The effect of Quantum Gravity on astrophysical Neutrino flavor observables.

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At the quantum level, an interaction of a neutrino with a graviton may trigger the collapse of the neutrino flavor eigenstate to a neutrino mass eigenstate. We will present that such an essentially quantum gravity effect may have strong consequences for neutrino oscillation phenomena in astrophysics due to the relatively large scattering cross section of relativistic neutrinos off massive sources of gravitational fields (the case of gravitational Bethe-Heitler scattering). This results in a new technique for the indirect detection of gravitons by measuring the flavor composition of astrophysical neutrinos.

A theoretical extrapolation of fundamental Quantum Mechanics concepts to Einstein's gravity suffers from major difficulties with quantization of space-time, ultraviolet behavior and non-renormalizability of the resulting theory. Typically quantum gravity effects are disregarded as being irrelevant at energy scales smaller than the Planck scale, $M_{Pl} \sim 10^{19}$ GeV. Due to suppression, quantum gravity effects are referred to as unobservable [1, 2].

We propose a new approach for indirect experimental studies of (local) quantum gravity interactions based upon an effect on neutrino oscillation observables of a neutrino interaction with an energetic graviton. This may happen in large-angle energetic gravitational Bremsstrahlung off an astrophysical neutrino passing through an external classical gravitational potential. This gravitational Bethe-Heitler (GBH) process can be considered in the quasi-classical approxi-

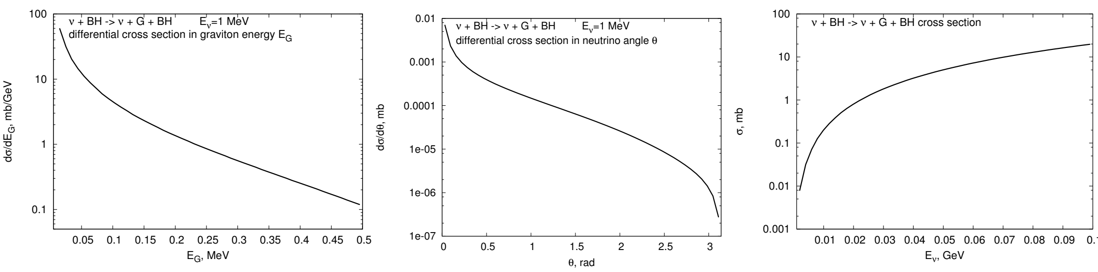

Figure 1: Differential cross section versus radiated graviton energy E_G (left), polar angle of the final-state neutrino θ_ν, and the integrated cross section as a function of incoming neutrino energy E_ν for GBH scattering of a neutrino off a massive object.

mation for large angle and/or large energy graviton emission (Born approximation). Such a process may happen with high probability, such as in the case of scattering off a massive source of gravitational field (star or a black hole). The neutrino interaction serves as a direct *quantum measurement* of the microscopic properties of the gravitational field at astrophysical scales.

Weakly-interacting neutrinos are an efficient carrier of information at astrophysical scales due to not being absorbed or scattered by interstellar mediums. This property of neutrinos enables us to utilize them for large-scale astrophysical *experiments*. In the identified experiment neutrinos *change* their quantum state due to a local quantum gravity process (in terms of local graviton coupling to a fundamental matter particle) and convey information about this process through the cosmological medium to Earth. Elementary particles in the mass basis are eigenstates of the Hamiltonian of quantum-gravitational interactions similar to how leptons and quarks are weak eigenstates in the flavor and CKM bases. The second important neutrino property that neutrino mass and flavor eigenstates are not the same.

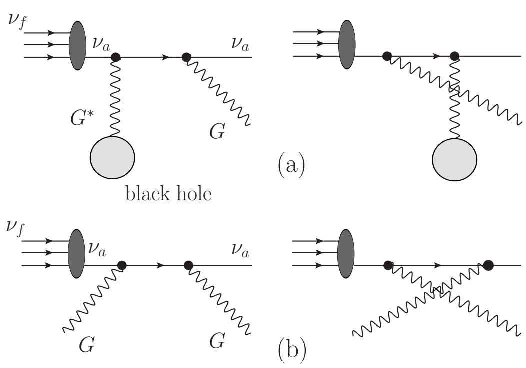

Figure 2: The quantum gravity processes which causes the decoherence of the neutrino flavor eigenstate ($f = e, \mu, \tau$) effectively converting it to a mass eigenstate ($a = 1, 2, 3$) – GBH scattering of neutrino off a massive object (a), and gravitational Compton scattering (b). The ellipse is a projection to a mass state and the circle is a classical source of gravitational field.

Consider a relativistic neutrino propagating in the gravitational potential of a static black hole. At the quantum level a graviton interacts only with a definite mass state (or gravitational mass eigenstate) $a = 1, 2$ or 3. Expressed equivalently definite mass eigenstates (propagating states) are conserved by the quantum gravity hamiltonian while superpositions, such as the flavor eigenstates, are not. Astrophysical neutrinos are initially produced in electroweak processes in a definitive flavor state, $f = e, \mu$ or τ, which are coherent superpositions of mass eigenstates. This neutrino is quantum mechanically observed by the energetic graviton as being in a definite mass state. This means that between the production in an astrophysical source and the detection in an Earth based detector, the neutrino exists in a definite mass state and has experienced *quantum decoherence*.

The neutrino is *converted* to mass state with a probability $P_{\nu_f \to \nu_a} = |\Psi_{\nu_f \to \nu_a}|^2$, given in terms of the corresponding wave function $\Psi_{\nu_f \to \nu_a}$ which projects out a flavor state ν_f onto a mass state ν_a and is typically expressed in terms of the corresponding PMNS mixing matrix element, $\Psi_{\nu_f \to \nu_a} = V_{af} e^{-i \frac{m_a^2}{2E_\nu} L}$.

We consider the case shown in Fig. 2(a), the graviton exchange is with negative momentum transfer squared $t = -q^2 < 0$ in the t-channel with the propagator stretched between the relativistic neutrino of mass m_ν and energy $E_\nu \gg m_\nu$ and a massive classical gravitational field source with mass $M \gg E_\nu$. The cross section has been calculated for the gravitational scattering of scalar particles with $M \gg m$ in [3]. We use their formula as a good approximation to estimate the neutrino-solar mass cross section numerically. In this case, as an order-ofmagnitude estimate, the GBH cross section at the Born level behaves as

$$\sigma_{\rm GBH} \sim \frac{M^2 E_\nu^2}{M_{Pl}^6}\,, \quad M \gg E_\nu \gg m_\nu\,, \tag{1}$$

and thus may not be very small since the Planck scale suppression can be largely eliminated by having a mass M of a heavy classical source in numerator. In particular, for a solar mass object $M \sim 10^{57}$ GeV, we have $M^2/M_{Pl}^6 \sim 1$ GeV^{-4}, so there is no significant suppression of the cross section for relativistic neutrinos.

Note that the Bethe-Heitler calculation in QED to first order gives the correct cross section for photon Bremsshtrahlung for extended objects such as a nucleus as shown in Ref. [4]. Similarly, we expect that the GBH result for a point-like classical source should be roughly correct to first order for extended objects, like a star or dark matter distribution.

The traditional source of decoherence typically referred to in astrophysical neutrino oscillations studies can be called *propagation decoherence*. Here the neutrino mass states have separated or dispersed so that they no longer interfere at large distances from the production point. This source of decoherence depends on the energy resolution of the detection process, the energy of the neutrino, the masses of the neutrino mass states, and details of the production and detection processes. Beyond the characteristic length the propagating neutrino mass states no longer interfere during the interaction process [5].

Note that while the flux due to *quantum decoherence* is a flux of pure mass eigenstates, that in the propagation decoherence case the flux is not of pure mass eigenstates, but rather decoherent (spatially separated) mass eigenstates. No quantum measurement of the state of these neutrinos has taken place, and the neutrino still exists as a superposition of mass states. These two situations are the same when detected in the case where the flux does not pass through matter; in the case where the flux passes through matter, the effect due to matter is different for the two cases. In the *quantum decoherence* case, the neutrino flux experiences regeneration as fluxes of neutrinos in pure mass eigenstates. In the propagation decoherence case, the neutrino flux experiences regeneration as a superposition of mass eigenstates.

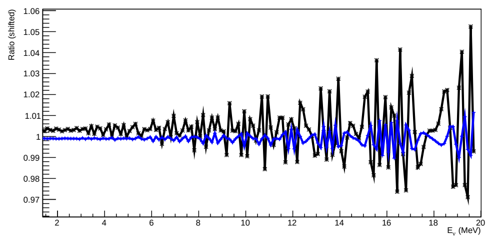

Figure 3: Ratio for neutrinos which have undergone propagation decoherence (blue) and neutrinos which have undergone graviton induced decoherence (black).

The theory of neutrino propagation, including neutrino propagation in medium and neutrino propagation where the neutrino experiences propagation decoherence, is well presented in [5] [6]. These papers give the essentials of neutrino propagation in matter and propagation decoherence, but no explicit formula is given for a neutrino which undergoes propagation decoherence and then experiences the Earth matter effect.

For simplicity consider just two regimes, the vacuum and the earth (with constant density) and two neutrino flavors. Due to the discontinuity at the earth's surface, the flavor amplitudes should be matched at the border between the two regimes. The flavor at the point before the density jump is used to determine the initial state [6].

The condition for the wave packet separation to be complete is given explicitly by [7]. They note that this is different than the effect due to averaging (Section 2.1) over the energy, despite the effect being computationally the same for vacuum[5]. We expect significant (> 1 km) wave packet separation for supernova more than 10 kpc distant.

The amplitude of the state at the boundary between regimes can be given by $A_{ee}^{dec} = \cos^2(\theta)\, e^{i\frac{3\pi}{4}} + \sin^2(\theta)\, e^{-i\frac{3\pi}{4}}$ and $A_{e\mu}^{dec} = \sin(\theta)\cos(\theta)\left(e^{-i\frac{3\pi}{4}} - e^{i\frac{3\pi}{4}}\right)$. These give the flavor amplitudes of a neutrino produced in a ν_e state which has travelled through vacuum and experienced wave packet separation when it reaches the Earth vacuum transition. An amplitude which depends on the phase between the wave packets would be incorrect for large wave packet separations. This amplitude is then projected to the new matter basis.

The ratio of neutrinos which have undergone *propagation decoherence* and at the same time propagated through a region of constant density to those which have only propagated through the vacuum is given by the following expression

$$\begin{aligned} R_p &= \Big(\cos(x_m)^2 (3+\cos(4\theta)) + (2+\cos(4\theta_m - 8\theta) + \cos(4\theta_m - 4\theta))\sin(x_m)^2 \\ &- 2\sin(2x_m)\sin(2\theta_m - 2\theta)\sin(2\theta)\Big)/(3+\cos(4\theta))\,. \end{aligned} \tag{2}$$

Analogically, the ratio of neutrinos which have undergone *quantum decoherence* in the presence of the matter effect (medium of constant density) to those which have propagated through the vacuum takes a different form

$$R_q = \frac{5+\cos(4\theta_m)+\cos(4\theta_m-4\theta)+\cos(4\theta)+4\cos(2x_m)\cos(2\theta)\sin(2\theta_m-2\theta)}{6+2\cos(4\theta)}\,. \tag{3}$$

A difference between the ratios R_p and R_q could be measurable and indicates the difference between *propagation decoherence* and *quantum decoherence* in the presence of the matter effect. Measurement of such a difference could serve as a clear example of graviton detection. In the presence of an additional jump in matter density the corresponding numerical results are presented in Fig.3, demonstrating that measurement of graviton induced decoherence is possible.

More details of this work can be found at [8] and this research was supported in part by the National Science Foundation under Grant No. NSF PHY11-25915 and by PROYECTO BASAL FB 0821 CCTVal and by Fondecyt (Grant No. 11130133).

References

[1] F. Dyson, *The World on a String*, review of *The Fabric of the Cosmos: Space, Time, and the Texture of Reality* by Brian Greene, New York Review of Books, Volume 51, Number 8, May 13, (2004);
F. Dyson, *Is a Graviton Detectable?*, Poincare Prize Lecture International Congress of Mathematical Physics Aalborg, Denmark, Aug. 6, 2012.

[2] T. Rothman and S. Boughn, Found. Phys. **36**, 1801 (2006).

[3] B. M. Barker, S. N. Gupta, J. Kaskas, Phys. Rev. 182 (1969) 1391-1396.

[4] H.K. Tseng, R.H. Pratt, Phys. Rev. A 19, 1525 (1979).

[5] M. Beuthe, Phys. Rept. **375**, 105 (2003) [hep-ph/0109119].

[6] M. Blennow and A. Y. Smirnov, Adv. High Energy Phys. **2013**, 972485 (2013) [arXiv:1306.2903].

[7] Y. Farzan and A. Y. Smirnov, Nucl. Phys. B **805**, 356 (2008) [arXiv:0803.0495].

[8] J. Miller and R. Pasechnik, [arxiv:1305.4430].