Low-Energy Tests of the Standard Model - \((g-2)_\mu\)
and Dark Photons

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Low-energy tests of the Standard Model provide complementary insights to Beyond Standard Model Physics. We review two topical issues, namely the status of the anomalous magnetic moment of the muon \((g-2)_\mu\) as well as searches for a hypothetical extra-U(1) GeV-scale particle beyond the Standard Model - the so-called Dark Photon.

1 Introduction

The discovery of the Higgs boson in 2012\cite{1} represents an impressive confirmation of the concepts of the Standard Model (SM) of particle physics. The last particle of the SM is now finally discovered, the production rate as well as the decay pattern of the Higgs particle seem to follow the predictions of the theory\cite{2}. While searches for Beyond Standard Model (BSM) physics are of course continued at the high-energy frontier\cite{3} and stringent mass bounds up to the TeV scale are established, measurements at the precision frontier\cite{4} provide complementary insights. By loop-induced processes, the experimental values of low-energy observables might indeed be affected by particles with very high masses. It was found that the mass scales of BSM particles, which are tested in low-energy experiments, do indeed exceed the mass scales tested at high energies by large factors in many cases.

In this paper we are going to discuss two topical subjects of the precision frontier, which have triggered an enormous amount of work both in experiment and theory in the past years. The anomalous magnetic moment of the muon \((g-2)_\mu\) is one of the few physics observables, in which for more than a decade a deviation between the SM theory and the direct experiment persists. New and improved measurements of \((g-2)_\mu\) at FNAL\cite{4} and JPARC\cite{5} are upcoming and it is hence a good moment to review the status of this precision quantity. Originally motivated by the dark sector and their relation to dark matter, it was realized that extra-U(1) gauge bosons beyond the ordinary photon - therefore often called Dark Photons - could indeed explain the deviation in the \((g-2)_\mu\) system mentioned above. Low-energy searches for the dark photon have been carried out as a consequence and will be presented in chapter 3.

There are of course many more low-energy tests of the SM ongoing beyond the ones covered in this paper. Flavour physics (see Ref.\cite{6}) played for instance an important role in particle

\footnote{often also denoted as the intensity frontier}
physics in the first decade of the 21st century and is continuing to do so in the LHC era. Searches for lepton flavour violation (LFV) have been carried out at flavour factories and at dedicated muon beam lines [7]. New LFV experiments are upcoming with the potential to improve upon existing results by orders of magnitude. As a legacy of the LEP-SLC era, there remain precision measurements of the electroweak mixing angle, \( \sin^2 \theta_W \), which plays a central role in the SM. Unfortunately, a discrepancy between LEP and SLC could never be clarified [8]. New low-energy experiments are currently being performed or are in the design stage with the goal to measure \( \sin^2 \theta_W \) at very low momentum transfer. Measurements of that kind do not only have the potential to resolve the LEP-SLC discrepancy, but have also the resolving power for New Physics contributions up to the highest mass scales in the multiple TeV range. Different measurements of \( \sin^2 \theta_W \), for instance in electron-electron scattering or electron-proton scattering, are also testing complementary BSM models.

2 The anomalous magnetic moment of the muon \((g - 2)_\mu\)

The gyromagnetic factors of the electron and muon \((g_l, l= e, \mu)\) belong to the best known quantities in physics, both experimentally and theoretically. [9] The high accuracy is indeed motivated by the fact, that calculations of \(g_l\) are very sensitive to loop corrections and hence allow for very accurate tests of the underlying theory.

The anomalous magnetic moment of the electron \(a_\mu \equiv (g - 2)_e/2\) – i.e. one half of the

![Figure 1: Comparison between the direct measurement of \((g - 2)_\mu\) (BNL-E821, blue) and several theoretical evaluations within the Standard Model (black). A discrepancy larger than 3 standard deviations is found.](image)

development of the \(g\)-factor from the Dirac value \(g_e = 2\) – has been measured a few years ago by Gabrielse with an accuracy of 1 part in \(10^{13}\) [10]. This accuracy is a test of the theory of quantum electrodynamics QED with unprecedented precision. The anomalous magnetic moment of the muon, \((g - 2)_\mu\), is known with less accuracy. It allows, however, to resolve effects not only of QED but also of weak and strong interactions and eventually of BSM contributions.

PANIC2014 53
Presently, the experimental and Standard Model values of \((g-2)_\mu\) differ by more than 3 standard deviations \([11] [12]\), see Fig. 1, which triggered many speculations whether this might be an indication of a missing contribution due to New Physics. In the following two subchapters we will briefly review the status of theory and experiment. We stress that the physics of \((g-2)_\mu\) is indeed testing an extremely wide class of New Physics models. Supersymmetric theories (SUSY), in which the masses of the SUSY particles are on the weak scale, could a priori explain the presently seen deviation in \((g-2)_\mu\) very nicely. There is however an increasing tension with SUSY mass limits from the LHC reaching now the TeV scale. Nevertheless, non-traditional SUSY models are still viable \([13]\). We will show later that light particles with very weak coupling to the SM, so-called Dark Photons, could explain the \((g-2)_\mu\) deviation very elegantly as well.

![Figure 2: Measured event yield of positrons by the BNL-E821 experiment. From the modulation the value of \((g-2)_\mu\) can be extracted.](image)

2.1 Experimental value

The most recent and most accurate experimental value of \((g-2)_\mu\) stems from a measurement at BNL. The E821 collaboration has improved the accuracy of the previous CERN measurement by a factor 14 and finds the following value:

\[
\alpha_{\mu}^{\text{exp}} = (11659208.9 \pm 5.4_{\text{stat}} \pm 3.3_{\text{syst}}) \cdot 10^{-10} \quad [14].
\] (1)

To achieve such an accuracy a high-intensity polarized muon beam is injected into a storage ring with known magnetic field. The muon spin is rotating around the momentum vector due to the \(\approx 0.1\%\) difference between the cyclotron and spin precession frequencies. After circling the ring many times, the muon decays into electrons plus neutrinos. Weak interaction guarantees a correlation between the electron flight direction and the original muon spin direction. As the decay electrons are detected in the experiment, the measured event yield shows a modulation proportional to the difference between the cyclotron and spin precession frequencies, i.e. proportional to \((g-2)_\mu\), see Fig. 2. Electric fields are required for a focusing of the muon beam, which complicates a precise extraction of \((g-2)_\mu\). As realized already in previous experiments
at CERN, these effects cancel if a so-called magic relativistic gamma value of the muon beam is used, which corresponds to a muon beam momentum of 3.09 GeV/c.

In 2013 the BNL $(g-2)_\mu$ ring was shipped to FNAL, where a new experiment is presently set up with the overall goal to improve the accuracy by a factor of 4 \[4\]. Apart from a higher muon flux compared to BNL, a series of additional improvements will lead to smaller systematic uncertainties. A second new experiment is in preparation at JPARC \[5\]. Differently from the BNL/FNAL approach, here the magic muon momentum will not be used, as no electric focussing fields are needed for the experiment. The solution of JPARC is the production of ultracold muons, which are then reaccelerated and injected into a 3 Tesla MRT magnet. The muon flux will be higher compared to the FNAL experiment and the overall goal is to achieve a similar accuracy as in the FNAL project.

Figure 3: Hadronic contributions to $(g-2)_\mu$: the hadronic vacuum polarization (left) and the hadronic Light-by-Light contribution (right).

2.2 Standard Model prediction

Given the experimental accuracy reported above, there are measurable contributions to $a_\mu$ not only from QED, but also from weak and strong interactions. These individual contributions are listed below:

\[
a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr} \quad \quad (2)
\]

\[
= [11658471.808 \pm 0.015 + 15.4 \pm 0.2 + 693.0 \pm 4.9] \cdot 10^{-10}
\]

\[
= [11659182.8 \pm 4.9] \cdot 10^{-10}
\]

The calculation of the by far dominating QED contribution was a heroic effort and has been pursued by Kinoshita and co-workers in the past decades \[15\]. An evaluation of up to 5 loops requires the calculation of more than 12,000 Feynman diagrams. The weak contribution has been computed up to NLO and is found to be many orders of magnitude smaller than the QED one \[16\]. Both the uncertainties of the QED and weak contributions are negligible in comparison to the experimental uncertainty. As can be seen from Equation 2, the bottleneck of the Standard Model prediction of $(g-2)_\mu$ is the hadronic contribution. It is split into two
parts, namely the Hadronic Vacuum Polarization HVP (see left Feynman diagram in Fig. 3) and the Hadronic Light-by-Light Scattering HLL (Fig. 3, right) contributions. It should be noted that both contribute to only 60 ppm of the absolute contribution, they however dominate completely the uncertainty.

Figure 4: Exclusive hadronic final states measured by BaBar via Initial State Radiation (ISR).

The leading order HVP contribution is related via a dispersion integral to experimental data on the cross section $e^+e^- \rightarrow \text{hadrons}$. Such a relation is based on unitarity and analyticity and is hence theoretically on safe grounds. Due to a kernel function in the dispersion integral, it comes out that low energy data of the hadronic cross section is particularly important. Indeed, the hadronic cross section below approximately 3 GeV is required with an accuracy on the level of 1%. This quest for accuracy triggered a series of cross section measurements at electron-positron facilities and led to the construction of the Novosibirsk colliders VEPP-2M and more recently of VEPP-2000 with the detectors CMD-2/CMD-3 and SND. Major new results on hadronic cross section data were achieved at the particle factories DAΦNE (experiment KLOE) and PEP-II (experiment BaBar). As those particle factories were designed to operate at a fixed center-of-mass energy, a classical energy scan is therefore impossible. A new and very successful method has however been worked out, which allows for cross section measurements by using events, in which one of the beam electrons/positrons has emitted a high-energetic photon (initial state radiation, ISR) [17]. Depending on the energy of the ISR photon, the available hadronic mass is reduced and the hadronic cross section can be extracted for all masses below the center-of-mass energy of the collider. A good knowledge of the QED radiative corrections is required for this radiative approach. These are calculated up to next-to-leading order within the PHOKHARA [18] Monte-Carlo event generator. An overview of hadronic cross section measurements of various exclusive hadronic states via ISR by the BaBar...
experiment is shown in Fig. 4. Essentially all channels up to 6 hadrons in the final state have been measured with systematic accuracies of few percent [17]. The two-pion final state \( e^+e^- \rightarrow \pi^+\pi^- \) plays a special role for \((g - 2)_\mu\). As can be seen in Fig. 4, the \( \rho \) resonance, which is almost entirely decaying into two pions, is dominating the cross section and hence is also playing a leading role in the dispersion integral for the HVP contribution to \((g - 2)_\mu\) with approximately 75% of the total contribution stemming from this channel. Unfortunately, the BaBar measurement of \( \sigma(e^+e^- \rightarrow \pi^+\pi^-) \) [20], which has a claimed systematic accuracy of 0.5%, shows quite some deviation from ISR-measurements of KLOE kloe, which claims a 0.8% accuracy for the most precise of its data sets. The deviation is in the order of 3% on the \( \rho \) peak and increases towards higher masses. Precision data points from Novosibirsk [21] [22] have larger statistical and systematic uncertainties and hence can confirm neither the BaBar nor the KLOE results. As a matter of fact, this deviation is dramatically limiting our knowledge of the HVP contribution and hence \( a_{\mu}^{\text{SM}} \). Presently, an average of the world data set for hadronic cross section measurements yields the following value for the LO-HVP contribution to \((g - 2)_\mu\): 
\[
 a_{\mu}^{\text{HVP}} = (692.3)_{-4.2}^{+4.2} \cdot 10^{-10}.
\]

The next important contribution beyond HVP is the HLbL contribution shown in Fig. 3, right. Here the leading subdiagram is shown, namely the coupling of photons to the pseudoscalar mesons \( \pi^0, \eta \), or \( \eta' \). So far hadronic models have been used for the calculation of the HLbL diagram. Although most groups report similar values for the absolute size of the HLbL contribution, the assumed uncertainties differ largely. The calculation with the lowest uncertainties stems from Prades, de Rafael, and Vainshtein [23]. They find the following value: 
\[
a_{\mu}^{\text{HLbL}} = (10.5)_{-2.6}^{+2.6} \cdot 10^{-10}.
\]
In most compilations of \((g - 2)_\mu\) this result is used.

Very recently new theoretical approaches have been proposed by two groups from Bern and Mainz, namely the use of dispersion relations [24] [25]. Form factor measurements of the two-photon coupling \( \gamma\gamma \rightarrow P \), where \( P \) is a one hadron or two hadron system, are therefore of special interest for the dispersive approaches. The B-factory experiments BELLE [26] and BaBar [27] have recently measured so-called single-tag form factors for the lightest pseudoscalar mesons, however data has been reported only at very large momentum transfer above 4 GeV\(^2\), while for the HLbL contribution measurements at low momentum transfer are required. In that kinematic range new spacelike measurements are expected from KLOE-II in Frascati and the BESIII experiment in Beijing. Important timelike measurements of the \( \eta \) form factor have recently been performed by the A2 collaboration in Mainz [28].

### 2.3 Conclusions \((g - 2)_\mu\)

With the persisting deviation between the SM prediction and the direct measurement of \((g - 2)_\mu\), an interpretation in terms of BSM physics is very tempting. It is good to know that new direct measurements of \((g - 2)_\mu\) with a factor 4 improved accuracy are underway at FNAL and JPARC and hopefully these projects will be able to report their results around the end of this decade. For the final interpretation of these experiments a reduction of the uncertainty of the SM prediction of \((g - 2)_\mu\) is highly desirable. Fortunately, new cross section measurements via the ISR technique are ongoing at the BESIII facility in China and new energy scan campaigns are performed at Novosibirsk. This will eventually help to clarify the discrepancies seen between hadronic cross section measurements from BaBar and KLOE for the \( 2\pi \) cross section and will hence improve our knowledge of the HVP contribution. Moreover, measurements of transition form factors are ongoing at several hadron and electron facilities around the world and together
with the new developments in theory will lead to a significant progress in the HLbL contribution, which otherwise might be the leading uncertainty of the SM on the long run. As discussed in a recent whitepaper [29], there is very good hope that all these developments will lead to a further reduction of a factor 2 of the SM prediction of \((g-2)_\mu\). The combined effort in theory and experiment will therefore tell us in few years from now, whether the hint for BSM physics becomes evidence.

Figure 5: Dark Photon coupling to the ordinary photon in a kinetic mixing model.

3 Dark Photons

Extra U(1) gauge bosons beyond the Standard Model photon appear in essentially all string compactifications as they result naturally from symmetry breaking mechanisms towards lower gauge symmetries. A search for such kind of hypothetical particles is carried out from the lowest energies – e.g. the search for axion or axion-like particles – up to the highest energies at the LHC. More recently, particles at the GeV mass scale were proposed by several authors as they might be connected with the following puzzles in particle and astroparticle physics:

- It was shown by Arkani-Hamed and collaborators [30] that a GeV-scale particle – which was dubbed Dark Photon – could explain a surprisingly large number of astrophysical anomalies such as for instance the positron excess in the cosmic ray spectrum.
- A Dark Photon of a very similar mass scale [31] could also explain the discrepancy seen between the Standard Model prediction of \((g-2)_\mu\) and the direct measurement, see previous chapter.

The simplest mechanism with which a Dark Photon could couple to SM matter – the kinetic mixing model – was proposed by Holdom [32] already in the eighties. As depicted in Fig. 5 such a coupling can be realized by introducing a loop of charged leptons, which couple to the Standard Model U(1) photon as well to the Dark Photon. Hereby a portal between the hypothetical Dark Sector and the Standard Model is established. Of course the coupling \(\alpha'\) must be extremely weak - much weaker than the coupling given by the electromagnetic fine structure constant \(\alpha_{em}\). There remain two unknown parameters of the model: the mass of the Dark Photon \(m_{\gamma'}\) and the coupling constant \(\alpha'\), which is also often expressed as \(\epsilon' = \sqrt{\alpha'/\alpha_{em}}\). In case dark matter particles couple to a Dark Photon, it would couple according to the kinetic mixing model to the Standard Model photon, which in turn decays into electron-positron pairs. Like this, a very elegant explanation for the positron excess is given. Regarding the \((g-2)_\mu\) puzzle, the Dark Photon would give rise to an additional exchange term, see Fig. 6, which is
missing in the SM calculation. The currently seen deviation in \((g - 2)_\mu\) can be expressed as a well-constraint parameter range for \(m_{\gamma'}\) and \(\epsilon'\). Taking into account constraints from various precision observables and from old beam dump experiments at FNAL and SLAC, the following parameter range would allow for a solution of the \((g - 2)_\mu\) discrepancy: \(20\) MeV \(< m_{\gamma'} < 200\) MeV and \(\epsilon' \approx 2 - 4 \cdot 10^{-3}\).

The possible existence of a GeV-scale Dark Photon triggered an enormous theoretical and experimental interest in the particle and nuclear physics community. In the following we will distinguish between electron scattering experiments and results from various hadron and e\(^+\)e\(^-\) accelerators. No significant signal of a Dark Photon has been found before and only 90\% confidence limits have been published.

![Figure 6: Hypothetical Dark Photon contribution to \((g - 2)_\mu\).](image)

### 3.1 Electron accelerator fixed target experiments

As Bjoerken and collaborators [33] have pointed out, low-energy electron accelerators in combination with high-resolution detectors are very well suited for Dark Photon searches. By scattering the electron beam on a nuclear target, the Dark Photon may be emitted in the initial or final state, see Fig. 7. Its coupling to an \(e^+e^-\) pair allows for an identification by looking for a bump in the \(e^+e^-\) invariant mass. The huge background is almost entirely given by QED processes, such as for instance Bethe-Heitler processes.

Successful pilot experiments have been carried out in 2011 at MAMI [34] (experiment A1) and JLAB [35] (APEX experiment) with electron beam energies of 0.9 GeV and 2.3 GeV, respectively. These runs could improve upon existing Dark Photon limits from BaBar (2009 results) in the mass range around 200 MeV. More recently, a very wide parameter range between approximately 40 MeV and 200 MeV was tested by MAMI with the high resolution spectrometer (HRS) setup A1 [36]. No significant signal was found and the \(\epsilon'\) parameter range down to \(10^{-3}\) was excluded, constraining a large part of the parameter range motivated by \((g - 2)_\mu\), see Fig. 8.

For the near and mid-term future several dedicated experiments are in preparation at JLAB.
The APEX experiment [37], which is using an existing HRS setup at JLAB, will extend the mass range covered by A1/MAMI towards higher masses and lower $\epsilon'$ values. The HPS experiment [38] will exploit a displaced vertex technique, which allows to test even lower values of $\epsilon'$. Finally, the Dark Light [39] experiment at the FEL accelerator at JLAB aims for testing the low mass region below the results already covered by A1. A new spectrometer setup at the MESA accelerator [40] in Mainz will also be able to cover this mass range.

3.2 Results from hadron and $e^+e^-$ accelerators

A search for the Dark Photon is of course possible in physics environments beyond the ones tested in electron scattering. We list here the most recent results, which have been obtained in the past five years. All these results are displayed in Fig. 8 and have been obtained by looking for a bump in the $e^+e^-$ or $\mu^+\mu^-$ invariant mass spectrum.

- The KLOE experiment at the $\phi$ factory DAΦNE in Frascati has searched for a bump in $\phi \rightarrow \eta e^+e^-$ events [41]. A constraint at higher masses existed already before from BaBar by similarly investigating $\Upsilon$ decays (BaBar 2009 [42]).

- The WASA@COSY collaboration has produced a huge statistics of $\pi^0$ events in proton proton scattering and has looked for the Dark Photon in $\pi^0 \rightarrow e^+e^-\gamma$ Dalitz events [43].

- A similar search strategy is possible in heavy ion collision and has been pursued by the HADES experiment at GSI. In addition to Dalitz decays of the $\pi^0$ also $\eta$ Dalitz decays as well as decays of baryons are used [44].

- The most stringent Dark Photon limits have recently been published by the BaBar collaboration at SLAC [45]. Using ISR events and investigating the $e^+e^-\gamma$ and $\mu^+\mu^-\gamma$ final states, a very competitive search for the Dark Photon becomes available. BaBar has analyzed the full data set of approximately 500 fb$^{-1}$ for this analysis and has obtained limits in the extremely wide mass range from 10 GeV down to threshold. Again no Dark Photon has been found and stringent constraints have been placed for the Dark Photon coupling to SM matter down to few $10^{-4}$, see Fig. 8. A similar strategy had been followed already before by KLOE below 1 GeV.

![Feynman diagrams depicting the production of the Dark Photon in electron-nucleus scattering.](image-url)
In the meantime also the Phenix experiment at RHIC has produced competitive exclusion limits by analyzing Dalitz decays of $\pi^0$ and $\eta$ [46]. Those results are not yet displayed in Fig. 8 and are further constraining the favoured parameter range of $(g - 2)_\mu$, such that after five years of active research the Dark Photon seems to be excluded as an explanation of the $(g - 2)_\mu$ discrepancy. Of course, this range may change with new results for the direct measurement of $(g - 2)_\mu$ and its SM prediction. It should be noted, that the relation of the Dark Photon to Dark Matter is still a very strong motivation. For this all the uncovered parameter space of Fig. 8 is of interest. In this context also more involved models are discussed in which either the Dark Photon is lighter than twice the electron mass or in which the Dark Photon coupling to SM matter is different from the one known from the ordinary photon. Also proposals have been brought forward to use electron accelerators as a source to produce a Dark Photon beam. This would be a unique way to search for light Dark Matter particles [47].

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