Why does black hole describe the deconfinement phase?

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In the gauge/gravity duality, the deconfinement transition in the gauge theory is identified with the formation of black hole in the dual gravity theory. By assuming this correspondence, many predictions on QGP have been made. In this talk, we justify this approach quantitatively, and also provide an intuitive understanding. Firstly we give quantitative evidence for this identification from the thermodynamic study of the supersymmetric theory. We show that string theory and gauge theory give the same answer, even at finite temperature, including the $1/N$ correction. Then we consider generic gauge theories and show that the deconfinement transition is the condensation of very long and self-intersecting QCD strings, which is analogous to the formation of a black hole in string theory.

1 Introduction

In the gauge/gravity duality conjecture [1], the deconfinement phase of the gauge theory is dual to a black hole geometry in the gravity side [2]. In the first part of this talk, we give quantitative evidence for this identification, by solving a concrete example in both gauge and gravity sides, at the level of the string theory (i.e. finite α' and finite g_s). In the second part, we give an intuitive way of understanding this correspondence, without referring to a sophisticated dictionary of the duality. Our argument does not assume the dual gravity description, and hence it is applicable to generic gauge theories including QCD.

2 Quantitative test of the gauge/gravity duality

Let us consider the $U(N)$ maximally supersymmetric Yang-Mills theory in flat (p+1)-dimensional spacetime ($p = 0, 1, 2$ and 3), whose action is given by

$$S = \frac{1}{g_{YM}^2}\int d^{p+1}x\ Tr\left\{\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(D_\mu X_i\right)^2 - \frac{1}{4}[X_i, X_j]^2\right\} + (fermions), \tag{1}$$

where μ, ν run from 1 to $p+1$ and X_i ($i = 1, 2, \cdots, 9-p$) are scalar fields. In [3], it has been conjectured that this theory is in the deconfining phase at any nonzero temperature, in the sense that the Polyakov loop has nonzero expectation value, and that it describes full string dynamics around the black p-brane. Near the horizon, the metric of the black p-brane geometry is given by

$$ds^2 = \alpha' \Bigg\{ \frac{U^{(7-p)/2}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + dy_{\parallel}^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{(7-p)/2}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N} U^{(p-3)/2} d\Omega_{8-p}^2 \Bigg\}, \qquad (2)$$

where the Yang-Mills coupling g_{YM} and the size of the gauge group N in the corresponding super Yang-Mills theory are used. A constant α' is the square of the string length, $(t, y_{\parallel})$ represent the $(p+1)$-dimensional extension of the brane, U and Ω are the radial and angular coordinate of the transverse directions, and U_0 is the place of the horizon. The Hawking temperature is

$$T_H = \frac{(7-p)U_0^{(5-p)/2}}{4\pi\sqrt{d_p g_{YM}^2 N}}, \qquad (3)$$

where $d_p = 2^{7-2p}\pi^{(9-3p)/2}\Gamma((7-p)/2)$. The string coupling constant is given by

$$g_s = (2\pi)^{2-p} g_{YM}^2 \left(\frac{d_p g_{YM}^2 N}{U^{7-p}} \right)^{\frac{3-p}{4}}. \qquad (4)$$

When $\lambda = g_{YM}^2 N$ is fixed, it behaves as $g_s \propto 1/N$, in the same way as in 't Hooft's identification. The Hawking temperature and the mass of the black brane are identified with the temperature and the energy $E \equiv -\partial \log Z / \partial \beta$ of the gauge theory, where Z is the partition function.

In this work we study the case of $p = 0$. The gauge theory is quantum mechanics of $N \times N$ matrices, which was originally proposed as the matrix model of M-theory [4]. It can be studied extensively by using the Monte Carlo method. In the gravity side, the black hole mass can be calculated by adding stringy corrections to the black 0-brane geometry shown above. The result is [5]

$$\frac{1}{N^2} E_{gravity} = (7.41\, T^{2.8} + a\, T^{4.6} + \cdots) + (-5.77\, T^{0.4} + b\, T^{2.2} + \cdots)\frac{1}{N^2} + O\left(\frac{1}{N^4}\right), \qquad (5)$$

where $T = \lambda^{-1/3} T_H$ is dimensionless effective temperature and a, b are unknown constants. The energy $E_{gravity}$ is also made dimensionless, by multiplying $\lambda^{-1/3}$. In the following, we simply set $\lambda = 1$ without loss of generality. Higher order terms in each power of $1/N$ represent to the α' corrections, which appear because strings are not point-like. If the gauge/gravity duality conjecture is correct, this expression must be reproduced from the matrix quantum mechanics.

The $O(N^0)$ terms of the dual gravity prediction (5) have been tested previously. In particular, the α' correction $a\, T^{4.6}$ has been confirmed with $a = -5.58(1)$, by looking at $T \gtrsim 0.5$ [7].

In order to study the $1/N$ correction, we must study rather small values of N. Here we study $N = 3, 4$ and 5. (That we have to take N small caused a technical problem which required a proper treatment; see [8].) We also have to study low temperature where the α' correction becomes small, because there are too many fitting parameters otherwise. At $0.08 \le T \le 0.11$, we estimated the coefficient of $1/N^2$ by using a fitting ansatz $E_{gauge} = 7.41T^{2.8} + c_1(T)/N^2 + c_2(T)/N^4$ for each fixed value of T. We confirmed that $c_1(T)$ is consistent with $-5.77T^{0.4}$. This is very strong evidence that the gauge/gravity duality holds at quantum gravity level.

3 Why the deconfinement phase describes the black hole

In this section, motivated by the numerical confirmation in Sec. 2, we give an intuitive explanation for the duality based on [6]. As a concrete example, let us consider pure $U(N)$ Yang-Mills theory. The Hilbert space is spanned by Wilson loops acting on the vacuum $|0\rangle$, as $W_C W_{C'} \cdots |0\rangle$. Here W_C represent the Wilson loop along a closed contour C. In the standard identification of the Feynman diagrams and the string world-sheet [9], the Wilson loop is naturally interpreted as the creation operator of the string.

In the large-N limit and at sufficiently strong coupling, the energy of the string is approximately proportional to its length. In the confinement phase, the energy is of order N^0 per unit volume, and hence a typical state is a finite-density gas of loops with finite length. In this gas, two loops can intersect with each other and combined to form a longer string. Alternatively, when a loop intersects with itself, it can be split into two shorter loops. However such joining and splitting are suppressed at large-N.

In the deconfinement phase, the energy density is of order N^2. In this phase the loops necessarily intersect $O(N^2)$ times. Although the interaction at each intersection is $1/N$-suppressed, small interactions at many intersections accumulate to a non-negligible amount. A standard entropic argument shows that typical state consists of finitely many very long and self-intersecting strings, whose lengths are of $O(N^2)$. In the string theory, it is natural to interpret such very long and self-intersecting strings as a black hole [10, 11]. In this sense, *the deconfinement transition can be understood as the formation of a 'black hole' through condensation of QCD strings.*

When we identify the long string with a black hole, fluctuations of the string near the horizon are regarded as open strings attached to black hole (Fig. 1). In terms of the gauge theory, these open strings are open Wilson lines which have N color degrees of freedom at their endpoints. Therefore, we can interpret that the black hole is made from N D-branes.

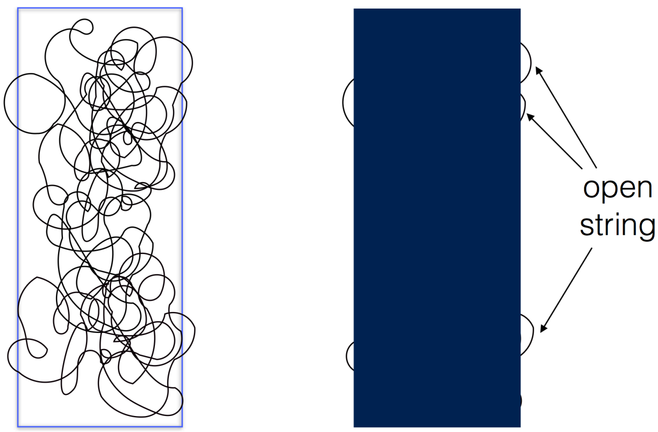

Figure 1: Closed string picture (left) and open string picture (right) [6].

In the Euclidean theory, the deconfinement is characterized by the condensation of the Polyakov loop, which can naturally be related to the black hole geometry in the gravity dual [2]. This can be explained from our picture as follows. First notice that the condensation of the

Polyakov loop is equivalent to the disappearance of the linear confinement potential between a pair of probe quark and anti-quark. In terms of strings, the linear potential in the confining phase appears because an open string connecting probes must be stretched as they are separated. In the deconfining phase, however, as soon as a short open string is introduced, it intersects with closed strings many times, and they immediately interact at one of the intersections to form a long open string (Fig. 2). Therefore, probes can be separated without stretching the open string, or equivalently, without costing energy. Note that the interaction is crucial for this argument; the deconfining phase cannot be described by free strings à la Hagedorn.

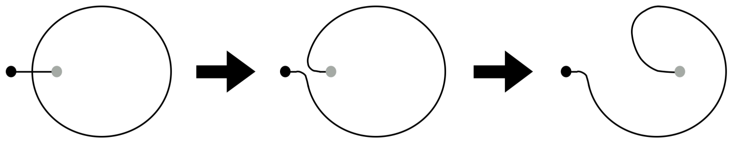

Figure 2: Deconfinement of a pair of probe quark and anti-quark [6].

In order to test this picture, we have estimated the deconfinment temperature of the lattice gauge theory with spatial lattice and continuum time [12]. We performed numerical simulation and confirmed the prediction.

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