

Statistical criteria for possible indications of new physics in tritium β -decay spectrum

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The method of quasi-optimal weights is applied to constructing (quasi-)optimal criteria for various anomalous contributions in experimental spectra. Anomalies in the spectra could indicate physics beyond the Standard Model (additional interactions and neutrino flavours, Lorenz violation etc.). In particular the cumulative tritium β -decay spectrum (for instance, in Troitsk- ν -mass, Mainz Neutrino Mass and KATRIN experiments) is analysed using the derived special criteria. Using the power functions we show that the derived quasi-optimal criteria are efficient statistical instruments for detecting the anomalous contributions in the spectra.

1 Introduction

Studying anomalies in experimental spectra extends our understanding of experimental setups. Besides anomalous contributions could also indicate new physics beyond the Standard Model. For instance, in tritium β -decay spectra possible additional interactions can lead to a step-like anomaly near the end-point while the existence of the fourth neutrino (with the mass of a few keV) induces a kink structure in the region of several keV from the end-point. Here we consider these two possible anomalous contributions.

The search for an anomaly should be based on a statistically reliable inference about presence or absence of the anomaly. Such inference is provided by special statistical criteria. One can construct the criteria according to each particular situation and accounting for some additional information about the theoretical model or experimental setup. The various approaches here are as follows:

1. Direct fit with additional parameters (the mass of neutrino and the mixing parameter, the amplitude and the position of the step) [1], [2], [3].
2. Searching for the kink with various filters [3].
3. Wavelet analysis [4].
4. Searching for special functional dependencies.
5. Constructing special statistical criteria for the heavy neutrino or the step accounting for the uncertainties of other parameters (based on the method of quasi-optimal weights [5]). In the paper we present two examples of construction of the special statistical tests.

2 Step-like anomaly in Troitsk- ν -mass spectrum

Any deviations at the very end of the spectrum have crucial influence on the estimations of the neutrino mass squared. The spectrum of the Troitsk- ν -mass experiment is cumulative. The numbers of electrons are measured for a set of energies (these numbers have Poisson distributions). After that the measured data points are fitted with the theoretical curve. There are four fitted parameters in the Troitsk- ν -mass spectrum. One of them is the neutrino mass squared.

The first data analysis [1], performed with the standard Minuit routines, yielded rather controversial result: the estimate of the neutrino mass squared lies far beyond physically relevant range, it appears to be large and negative. This was interpreted as due to an excess of electrons near the end-point energy of the tritium β -decay spectrum; in cumulative spectra, such an excess takes the form of a step. Such a step is described by two parameters, the height and the position. Including these into the fit, a satisfactory value for the neutrino mass squared was obtained.

The recently finished new analysis [2] (exploiting the method of the quasi-optimal weights [5] and improved theoretical model of the experimental setup as well), yielded physically relevant values (within errors) of the neutrino mass squared while the step-structure has not been accounted for. The goodness-of-fit test included into the fitting procedure is not tuned to feel the anomalous contributions of this step-like form. It will be nice to have convenient, robust statistical criteria, particularly targeted to the described anomaly. We also should take into account that the position of the step is unknown and even may vary in time. We have constructed three special criteria and with them one can perform the standard procedure of the test of hypotheses [6], [7]. The null-hypothesis is that the height of the step is zero, the alternative - the height is positive. We use the fit from the new analysis [2].

The first criterion is constructed via routines of the method of quasi-optimal moments. And it is by construction the Locally Most Powerful (LMP) one. Locally here means near the null-hypothesis. The distributions for the experimental counts are $f_i(N) = \frac{\mu_i^N e^{-\mu_i}}{N!}$, where $\mu_i \rightarrow \begin{cases} \mu_i + \Delta_-, & i > m \\ \mu_i + \Delta_+, & i \leq m \end{cases}$, i stands for the number of an experimental point. Here m is defined by the inequality $E_m \leq E_{st} \leq E_{m+1}$. Constructing the weights

$$\omega_i^+(N) = \frac{\partial \ln f_i}{\partial \Delta_+} = \begin{cases} 0, & i > m \\ \frac{N}{(\mu_i + \Delta_+)} - 1, & i \leq m \end{cases}, \omega_i^-(N) = \frac{\partial \ln f_i}{\partial \Delta_-} = \begin{cases} 0, & i \leq m \\ \frac{N}{(\mu_i + \Delta_-)} - 1, & i > m \end{cases}$$

and solving the corresponding equations $h^{exp} = \sum_i \omega_i = 0$ one obtains the statistics of the LMP

criterion $\bar{\Delta} = \Delta_+ - \Delta_-$ - the estimate for the height of the step. $\bar{\Delta}$ can be also presented as a weighted sum of experimental counts $\bar{\Delta} = \sum_{i=1}^M w_i \cdot N_i$

Recalling the uncertainty of the step position it is useful to decrease the sensitivity of our criteria to the position of the step, even loosing some sensitivity to the step itself. For this we slightly change the weights in the sum of the LMP test (see Fig. 1), to suppress the values near the position of the step, saving the properties of the LMP test in the rest areas. The

corresponding statistics is $S_{q-opt} = \sum_i^k w_i \cdot \xi_i$, where $\xi_i = \frac{N_i - \mu_i}{\sqrt{\mu_i}}$ and $w_i = \begin{cases} \frac{(m-i)}{m}, & i \leq m, \\ \frac{(m-i)}{M-m}, & i > m. \end{cases}$

One more criterion, $S_{pair} = \sum_i \xi_i \cdot \xi_{i+1}$, constructed somehow speculatively, exploits the following idea: if the anomaly is a deviation of several neighbour points to one side of the

fitting curve (Fig. 3) than it will increase the value of the statistics S . Thus the pairwise neighbours' correlations test can be used as a criterion for rather general class of anomalies.

We can compare all these criteria using the standard tool of Power Functions. The power function is simply the probability of a criterion to reject the null hypothesis while it is in fact false. As one can see on Fig. 4 the LMP (1) is the best here, the quasi-optimal (2) is slightly less powerful, the pairwise neighbours correlations test (3) comes third and the conventional tests (4,5) are the least sensitive. The situation changes if the assumed position of the step is not correct. The left graph in Fig. 5 shows that the LMP test (1) is losing its sensitivity rather rapidly while two other special tests remain rather powerful.

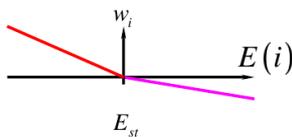


Figure 1: The quasioptimal weights for the step-like anomaly searches

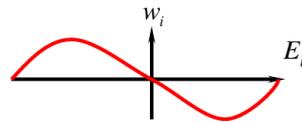


Figure 2: The quasioptimal weights for the heavy neutrino searches

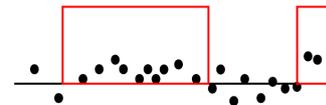


Figure 3: The idea of the pairwise neighbours correlations criterion

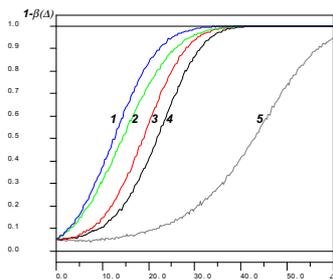


Figure 4: The power functions for the special and conventional criteria

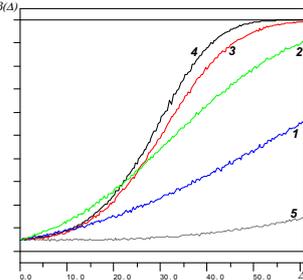
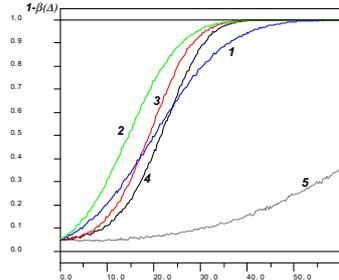


Figure 5: The power functions of the five criteria for the cases when the actual step position is shifted from the assumed position E_m by 12 eV (left) and 25 eV (right)

3 Heavy neutrino searches

Similarly to the case of step-like anomaly one can derive special criteria for the search of a heavy neutrino in β -decay spectrum

$$\frac{d\Gamma}{dE} = \sin^2 \theta \left(\frac{d\Gamma}{dE} \right)_{m_{keV}} + \cos^2 \theta \left(\frac{d\Gamma}{dE} \right)_{m_{light}} .$$

The spectrum $\frac{d\Gamma}{dE}(E_i, m) = S_{i,m}$ is again measured in a number of points with various retarding potentials. The spectrum is defined by the mixing parameter $U^2 = \sin^2 \theta$ and the mass of the

heavy neutrino m_{keV} . The mass of the light neutrino is considered to be $m_{light} = 0$. Thus the theoretical means are $\mu'_i = U^2 S_{i,m_H} + (1 - U^2) S_{i,m=0}$. The first test (the LMP) is obtained via the method of quasi-optimal weights and it is by construction the most sensitive one in case when the mass of the heavy neutrino is well-known. The corresponding weights are as follows:

$$\omega_i = \frac{\partial \ln f_i}{\partial U^2} = \left(\frac{N}{U^2 S_{i,m_H} + (1 - U^2) S_{i,m=0}} - 1 \right) \cdot (S_{i,m_H} + S_{i,0}).$$

Using these weights one constructs an equation $\frac{1}{M} \sum_{i=1}^M \omega_i = 0$ for the mixing parameter estimate $\widehat{U^2}$ – the statistics of the LMP criterion.

To reduce the sensitivity of our test to the mass we modify the weights (for instance, as shown in Fig. 2) to obtain the quasi-optimal criterion $S_{q-opt} = \sum_i^k w_i \cdot \xi_i$. It is more robust and require no information about the exact mass of the additional neutrino. The universal pairwise neighbours correlatiosn test can be exploited in the case of kink searches as well.

4 Conclusions

We illustrated the new approach to the search for anomalies in experimental spectra with account for the parameters with uncertainties (the position of the step-like anomaly and the mass of the additional neutrino in our examples). We showed that the Locally Most Powerful criterion for each anomalous contribution can be constructed via the method of quasi-optimal moments. Than the LMP test can be tuned to reduce the influence of the unknown parameters of the spectra. With the help of the power functions the constructed criteria are proved to be more efficient in searches for the specific anomalous contributions. The next step is to compare the sensitivity of the constructed tests with the wavelet analysis [4], direct fitting and search for the kink with filters [3]. The approach appears to be useful for the future searches of the heavy neutrino in Troitsk [8], [9] and Karlsruhe [3].

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