_1, A_2, a)^T$, of energy E propagating along the x_3 axis in a cold plasma with homogeneous magnetic field can be written as [23]

$$\left(i \frac{\mathrm{d}}{\mathrm{d}x_3} + E + \mathcal{M}_0 \right) \Psi(x_3) = 0, \tag{1}$$

where the mixing is induced by off-diagonal elements of the mixing matrix $\mathcal{M}_0$. The resulting photon-ALP oscillations are similar to neutrino oscillations and we denote the mixing angle by α (see, e.g. Ref. [2] for the full expressions for $\mathcal{M}_0$ and α). Equation (1) can be solved with the transfer matrix $\mathcal{T}$, so that $\Psi(x_3) = \mathcal{T}(x_3, 0; \psi; E)\Psi(0)$, where ψ denotes the angle between the transversal magnetic field and the photon polarisation state along x_2 [3]. With the eigenvalues λ_j, $j = 1, 2, 3$, of the mixing matrix and introducing the notation $s_\theta = \sin\theta$ and $c_\theta = \cos\theta$, the transfer matrix can be written as [3]

$$\mathcal{T} = \sum_{j=1}^{3} e^{i\lambda_j x_3} \mathcal{T}_j \qquad \text{with} \qquad \mathcal{T}_1 = \begin{pmatrix} c_\psi^2 & -s_\psi c_\psi^2 & 0 \\ s_\psi c_\psi & s_\psi^2 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{T}_2 = \begin{pmatrix} s_\psi^2 s_\alpha^2 & c_\psi s_\psi s_\alpha^2 & -s_\psi c_\alpha s_\alpha \\ c_\psi s_\psi s_\alpha^2 & c_\psi^2 s_\alpha^2 & -c_\psi c_\alpha s_\alpha \\ -s_\psi c_\alpha s_\alpha & -c_\psi c_\alpha s_\alpha & c_\alpha^2 \end{pmatrix}, \quad \mathcal{T}_3 = \begin{pmatrix} s_\psi^2 c_\alpha^2 & c_\psi s_\psi c_\alpha^2 & s_\psi c_\alpha s_\alpha \\ c_\psi s_\psi c_\alpha^2 & c_\psi^2 c_\alpha^2 & c_\psi c_\alpha s_\alpha \\ s_\psi c_\alpha s_\alpha & c_\psi c_\alpha s_\alpha & s_\alpha^2 \end{pmatrix}. \tag{2}$$

For N consecutive domains with angle ψ_n in each domain, it can be shown that the total transfer matrix is given as a product over all domains,

$$\mathcal{T}(x_{3,N}, \ldots, x_{3,1}; \psi_N, \ldots, \psi_1; E) = \prod_{n=1}^{N} \mathcal{T}(x_{3,n+1}, x_{3,i}; \psi_n; E). \tag{3}$$

Present γ-ray experiments cannot measure the polarisation. Therefore, one has to generalise the problem at hand to the density matrix formalism, where $\rho = \Psi \otimes \Psi^\dagger$ [21]. The probability for an initially unpolarised photon beam, $\rho_{\mathrm{unpol}} = 1/2\,\mathrm{diag}(1,1,0)$, to oscillate into an ALP, $\rho_{aa} = \mathrm{diag}(0,0,1)$ is then given by [21]

$$P_{a\gamma} = \mathrm{Tr}\left(\rho_{aa} \mathcal{T} \rho_{\mathrm{unpol}} \mathcal{T}^\dagger \right). \tag{4}$$

The oscillation probability will take some value $0 \leqslant P_{a\gamma} \leqslant 1/2$ [20] depending on the realisation of the angles $\{\psi_n\}$ and the mixing angle. Interestingly, realisations exist for which $P_{a\gamma} = 0$ even though $\alpha > 0$. To show this we assume an even number of domains where $\psi = c\pi$ in one half of the domains and $(c+1)\pi$ in the other half (where c is a real non-zero number) ordered

randomly. A straightforward calculations shows that the commutator of the transfer matrices $C = [\mathcal{T}(\psi = (c+1)\pi), \mathcal{T}(\psi = c\pi)]$ is an anti-symmetric matrix with entries

$$C_{13} = -C_{31} \quad = \quad \frac{1}{2}\left(e^{i\lambda_2 x_3} - e^{i\lambda_3 x_3}\right)^2 s_{c\pi}s_{4\alpha} \tag{5}$$

$$C_{23} = -C_{32} \quad = \quad \frac{1}{2}\left(e^{i\lambda_2 x_3} - e^{i\lambda_3 x_3}\right)^2 c_{c\pi}s_{4\alpha} \tag{6}$$

and zero in all other entries. The matrix elements of the product $\mathcal{T}(\psi = (c+1)\pi)\mathcal{T}(\psi = c\pi)$ that induce mixing (i.e., the $i3$, $i = 1, 2$ elements in the current basis) are found to be equal to $2C_{i3}$. Above a critical energy E_{crit} the mixing becomes independent of energy. If in addition the mixing is strong so that $\alpha \to \pi/4$, the commutator and the mixing inducing matrix elements vanish. With the commutator equal to zero we can now combine all pairs of $c\pi$ and $(c+1)\pi$ transfer matrices and see that the resulting product of all matrices given in Eq. (3) does not induce any photon-ALP mixing.

As an example, we show this behaviour in Fig. 1, in which we assume magnetic-field parameters found in galaxy clusters. The conversion probability is calculated numerically following Eq. (4). Above the critical energy the probability goes to zero, however, around the critical energy oscillations still occur. Our findings still hold even if photon absorption is included as it is the case for conversions in the intergalactic magnetic field. However, as this magnetic field evolves with redshift, not all realisations lead to a conversion probability exactly equal to zero for all random permutations.

3 Conclusions

As shown in the previous section, the photon-ALP conversion probability can be exactly zero in special configurations of a turbulent magnetic field given that (a) it is modelled with a simple cell-like structure and (b) that the mixing occurs in the strong mixing regime, i.e. at energies $> E_{\mathrm{crit}}$ and $\alpha \to \pi/4$. Oscillations around the critical energy still occur making spectral features at this energy a universal prediction of photon-ALP oscillations. The absence of such signatures in γ-ray spectra has already been used to constrain the photon-ALP coupling [1]. In more realistic models of the turbulent field (that use, e.g., a Kolmogorov turbulence spectrum) we do not have the freedom to choose the ψ angles (see, e.g. Ref. [19]) and we cannot easily construct a scenario with vanishing mixing as done here. Utilizing such models, it can be shown that the future Cherenkov Telescope Array will be sensitive to detect a boost in the photon flux for photon-ALP couplings $\gtrsim 2 \times 10^{-11}\,\mathrm{GeV}^{-1}$ and ALP masses $\lesssim 100\,\mathrm{neV}$ [17], the same parameters that could explain evidence for a reduced opacity for VHE γ-rays [18].

Acknowledgments

References

[1] A. Abramowski, F. Acero, F. Aharonian, F. Ait Benkhali, A. G. Akhperjanian, E. Angüner, G. Anton, S. Balenderan, A. Balzer, A. Barnacka, and et al. Constraints on axionlike particles with H.E.S.S. from the irregularity of the PKS 2155-304 energy spectrum. Phys. Rev. D, 88(10):102003, 2013.
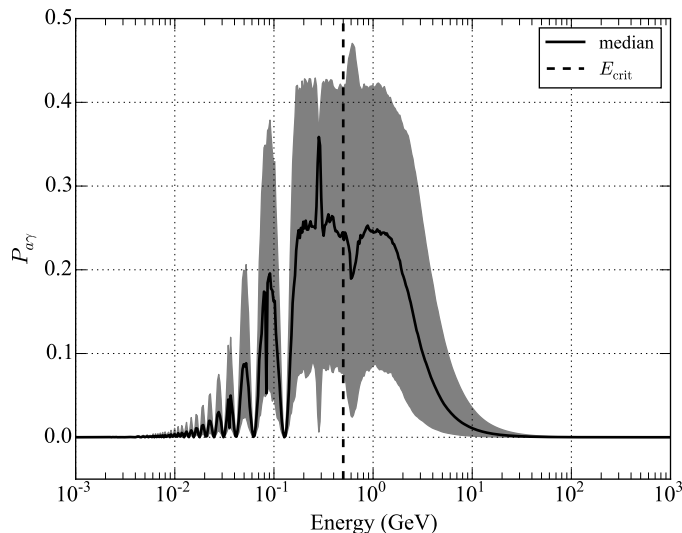
Figure 1: Conversion probability in 30 domains with $L_{\mathrm{coh}} = 10\,\mathrm{kpc}$ and $B = 1\,\mu\mathrm{G}$. Half of the domains have $\psi = 0$ while in the other half $\psi = \pi$ is chosen. The solid line shows the median of $P_{a\gamma}$ for 1000 random permutations of the angles. The shaded area gives the probability in the 68 % interval around the median. The dashed line shows E_{crit} above which $\alpha \to \pi/4$. An ALP mass of $1\,\mathrm{neV}$ and a photon-ALP coupling of $5 \times 10^{-11}\,\mathrm{GeV}^{-1}$ are assumed.

[2] N. Bassan, A. Mirizzi, and M. Roncadelli. Axion-like particle effects on the polarization of cosmic high-energy gamma sources. *JCAP*, 5:10, 2010.

[3] A. de Angelis, G. Galanti, and M. Roncadelli. Relevance of axionlike particles for very-high-energy astrophysics. Phys. Rev. D, 84(10):105030, 2011.

[4] A. De Angelis, O. Mansutti, M. Persic, and M. Roncadelli. Photon propagation and the very high energy γ-ray spectra of blazars: how transparent is the Universe? MNRAS, 394:L21–L25, 2009.

[5] A. de Angelis, M. Roncadelli, and O. Mansutti. Evidence for a new light spin-zero boson from cosmological gamma-ray propagation? Phys. Rev. D, 76(12):121301, 2007.

[6] A. Domínguez, J. R. Primack, D. J. Rosario, et al. Extragalactic background light inferred from AEGIS galaxy-SED-type fractions. MNRAS, 410:2556–2578, 2011.

[7] A. Domínguez, M. A. Sánchez-Conde, and F. Prada. Axion-like particle imprint in cosmological very-high-energy sources. *JCAP*, 11:20, 2011.

[8] E. Dwek and F. Krennrich. The extragalactic background light and the gamma-ray opacity of the universe. *Astroparticle Physics*, 43:112–133, 2013.

[9] G. G. Fazio, M. L. N. Ashby, et al. Number Counts at 3 μm < λ < 10 μm from the Spitzer Space Telescope. ApJS, 154:39–43, 2004.

[10] R. C. Gilmore, R. S. Somerville, J. R. Primack, and A. Domínguez. Semi-analytic modelling of the extragalactic background light and consequences for extragalactic gamma-ray spectra. MNRAS, 422:3189–3207, 2012.

[11] M. G. Hauser, R. G. Arendt, et al. The COBE Diffuse Infrared Background Experiment Search for the Cosmic Infrared Background. I. Limits and Detections. ApJ, 508:25–43, 1998.

[12] D. Horns, L. Maccione, M. Meyer, A. Mirizzi, D. Montanino, and M. Roncadelli. Hardening of TeV gamma spectrum of active galactic nuclei in galaxy clusters by conversions of photons into axionlike particles. Phys. Rev. D, 86(7):075024, 2012.

[13] D. Horns and M. Meyer. Indications for a pair-production anomaly from the propagation of VHE gamma-rays. *JCAP*, 2:33, 2012.

[14] J. Jaeckel and A. Ringwald. The Low-Energy Frontier of Particle Physics. *Annual Review of Nuclear and Particle Science*, 60:405–437, 2010.

[15] T. M. Kneiske and H. Dole. A lower-limit flux for the extragalactic background light. A&A, 515:A19+, 2010.

[16] P. Madau and L. Pozzetti. Deep galaxy counts, extragalactic background light and the stellar baryon budget. MNRAS, 312:L9–L15, 2000.

[17] M. Meyer and J. Conrad. Sensitivity of the Cherenkov Telescope Array to the detection of axion-like particles at high gamma-ray opacities. *ArXiv e-prints*, 2014.

[18] M. Meyer, D. Horns, and M. Raue. First lower limits on the photon-axion-like particle coupling from very high energy gamma-ray observations. Phys. Rev. D, 87(3):035027, 2013.

[19] M. Meyer, D. Montanino, and J. Conrad. On detecting oscillations of gamma rays into axion-like particles in turbulent and coherent magnetic fields. *ArXiv e-prints*, 2014.

[20] Manuel Meyer. *The Opacity of the Universe for High and Very High Energy γ-Rays*. PhD thesis, University of Hamburg, 2013. `http://inspirehep.net/record/1254304`.

[21] A. Mirizzi and D. Montanino. Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields. *JCAP*, 12:4, 2009.

[22] A. Mirizzi, G. G. Raffelt, and P. D. Serpico. Signatures of axionlike particles in the spectra of TeV gamma-ray sources. Phys. Rev. D, 76(2):023001, 2007.

[23] G. Raffelt and L. Stodolsky. Mixing of the photon with low-mass particles. Phys. Rev. D, 37:1237–1249, 1988.

[24] G. I. Rubtsov and S. V. Troitsky. Breaks in gamma-ray spectra of distant blazars and transparency of the Universe. *ArXiv e-prints*, 2014.

[25] M. Simet, D. Hooper, and P. D. Serpico. Milky Way as a kiloparsec-scale axionscope. Phys. Rev. D, 77(6):063001, 2008.

[26] F. Tavecchio, M. Roncadelli, and G. Galanti. Photons into axion-like particles conversion in Active Galactic Nuclei. *ArXiv e-prints*, 2014.

[27] F. Tavecchio, M. Roncadelli, G. Galanti, and G. Bonnoli. Evidence for an axion-like particle from PKS 1222+216? Phys. Rev. D, 86(8):085036, 2012.