On a four dimensional formulation for dimensionally regulated amplitudes

A.R. Fazio

Departamento de Fisíca, Universidad Nacional de Colombia
Ciudad Universitaria, Bogotá, D.C. Colombia

DOI: http://dx.doi.org/10.3204/DESY-PROC-2014-04/149

We propose a pure four-dimensional formulation (FDF) of the d-dimensional regularization of one-loop scattering amplitudes. In our formulation particles propagating inside the loop are represented by massive internal states regulating the divergences. We present explicit representations of the polarization and helicity states of the four-dimensional particles propagating in the loop. They allow for a complete, four-dimensional, unitarity-based construction of d-dimensional amplitudes. Finally we show how the FDF allows for the recursive construction of d dimensional one-loop integrands, generalizing the four-dimensional open-loop approach.

1 Introduction

The recent development of novel methods for computing one-loop scattering amplitudes in gauge field theory has been highly stimulated by a deeper understanding of their kinematics enforced by on-shellness [1][2] and generalized unitarity [3][4]. Analyticity and unitarity of scattering amplitudes have then been strengthened by the complementary classification of the mathematical structures present in the residues of singular points.

The use of unitarity cuts and complex momenta for on-shell internal particles turned unitarity based methods into very efficient tools for computing scattering amplitudes. These methods exploit two general properties of scattering amplitudes such as unitarity and analyticity: the former granting that amplitudes can be reconstructed from the knowledge of their generalized singularity structures; the latter granting that the residues at singular points factorize in the product of simpler amplitudes [5][6][7].

However one-loop scattering amplitudes arising from a dimensionally regulated theory are the sum of one part containing polylogarithms, the so called "cut constructible" part, and the rational part, which is a rational function of the external spinors and polarizations. Contrarily to the cut-constructible, the rational part cannot be detected in four dimensions.

Based on the paper [8] this talk addresses the possibility of fully reconstructing a one loop amplitude in its cut-constructible and rational part in quantum chromodynamics (QCD) by just gluing tree level amplitudes. Such trees will be obtained by extending the definition of the helicity eigenstates entering the state sum in the propagators of quark and gluons, whithout leaving the four space-time dimensions.

This point of view combines the generalized unitarity cuts in $d = 4 - 2\epsilon$ dimensions with the Four Dimensional Helicity Scheme (*FDH*). The d-dimensional unitarity cuts detect also the rational part by generalized on-shell conditions and generalized residues [9]. The former imply the vanishing of massive denominators, where the mass term depends both on the physical mass (vanishing or not) of the particle across the cut and on the effective mass parameter encoding the extra-dimensional dependence. Generalized residues computed on the cuts are generated by tree level amplitudes, which depend on the effective mass parameter, hence from the extra dimensions regulating the integrals, either from the generalized polarizations vectors associated to the cut particles, or from the extended algebra of the metric tensor and of the Dirac matrices in the definition of the Feynman rules.

In order to compute the constituting blocks of tree level amplitudes by using the helicity spinor formalism the *FDH* scheme will be used [10], in which the external particles are described by four dimensional Lorentz labels (momenta and helicities) and the internal particles (the so called "unobserved") have still the same numbers of helicity states like in four dimensions. In *FDH* scheme the momenta of the unobserved particles are kept in d dimensions as well as the metric tensor and the Dirac matrices, therefore in diagrammatic computations the algebraic manipulations are implemented by separating the four dimensional algebra from the extra-dimensional one.

In this talk we show that dimensionally regularized one-loop QCD amplitudes in *FDH* scheme can be simply calculated by generalizing the helicity eigenstates of the unobserved particles, by including an effective mass parameter in a pure four-dimensional formalism. The generalized four dimensional polarizations and propagators should be used for tree level and one loop computations avoiding any special decomposition of the particle running around the loop. We want to demonstrate that by an appropriate generalization of the cutted internal legs no supersymmetric decomposition [11] will be needed neither the introduction of new particles and new interactions [12] to afford separately the computation of the cut constructible and the rational part of a scattering amplitude.

2 Generalized internal legs

In this section we are going to provide the explicit expression of the cut legs of a one-loop amplitude involving fermion or vector particles in the loop. Those wave functions will be needed to compute the tree amplitudes to be merged in the reconstruction of the S-matrix elements by unitarity. Their dynamics is described by a pure four dimensional quantum field theory dual to the dimensionally regularised one. The following explicit construction of generalized spinors for fermions and polarization vectors for gluons is suitable for a numerical implementation of such an on-shell procedure of computation. In the following discussion we will decompose a d-dimensional momentum $\bar{\ell}$ as follows

$$\bar{\ell}^\alpha = \ell^\alpha + \mu^\alpha \qquad \bar{\ell}^2 = \ell^2 - \mu^2 = m^2, \tag{1}$$

while its four-dimensional component ℓ will be expressed in terms of the massless momenta $\ell^\flat$ and q_ℓ as

$$\ell = \ell^\flat + \hat{q}_\ell\,, \qquad \hat{q}_\ell \equiv \frac{m^2 + \mu^2}{2\,\ell \cdot q_\ell} q_\ell\,. \tag{2}$$

Spinors – The legs of the cut fermion propagtors in the loop have to fulfill the following completness relation [8]

$$\sum_{\lambda=\pm} u_\lambda(\ell)\,\bar{u}_\lambda(\ell) = \ell\!\!\!/ + i\mu\gamma^5 + m \tag{3}$$

which is satisfied by the following four dimensional spinors

$$u_+(\ell) = \left|\ell^\flat\right\rangle - \frac{(m - i\mu)}{\left[\ell^\flat\, q_\ell\right]} \left|q_\ell\right], \quad u_-(\ell) = \left|\ell^\flat\right] - \frac{(m + i\mu)}{\left\langle\ell^\flat\, q_\ell\right\rangle} \left|q_\ell\right\rangle. \tag{4}$$

Polarization vectors – In the light-cone gauge the d-dimensional polarization vectors fulfill the following relation

$$\sum_{i=1}^{d-2} \varepsilon^{\alpha}_{i\,(d)}\left(\bar{\ell},\bar{\eta}\right)\varepsilon^{*\beta}_{i\,(d)}\left(\bar{\ell},\bar{\eta}\right) = -\bar{g}^{\alpha\beta} + \frac{\bar{\ell}^\alpha\ \bar{\eta}^\beta + \bar{\ell}^\alpha\,\bar{\eta}^\beta}{\bar{\ell}\cdot\bar{\eta}} - \frac{\bar{\eta}^2\bar{\ell}^\alpha\bar{\ell}^\beta}{(\bar{\eta}\cdot\bar{\ell})^2}, \tag{5}$$

where $\bar{\eta}$ is an arbitrary d-dimensional momentum such that $\bar{\ell}\cdot\bar{\eta} \neq 0$. Gauge invariance in d dimensions guarantees that the unitarity cuts are independent of $\bar{\eta}$. Assuming a four dimensional description we can take the vector μ fixed. The choice $\bar{\eta}^\alpha = \mu^\alpha$ allows for disentangling the four-dimensional contribution from the d-dimensional one:

$$\sum_{i=1}^{d-2} \varepsilon^{\alpha}_{i\,(d)}\left(\bar{\ell},\bar{\eta}\right)\varepsilon^{*\beta}_{i\,(d)}\left(\bar{\ell},\bar{\eta}\right) \;=\; \left(-g^{\alpha\beta} + \frac{\ell^\alpha\ell^\beta}{\mu^2}\right) - \left(\tilde{g}^{\mu\nu} + \frac{\mu^\alpha\mu^\beta}{\mu^2}\right). \tag{6}$$

The first term is related to the cut propagator of a massive gluon whose polarization vectors are

$$\varepsilon^\mu_+(\ell) = -\frac{\left[\ell^\flat\left|\gamma^\mu\right|\hat{q}_\ell\right\rangle}{\sqrt{2}\mu} \quad \varepsilon^\mu_-(\ell) = -\frac{\left\langle\ell^\flat\left|\gamma^\mu\right|\hat{q}_\ell\right]}{\sqrt{2}\mu} \quad \varepsilon^\mu_0(\ell) = \frac{\ell^{\flat\mu} - \hat{q}^\mu_\ell}{\mu}. \tag{7}$$

3 Open loop

The FDF of d-dimensional one-loop amplitudes is compatible with methods generating recursively the integrands of one-loop amplitudes and leads to the complete reconstruction of the numerator of Feynman integrands as a polynomial in ℓ^ν and μ. Our scheme allows for a generalization of the current implementations of these techniques, reconstructing only the four-dimensional part of the numerator of the integrands, which is polynomial in ℓ. In the following we describe how the open-loop technique [13] has to be generalized within the FDF scheme. The subtrees $w^\beta(i)$ recursively merged by connecting their cut lines to vertices and propagators have the following form

$$w^\beta(i) = \frac{X^\beta_{\gamma\delta}(i,j,k)w^\gamma(j)w^\delta(k)}{p_i^2 - m_i^2 + i\epsilon}, \tag{8}$$

where $\frac{X^\beta_{\gamma\delta}}{p_i^2 - m_i^2 + i\epsilon}$ describes a vertex connecting i, j, k to a propagator attached to i. For one loop amplitudes the numerators of Feynman integrals can be computed by tree-level techniques. For the open loop with indices α and β where a single propagator has been cutted and the denominator stripped out, the numerator of the amplitude's integrand satisfies the recursive relation

$$\mathcal{N}^{\beta}{}_{\alpha}\left(\mathcal{I}_n, \ell, \mu\right) = X^{\beta}_{\gamma\delta}\left(\mathcal{I}_n, i_n, \mathcal{I}_{n-1}\right) \mathcal{N}^{\gamma}{}_{\alpha}\left(\mathcal{I}_{n-1}, \ell, \mu\right) w^{\delta}\left(i_n\right) \tag{9}$$

where w^{δ} is the expression related to the tree-level topology i_n. To achieve the Feynman diagrams expressions in FDH scheme by our FDF formulation the vertices $X^{\beta}_{\gamma\delta}$ are obtained by the Feynman rules in [8]

$$X^{\beta}_{\gamma\delta} = Y^{\beta}_{\gamma\delta} + \ell^{\nu}\, Z^{\beta}_{\nu;\,\gamma\delta} + \mu\, W^{\beta}_{\gamma\delta}\,. \tag{10}$$

Therefore the tensor coefficients of the covariant decomposition of the numerator in a given topology are obtained by the recursive relation

$$\mathcal{N}^{[a]\,\beta}_{\nu_1\cdots\nu_j;\,\alpha}\left(\mathcal{I}_n\right) = \left[Y^{\beta}_{\gamma\delta}\,\mathcal{N}^{[a]\,\gamma}_{\nu_1\cdots\nu_j;\,\alpha}\left(\mathcal{I}_{n-1}\right) + Z^{\beta}_{\nu_1;\,\gamma\delta}\,\mathcal{N}^{[a]\,\gamma}_{\nu_2\cdots\nu_j;\,\alpha}\left(\mathcal{I}_{n-1}\right) + W^{\beta}_{\gamma\delta}\,\mathcal{N}^{[a-1]\,\gamma}_{\nu_1\cdots\nu_j;\,\alpha}\left(\mathcal{I}_{n-1}\right)\right] w^{\delta}(i_n).$$

The recursive generation of integrands within the FDF can be suitably combined with public codes like Samurai and Ninja.

4 Conclusions and outlook

A four-dimensional formulation (FDF) of dimensional regularization of one-loop scattering amplitudes has been applied to generalized unitarity techniques. At one loop the cut-constructible part and the rational part of scattering amplitudes have been computed by the same on-shell methods. The inclusion of the fermion mass and the two loop case will be analysed elsewhere.

References

[1] F.Cachazo, P. Svrcek and E. Witten, JHEP 09 006 (2004).

[2] R. Britto, F. Cachazo, B. Feng, Nucl. Phys. B715 499 (2005).

[3] Z. Bern, L.J. Dixon, D.C. Dunbar, D.A. Kosower, Nucl. Phys. B425 217 (1994).

[4] R. Britto, F. Cachazo, B. Feng, Nucl. Phys. B725 275 (2005).

[5] C. F. Berger and D. Forde, Ann.Rev.Nucl.Part.Sci. 60,181 (2010).

[6] R. Britto, J.Phys. A44, 454006 (2011), 34 pages.

[7] R. Ellis, Z. Kunszt, K. Melnikov, and G. Zanderighi, Phys.Rept. 518 (2012) 141-250 (2011).

[8] A.R. Fazio, P. Mastrolia, E. Mirabella, W.J. Torres, arXiv:14044783 [hep-ph].

[9] Z. Bern and A. G. Morgan, Nucl. Phys., B467 479 (1996).

[10] A. Signer, D. Stockinger, Nucl.Phys. B808 88 (2009).

[11] A. Brandhuber, S. McNamara, B. J. Spence, G. Travaglini, JHEP 0510 11 (2005).

[12] R. Pittau, JHEP 1202 29 (2012).

[13] F. Cascioli, P. Maierhofer, S. Pozzorini, Phys. Rev. Lett. 108 111601 (2012).