Detailed study of the $K_{e4}$ decay mode properties from the NA48/2 experiment at CERN

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The NA48/2 Collaboration at CERN has accumulated an unprecedented statistics of rare semileptonic four-body $K_{e4}$ decay modes $K_{\pm} \rightarrow \pi^{\pm} \pi^{\mp} e^{\pm} \nu$ and $K_{\pm} \rightarrow \pi^{0} \pi^{0} e^{\pm} \nu$, with nearly 1% background contamination. Final results from the analyses of the full $K_{e4}$ data samples are described. The most accurate measurements to date of form factors and branching ratios have been achieved by NA48/2, bringing new inputs to low energy QCD description and stringent tests of predictions from Chiral Perturbation Theory and Lattice QCD.

1 Introduction

The NA48/2 experiment at CERN has collected the largest world sample of charged kaon decays with the main goal of searching for direct CP violation \cite{1}. The beam line was designed to deliver simultaneous $K^+$ and $K^-$ beams at 60 GeV/c central momentum, produced by 400 GeV/c primary protons from the CERN SPS impinging on a beryllium target. Charged particle momenta were measured by a magnetic spectrometer consisting of four drift chambers (DCH) and a dipole magnet. The spectrometer was located in a tank filled with helium at atmospheric pressure and followed by a scintillator trigger hodoscope. A liquid Krypton electromagnetic calorimeter was exploited to measure the energy of electrons and photons. A hadron calorimeter and a muon veto system, essential to distinguish muons from pions, were located further downstream. Details of the experimental apparatus are available in \cite{2}.

2 $K_{e4}$ decay mode properties

The study of semileptonic four-body $K_{e4}$ decays is extremely interesting due to the well-known Standard Model (SM) electroweak amplitude responsible for the leptonic part and the small number of hadrons in the final state. In the non-perturbative QCD regime, at energies below 1 GeV, the developments of Chiral Perturbation Theory (ChPT) \cite{3} and Lattice QCD \cite{4} have reached precision levels competitive with the most accurate experimental results.

The NA48/2 experiment has collected high statistics samples of $K^\pm \rightarrow \pi^{\mp} \pi^{\pm} e^{\pm} \nu$ [$K_{e4}^{\pm\mp}$] and $K^\pm \rightarrow \pi^{0}\pi^{0} e^{\pm} \nu$ [$K_{e4}^{00}$] candidates, several orders of magnitude larger than the world ones. The latest NA48/2 results from the study of $K_{e4}^{\pm\mp}$ decays \cite{5}\cite{6} and new results from $K_{e4}^{00}$ data analysis \cite{7} will be reviewed.
2.1 $K_{e4}$ Form Factors and Branching Ratios

$K_{e4}$ decay kinematics is conveniently described by five variables [8]: the squared dipion invariant mass $S_\pi = M_\pi^2$, the squared dilepton invariant mass $S_e = M_e^2$, the angle $\theta_\pi$ of the pion in the dipion rest frame with respect to the flight direction of the dipion in the kaon rest frame, the angle $\theta_e$ of the lepton in the dilepton rest frame with respect to the flight direction of the dilepton in the kaon rest frame, and the angle $\phi$ between dipion and dilepton rest frames.

$K_{e4}$ decay amplitude is given by the product of a leptonic weak current and a $(V$-$A)$ hadronic current, expressed in terms of three axial-vector ($F, G$ and $R$) and one vector ($H$) complex form factors. Due to the smallness of the electron mass, $R$ does not contribute to $K_{e4}$ decay rates. In the isospin symmetry limit, hadronic form factors can be developed in partial wave expansion and expressed in term of $S$- and $P$-wave components (the $D$-wave contribution can be neglected): $F = F_e e^{i\delta} + F_p e^{i\delta} \cos \theta_\pi$, $G = G_p e^{i\delta}$, $H = H_p e^{i\delta}$.

Considering a unique phase $\delta_p$ for all the $P$-wave form factors in absence of CP violating weak phases, the decay probability depends on the real form factor magnitudes $F_s, F_p, G_p, H_p$ and a single phase shift $\delta = \delta_s - \delta_p$.

High precision measurements of $K_{e4}^{+}$ and $K_{e4}^{0}$ hadronic form factors and branching ratios (BR) have been published by NA48/2 [5][6][7]. Decay rates are measured relative to the $K_{3\pi}$ normalization channels, $K^+ \rightarrow \pi^+ \pi^- \pi^0$ [$K_{e4}^{+}$] and $K^0 \rightarrow \pi^0 \pi^0 \pi^0$ [$K_{e4}^{0}$] for the $K_{e4}^{+}$ and $K_{e4}^{0}$ one. The main background sources are given, respectively, by $K_{3\pi}$ and $K_{3\pi}$ events with a fake electron due to charged pion misidentification or with $\pi^0 \rightarrow e^+ e^- \nu \bar{\nu}$ decays.

The measurements of $\text{BR}(K_{e4}^{+})$ and $\text{BR}(K_{e4}^{0})$ are obtained from the number of reconstructed signal, background and normalization events, corrected for different acceptances and trigger efficiencies in the signal and normalization modes. The PDG [9] values are used for $\text{BR}(K_{e4}^{+})$ and $\text{BR}(K_{e4}^{0})$. Since in both $K_{e4}$ analyses the topologies of candidate and normalization events are similar in terms of the number of detected charged ($e^+$ or $e^-$) and neutral ($\pi^0\pi^0$ or $\pi^+\pi^-$) particles, signal and normalization samples are collected concurrently employing the same trigger logic and common selections as far as possible. This leads to the partial cancellation of systematic effects induced by imperfect beam description, local detector and trigger inefficiencies and makes the measurement independent on the absolute kaon flux measurements. A detailed Monte Carlo simulation has been developed to include full detector geometry, DCH alignment, local inefficiencies and beam properties.

$K_{e4}^{+}$ decays. Nearly $1.13 \times 10^6$ $K_{e4}^{+}$ candidates with about 0.6% background contamination have been reconstructed by NA48/2. The form factor analysis is performed in the five-dimensional space of the kinematic variables [8] to take into account the precise knowledge of experimental acceptance and resolution. The large data statistics allows to define grids of equal population five-dimensional boxes. Hadronic form factors and their dependence on energy are obtained by adjusting the expected number of simulated events to the observed data events. Positive and negative kaon charges are analysed separately to account for the different geometries. The results are consistent for both kaon charges and are combined according to their statistical error. Form factors are expressed as Taylor series expansion of $q^2 = S_e/(4m_e^2) - 1$ and $y^2 = S_e/(4m_e^2)$ dimensionless invariants [10]. Only relative form factors can be measured, normalized to the overall scale factor $F_s = F_s(q^2 = 0, S_e = 0)$. The $S$- and $P$-wave form factors and their variation with energy have been measured [5][6]. $F_s$ is described by one curvature and two slopes: $F_s = f_s[1 + (f_s^{(1)} / f_s)q^2 + (f_s^{(2)} / f_s)q^2 + (f_s^{(3)} / f_s)q^3]$; $G_p$ by one slope and an offset: $G_p = f_p[(g_p / f_s) + (\gamma_p / f_s)q^2]$; $F_p$ and $H_p$ by constants. The first evidence for a negative 5% contribution from $F_p$ and for a $S_e$ dependence of $F_s$ have been established.
The BR value, evaluated from 16 statistically independent subsamples separately for $K^+$ and $K^-$, is $\text{BR}(K_{e4}^{00}) = (4.257 \pm 0.004_{\text{stat}} \pm 0.016_{\text{syst}} \pm 0.031_{\text{ext}}) \times 10^{-5}$, inclusive of $K_{e4}$ decays. The 0.8% total error is dominated by the external uncertainty from the PDG value of the normalization mode $\text{BR}(K_{3\pi}^{00})$.

The BR measurement allows assigning absolute values to the relative form factors. The overall form factor normalization is $f_s = 5.705 \pm 0.017_{\exp} \pm 0.031_{\text{ext}}$, ($\sigma_{\text{exp}} = \sigma_{\text{stat}} \oplus \sigma_{\text{syst}}$). The main error is the external one, due to uncertainties on the PDG values of kaon lifetime, CKM matrix element $|V_{us}|$ and $\text{BR}(K_{3\pi}^{00})$ [9]. The 0.6% total relative precision improves previous results of a factor of 2 to 4.

$K_{e4}^{00}$ decays. An unprecedented statistics of 65210 $K_{e4}^{00}$ candidates has been collected by NA48/2, with (1.00±0.02)% background [7]. The formalism for studying $K_{e4}^{00}$ decays is simpler because of two identical particles, $\pi^0\pi^0$, in the final state. As $G$ and $H$ form factors are antisymmetric in the exchange of the two pions, they do not contribute to the decay probability. At leading order only the S-wave component of the partial wave expansion is present and the differential rate depends on a single complex hadronic form factor $F = F_s e^{i\delta_s}$ with magnitude $F_s$, whose variation with $(S_x, S_y)$ has been studied. A data sample free of radiative effects is selected. $F_s$ is estimated by fitting the data in the $(S_x, S_y)$ plane where the event density is proportional to $F_s^2$. A grid is defined with equal population bins. A sample of Monte Carlo simulated events, including acceptance and resolution effects, trigger efficiency, radiative corrections and a constant $F_s$ value is distributed over the same grid as the data. A formalism based on the $q^2$ and $y^2$ variables has been used for a direct comparison with $K_{e4}^{00}$ results.

Fig. 1(a) shows the ratio of the $q^2$ distributions for data and simulated events in equal population bins. Above $q^2=0$ the distribution is similar to the $K_{e4}^{00}$ one, while it is depleted at negative values. The line corresponds to the empirical description using the best fit-parameters: a degree-2 polynomial for positive $q^2$ values and a cusp-like function below zero. Only statistical errors are represented. A possible interpretation of the observed deficit of events for negative $q^2$ values can be related to final state charge exchange scattering processes $(\pi^+\pi^- \rightarrow \pi^0\pi^0)$ in the $K^{++}$ mode, as observed in the $K_{3\pi}^{00}$ mode analysis [11]. A more elaborate description of the $K_{e4}^{00}$ amplitude is needed to extract more information on physical quantities from the result reported here. Fig. 1(b) shows the comparison of form factor results for $K_{e4}^{00}$ and $K_{e4}^{00}$ data in the $(q^2, y^2)$ formulation, displayed in the $(f'_e/f_e, f''_e/f_e)$ plane. All contours are 68% C.L., errors are statistical only. The smaller area in the charged mode is due to the larger statistics. The correlations between fitted parameters are very similar and the results for $K_{e4}^{00}$ and $K_{e4}^{00}$ decays are consistent within statistical errors.

Exploiting a model independent form factor description, $\text{BR}(K_{e4}^{00})$, inclusive of radiative decays, has been measured with respect to the $K_{3\pi}^{00}$ normalization mode. The global result is $\text{BR}(K_{e4}^{00}) = (2.552 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}} \pm 0.032_{\text{ext}}) \times 10^{-5}$, given by the combination of the values obtained for ten statistically independent sub-samples. The 1.4% relative precision is dominated by the external uncertainty from the normalization mode $\text{BR}(K_{3\pi}^{00})$ [9]. This result improves the current world average BR by more than one order of magnitude.

Both total rate and form factor descriptions are used to obtain the absolute form factor value. A long distance electromagnetic correction $\delta_{EM}$ to the total rate, not yet available in the literature, should be taken into account. The absolute form factor value is $(1 + \delta_{EM}) \times f_s = 6.079 \pm 0.012_{\text{stat}} \pm 0.027_{\text{syst}} \pm 0.046_{\text{ext}}$. The main external uncertainty is due to the errors on the PDG values of kaon lifetime, CKM matrix element $|V_{us}|$ and $\text{BR}(K_{3\pi}^{00})$ [9]. A difference from the $K_{e4}^{00}$ result has been observed, statistically significant as experimental errors are mostly uncorrelated. A more precise theoretical description of the $K_{e4}^{00}$ mode, including radiative,
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