Astroparticles and extra dimensions

A. Nicolaidis

Theoretical Physics Department
Aristotle University of Thessaloniki
54124 Thessaloniki, Greece
nicolaid@auth.gr

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In theories with large extra dimensions, besides the graviton living in the bulk, particles which are singlets under the standard model may live in extra dimensions. A singlet neutrino can live in the bulk and its mixing with a standard flavor neutrino offers unconventional patterns of neutrino matter oscillations. These oscillations depend upon two parameters: the brane-bulk coupling ξ and the effective mass μ of the flavor neutrino inside matter. With a 1 km^3 neutrino telescope, extra dimensions with a radius down to 1 μm can be tested directly. An axion particle can live also in extra dimensions. We consider the photon axion mixing and analyze the eigenvalues and eigenstates of the mixing matrix. A resonance condition for the total conversion of a high energy photon into a Kaluza-Klein (KK) axion state is established. This resonant transition may provide a plausible explanation for the transparency of the universe to energetic photons. If the brane we live in is curved, then there are shortcuts through the bulk, which the axion can take. We suggest that such axionic shortcuts are at the root of the dispersion of time arrival of photons.

1 Introduction

The standard model (SM) of strong and electroweak interactions has been extremely successful. It provides a consistent theoretical framework within which we can analyze and understand all available experimental data. Still we know that it cannot be regarded as the final theory. Any attempt to include quantum gravity leads to a unified theory where two disparate scales coexist: the electroweak scale ($M_W \sim 1$ TeV) and the Planck scale ($M_{Pl} \sim 10^{19}$ GeV). Quantum radiative corrections then, especially in the scalar sector (Higgs field) of the theory, tend to mix the scales and, without an incredible amount of fine-tuning, will always equalize them (the hierarchy problem). A novel approach has been suggested to alleviate the hierarchy problem [1]. Our four-dimensional world is embedded in a higher dimensional space with D dimensions ($D = 4+n$). While the SM fields are constrained to live on the 4-dimensional brane, gravity can freely propagate in the higher-dimensional space (bulk). The fundamental scale M_f of gravity in D dimensions is related to the observed 4-dimensional Planck scale M_{Pl} by

$$M_{Pl}^2 = M_f^{2+n} V_n, \tag{1}$$

where V_n is the volume of the extra space. For a torus configuration

$$V_n = (2\pi)^n R_1 R_{2...} R_n, \tag{2}$$

with R_i $(i = 1, 2, ...n)$ the radii of extra dimensions. Then for a sufficiently large volume V_n the fundamental scale of gravity M_f can become as low as M_W . In this radical way the hierarchy problem ceases to exist as such.

Besides the graviton, fields which are standard-model singlets, like a sterile neutrino [2-4] or an axion [5-7] can freely propagate in the bulk. These particles accrue then an infinite tower of Kaluza-Klein (KK) excitations and the issue of mass eigenstates and mixing has to be revisited.

In the next part we consider how the Yukawa coupling of the standard lepton doublet, the Higgs scalar and the right-handed bulk neutrino, will provide a mixing between the left handed neutrino of the standard model and the KK modes. We focus our attention on neutrino oscillations, inside matter. Compared to the usual oscillations, novel features appear since now we have a coupled system of infinite degrees of freedom. The pattern of oscillations depends upon two parameters: the coupling ξ between the left-handed neutrino and the KK states and the effective mass μ of the flavor neutrino. We study in detail this dependence.

In the third part we consider the photon-axion mixing, induced by a magnetic field. Compared to the usual oscillations, novel features appear linked to the presence of the new scale, the radius of the extra dimension (or its inverse, the mass of the Kaluza-Klein excitation). We study in detail the eigenvalues and eigenstates of the mixing matrix and establish the resonance condition for the total conversion of a high energy photon into a KK axion state. We examine also the astrophysical implications of our formalism, notably the production and propagation of photons in sites of strong magnetic fields (neutron stars, GRBs). A photon-generated KK axion can take a “shortcut” through the bulk and appear earlier, compared to a photon traveling along a geodesic on the brane. We quantify this effect and analyze its relevance for the timing of photons observed by the MAGIC telescope during an activity of Markarian 501. At the end we present our conclusions.

2 Sterile neutrino in large extra dimensions

There is already an extensive literature on neutrinos living in extra dimensions [8–20]. We start by considering the action for a massless bulk fermion Ψ living in 5 dimensions

$$S_\Psi = \int d^4xdy\bar{\Psi}\Gamma^\mu\partial_\mu\Psi, \qquad M = 0, 1 \dots 5. \tag{3}$$

Ψ is decomposed as

$$\Psi = \begin{pmatrix} N_R \\ N_L \end{pmatrix}, \tag{4}$$

with each component admitting a Fourier expansion

$$N(x, y) = \frac{1}{\sqrt{2\pi R}} N_0(x) + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left(N_n(x) \cos\left(\frac{ny}{R}\right) + \hat{N}_n(x) \sin\left(\frac{ny}{R}\right) \right). \tag{5}$$

Thus at the brane the bulk fermion appears as a KK tower. Since the momentum in the extra compact dimension is quantized, each KK mode appears as having a mass n/R $(n = 1, 2, \dots)$. Consider now the coupling of the standard left-handed lepton doublet to the bulk neutrino

$$\frac{h}{\sqrt{M_f}} \bar{L} H N_R \delta(y), \tag{6}$$

where H is the Higgs scalar doublet and h a dimensionless Yukawa coupling. After the Higgs field develops a vacuum expectation value (v), we get the mass terms

$$m\bar{\nu}_L\left(N_{R0}+\sqrt{2}\sum_{n=1}^{\infty}N_{Rn}\right), \tag{7}$$

with $m \sim hvM_f/M_{Pl}$. Notice that for $M_f \sim 1$ TeV, $h \sim 1$, we obtain $m \sim 10^{-4}$ eV [8-13]. Also, the left-handed zero mode N_{L0} decouples from the spectrum and remains massless. Denoting

$$\Psi_R = \begin{pmatrix} N_{R0} \\ N_{Ri} \end{pmatrix}, \qquad \Psi_L = \begin{pmatrix} \nu_L \\ N_{Li} \end{pmatrix}, \quad i = 1, 2, \ldots, \tag{8}$$

the mass term is $\bar{\Psi}_L M \Psi_R$ with the mass matrix M given by [10]

$$M = \begin{pmatrix} m & \sqrt{2}m & \sqrt{2}m & \cdots \\ 0 & 1/R & 0 & \cdots \\ 0 & 0 & 2/R & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{9}$$

The evolution of the neutrino states is determined by the Hamiltonian

$$H = \frac{1}{2E_\nu}MM^T. \tag{10}$$

We define $\xi = mR$. For a muon neutrino traveling inside matter the effective mass takes the form $\mu = \sqrt{2}E_v R^2 G_F N_n$ with N_n the neutron density. The eigenvalues λ_n^2 of the mass matrix satisfy the equation

$$\left[\mu - \lambda^2 + \xi^2\left(\lambda\pi\right)\cot\left(\lambda\pi\right)\right]\prod_{n=1}^{\infty}\left(n^2-\lambda^2\right) = 0. \tag{11}$$

A resonance occurs whenever the condition is satisfied

$$\mu_R = \lambda_n^2 \tag{12}$$

For $n = 1$, the condition becomes [20]

$$\left(\frac{\rho}{10\frac{g}{cm^3}}\right)\left(\frac{E_\nu}{100\,GeV}\right)\left(\frac{R}{1\,\mu m}\right) \simeq 1, \tag{13}$$

with ρ the density of the medium, E_ν the neutrino energy and R the radius of the large extra dimension.

Notice at figure 1, that an impressive resonance structure appears and by reading the energy and using Equ.(13) we can extract the value of the radius R.

3 Axion in large extra dimensions

Pecci-Quinn (PC) solution to the strong CP problem in QCD [7], predicts the existence of a neutral, spin-zero pseudoscalar particle, the axion. Axions and photons oscillate into each other in an external magnetic field [21, 22] due to the interaction term

$$\mathcal{L}_{int} = \frac{1}{f_{PQ}}\alpha F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{4}{f_{PQ}}\alpha\vec{E}\cdot\vec{B} \tag{14}$$

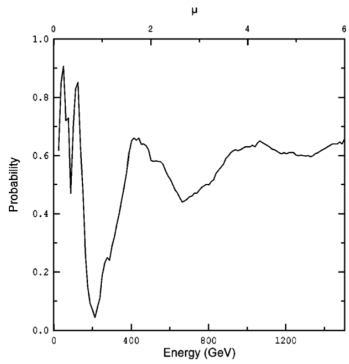

Figure 1: The probability $P\left(\nu_\mu \rightarrow \nu_\mu\right)$ for a neutrino traversing 10649 Km through the mantle of the Earth (with density $\rho \sim 4.4\, g/cm^3$), as a function of energy.

where $F_{\mu\nu}$ is the electromagnetic field tensor, $\tilde{F}_{\mu\nu}$ is its dual, α is the axion field. The mass m_{PQ} of the standard axion, as well as the axion-photon coupling is inversely proportional to the scale f_{PQ}. For a recent account of the axion searches see ref [23].

To simplify the calculation for the higher-dimensional case, we consider one extra compact dimension y and a singlet axion field $\alpha(x^\mu, y)$. Projected into the brane the axion field will appear as a collection of KK modes $a_n(x^\mu)$, each having a mass $m_n = \frac{n}{R}$, where R is the compactification radius. The coupling of the KK axions to the photon is universal [5, 6, 24, 25]

$$\mathcal{L}_{int} = \frac{1}{f_{PQ}} \sum_n \alpha_n F_{\mu\nu} \tilde{F}_{\mu\nu} \tag{15}$$

The mixing matrix M between the photon state $A_{||}$ parallel to the magnetic field B, the standard PQ axion α_0 and the KK axions α_n is [6]

$$M = \begin{pmatrix} \Delta_\gamma & \Delta_B & \Delta_B & \cdots & \Delta_B \\ \Delta_B & \Delta_0 & 0 & \cdots & 0 \\ \Delta_B & 0 & \Delta_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta_B & 0 & 0 & & \Delta_N \end{pmatrix} \tag{16}$$

where

$$\Delta_\gamma = \frac{\omega_{pl}^2}{2E}, \quad \Delta_0 = \frac{m_{PQ}^2}{2E}, \quad \Delta_n = \frac{n^2}{2ER^2}, \quad \Delta_B = \frac{4B}{f_{PQ}} \tag{17}$$

The plasma frequancy is $\omega_{pl}^2 = (4\pi\alpha n_e)/m_e$ for an electron density n_e.

The next step is to establish the eigenvalues and the eigenvectors of the mixing matrix M. The resonant condition for the transition of the photon to the first KK axion (the most dominant transition) takes the form, in actual units [27]

$$\left(\frac{E}{500\, GeV}\right)\left(\frac{R}{10^{-3}\, cm}\right)^2\left(\frac{B}{10^7 G}\right)\left(\frac{10^{12}\, GeV}{f_{PQ}}\right) = 1.0 \tag{18}$$

Within this scheme, photons of high energy produced in an active nucleus are transformed through an MSW type resonance into KK axions, which travel unimpeded before being reconverted back into photons. For photons not satisfying the resonance condition, the probability to transit as KK axions is reduced and therefore these photons are limited by the opacity of two photon annihilation into an electro-positron pair. Our approach offers a plausible explanation for the transparency of the universe to energetic photons [26]. Photons of the appropriate energy, are transformed into KK axions, travel freely in the bulk space, before returning back into the brane and observed again as photons [27]. The same mechanism may provide also high energy photons escaping the GZK cutoff [28].

The advent of imaging atmospheric Cerenkov telescope like HESS, MAGIC, VERITAS, FERMI, CTA, allows the detection of photons from astrophysical sources (neutron stars, GRB, AGN) in the high energy window from 100 GeV to few TeV .The MAGIC telescope analyzed the timing of photons originating from the Mkn 501 source [29]. It was found that the photons in the $0.25-0.6\,TeV$ energy range precede by 4 minutes the photons in the $1.2-10\,TeV$ energy band. The observed features might be possible to be explained within an astrophysical context [30]. A particle physics solution, within the framework of quantum gravity [31, 32] has been suggested also. Quantum fluctuations of space-time lead to a speed of light dependent upon energy, thus creating a dispersion in the time arrival of photons [33].

The transition of a photon to a KK axion, offers another alternative to analyze the observed dispersion in time arrival of high energy photons [27]. A particle, travelling from a point on the brane to another point on the brane, may take a "shortcut" by following a geodesic in the bulk and arriving earlier compared to a particle which follows a geodesic on the brane [34]. In our case a photon, transformed into a KK axion traveling through the bulk, may reappear earlier on the brane, compared to a photon stuck in the brane. There are a number of ways shortcuts emerge in theories with extra dimensions. In a brane containing matter and energy, self-gravity will induce a curvature to the brane, so that the brane becomes concave towards the bulk in the null direction. Then we can find geodesics in the bulk propagating signals faster compared to the geodesics in the brane [34]. A phenomenology, with a sterile neutrino shortcuting through the bulk has been developed [35, 36], accounting for all neutrino oscillation data. A $1+1$ dimensional toy model may exhibit the expected behavior [35]. In a Minkowski metric

$$ds^2 = dt^2 - dx_1^2 - dx_2^2 \tag{19}$$

the curved brane is represented by

$$x_2 = A\sin(kx_1) \tag{20}$$

while the bulk geodesic is given by $x_2 = 0$. A signal transmitted through the bulk will appear as having a superluminal speed. Equivalently a difference in time arrival will be observed given by

$$\frac{t_\gamma - t_\alpha}{t_\gamma} \simeq \left(\frac{Ak}{2}\right)^2 \tag{21}$$

where t_γ is the time it takes for the photon in the brane and t_α is the corresponding time for the photon which uses the axionic shortcut. Within our model, the photons having the resonance energy, eq.(18), use the axionic shortcut and arrive earlier compared to the other photons of different energies which propagate along the brane. From the MAGIC experimental data we infer that the $0.25-0.6\,TeV$ photon energy brackets the aforementioned resonance energy. The amount of the time difference is determined by the brane shape-parameter Ak and the rauge of the magnetic field B. The magnetic field near the core of a GRB or a blazar reaches high values. We adopt the average value of $B = 10^7\,G$. By appealing to the Hillas criterion [37] we obtain a range for the magnetic field of the order of $10^9\,km$. Then the set of values $R = 10^{-3}\,cm$, $f_{PQ} = 10^{12}\,GeV$, $B = 10^7\,G$, $E = 500\,GeV$, $\Delta t = 4\,min$, implies for the shape parameter the value $Ak \simeq 1$ [27]. Monitoring the astrophysical sites, where high energy photons are produced in the presence of strong magnetic fields, might be revealing. We might observe the disappearance of high energy photons connected to the photon-KK axion transition. Or, we might notice the sudden appearance of high energy photons, connected to the intrusion of KK axions from the bulk into the brane.

4 Conclusions

We analyzed the situation where the left-handed muon neutrino of the standard model mixes with a right-handed neutrino experiencing a large extra space dimension. This mixing induces neutrino oscillations, even if the neutrino itself is massless. We encountered novel features compared to the standard oscillations, since the mixing involves an infinity of KK states. A rich resonance structure appears accompanied by many spikes. We emphasized that the 1 km^3 neutrino telescope may be used as an instrument to explore the bulk space and establish unambiguously the existence of extra dimensions with $R > 1\,\mu m$.

We studied also the photon-axion oscillation in the presence of extra dimensions. Next to the standard scales, a new scale is introduced, the size R of the extra dimension, or its inverse the KK mass excitation. An MSW-type resonance occurs at high energies between the photon and the KK axion. This resonance transition $A\,(\gamma \to \alpha \to \gamma)$ allows the photon to travel unimpeded from a dense and opaque medium, offering the ground for a rich phenomenology. It is highly plausible that our brane is curved creating the possibility of shortcut through the bulk. We suggested that an axionic shortcut may be at the origin of the dispersion in the time arrival of the photons observed by the MAGIC telescope.

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