Numerical Analysis of the $^1S_0$ Pairing Gap in Neutron Matter

S. Maurizio[1], *J. W. Holt*[2], *P. Finelli*[3]

[1] Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, CH-3012 Bern, Switzerland
[2] Department of Physics, University of Washington, Seattle, WA 98195-1560
[3] Department of Physics and Astronomy, University of Bologna, I-40126 Bologna, Italy and INFN, Bologna section, I-40127 Bologna, Italy

DOI: http://dx.doi.org/10.3204/DESY-PROC-2014-04/66

In a recent paper [1] we studied the behavior of the pairing gaps Δ_F as a function of the Fermi momentum k_F for neutron and nuclear matter in all relevant angular momentum channels where superfluidity is believed to naturally emerge. The calculations employed realistic chiral nucleon-nucleon potentials [2, 3] with the inclusion of three-body forces and self-energy effects. In this contribution we perform a numerical analysis of Khodel's method [6] for the singlet case.

1 Khodel's method

In this section we explain the method employed to solve the BCS equations by partial-wave decomposition. The BCS equation reads in terms of the NN potential $V(\mathbf{k},\mathbf{k}') = \langle \mathbf{k}\,|V|\,\mathbf{k}'\rangle$ as follows

$$\Delta\left(\mathbf{k}\right) = -\sum_{\mathbf{k}'}\langle \mathbf{k}\,|V|\,\mathbf{k}'\rangle \frac{\Delta\left(\mathbf{k}'\right)}{2E\left(\mathbf{k}'\right)}\,, \tag{1}$$

with $E(\mathbf{k})^2 = \xi(\mathbf{k})^2 + |\Delta(\mathbf{k})|^2$ and where $\xi(\mathbf{k}) = \varepsilon(\mathbf{k}) - \mu$, $\varepsilon(\mathbf{k})$ denotes the single-particle energy and μ is the chemical potential. We can decompose both the interaction and the gap function

$$\langle \mathbf{k}\,|V|\,\mathbf{k}'\rangle = 4\pi\sum_l (2l+1) P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}') V_l(k,k') \tag{2}$$

$$\Delta(\mathbf{k}) = \sum_{lm}\sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\hat{\mathbf{k}})\Delta_{lm}(k)\,, \tag{3}$$

where $Y_{lm}(\hat{\mathbf{k}})$ denotes the spherical harmonics, l and m are the quantum numbers associated with the orbital angular momentum and its projection along the z axis and $P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')$ refers to the Legendre polynomials. After performing an angle-average approximation we have the following equation for any value of l

$$\Delta_l^j(k) = \sum_{l'}\frac{(-1)^\Lambda}{\pi}\int dk'\; V_{ll'}^j(k,k')\frac{\Delta_{l'}^j(k')}{E(k')}k'^2\,, \tag{4}$$

where $\Lambda = 1 + (l - l')/2$, j refers to the total angular momentum ($\boldsymbol{J} = \boldsymbol{l} + \boldsymbol{S}$) quantum number including spin $\boldsymbol{S}$ and now $E(k)^2 = \xi(k)^2 + \sum_{jl} \Delta_l^j(k)^2$. Gaps with different l and j are coupled due to the energy denominator but we assume that different components of the interaction mainly act on non-overlapping intervals in density. To solve Eq. (4), we follow the approach suggested by Khodel *et al.* [6] that has been proven to be stable even for small values of the gap and to require only the initial assumption of a scale factor δ (results will be δ-independent, as will be shown in Sect. 2). We define an auxiliary potential W according to

$$W_{ll'}(k,k') = V_{ll'}(k,k') - v_{ll'}\phi_{ll'}(k)\phi_{ll'}(k') \,, \tag{5}$$

where $\phi_{ll'}(k) = V_{ll'}(k,k_F)/V_{ll'}(k_F,k_F)$ and $v_{ll'} = V_{ll'}(k_F,k_F)$ so that $W_{ll'}(k,k')$ vanishes on the Fermi surface. The coupled gap equations can be rewritten as

$$\Delta_l(k) - \sum_{l'} (-1)^{\Lambda} \int d\tau' \; W_{ll'}(k,k') \frac{\Delta_{l'}(k')}{E(k')} = \sum_{l'} D_{ll'}\phi_{ll'}(k) \,, \tag{6}$$

where $d\tau = k^2 dk/\pi$ and the coefficients $D_{ll'}$ satisfy

$$D_{ll'} = (-1)^{\Lambda} v_{ll'} \int d\tau \; \phi_{ll'}(k) \frac{\Delta_{l'}(k)}{E(k)} \,. \tag{7}$$

The gap is defined as follows

$$\Delta_l(k) = \sum_{l_1 l_2} D_{l_1 l_2} \chi_l^{l_1 l_2}(k) \,, \tag{8}$$

where

$$\chi_l^{l_1 l_2}(k) - \sum_{l'} (-1)^{\Lambda} \int d\tau' \; W_{ll'}(k,k') \frac{\chi_{l'}^{l_1 l_2}(k')}{E(k')} = \delta_{ll_1}\phi_{l_1 l_2}(k) \,, \tag{9}$$

and $\delta_{ll'}$ is the scale factor. The property that $W_{ll'}(k,k')$ vanishes on the Fermi surface ensures a very weak dependence of $\chi_l^{l_1 l_2}(k)$ on the exact value of the gap so that, in first approximation, it is possible to rewrite the previous equation (9) as

$$\chi_l^{l_1 l_2}(k) - \sum_{l'} (-1)^{\Lambda} \int d\tau' \; W_{ll'}(k,k') \frac{\chi_{l'}^{l_1 l_2}(k')}{\sqrt{\xi^2(k') + \delta^2}} = \delta_{ll_1}\phi_{l_1 l_2}(k) \,. \tag{10}$$

We use this equation to evaluate $\chi_l^{l_1 l_2}(k)$ initially by matrix inversion, then we use this function to self-consistently evaluate $D_{ll'}$. Finally, we solve the system given by Eqs. (7)–(9) in a self-consistent procedure as shown in Fig. 1 (left panel). We always assumed $\mu = \varepsilon_F$ and adopted the relativistic version of the single-particle energy $\varepsilon(k) = \sqrt{k^2 + M_N^2}$, where M_N is the nucleon mass. For the pairing potential $V(p,k)$ we introduce the following ansatz:

$$V(p,k) = V_{2\mathrm{B}}(p,k) + \sum_m V_{3\mathrm{B}}(p,k,m) \simeq V_{2\mathrm{B}}(p,k) + V_{2\mathrm{B}}^{\mathrm{eff}}(k_F,p,k) \,, \tag{11}$$

where $V_{2\mathrm{B}}$ is the NN potential [2] at N3LO order in the chiral expansion and the three-body potential is approximated by an effective two-body density-dependent potential V_{2B}^{eff} derived by Holt *et al.* in Refs. [4, 5]. When considering self-energy effects, we simply perform the transformation $M_N \to M_N^*$ using the effective mass obtained by Holt *et al.* in Ref. [7] using a density matrix expansion technique.

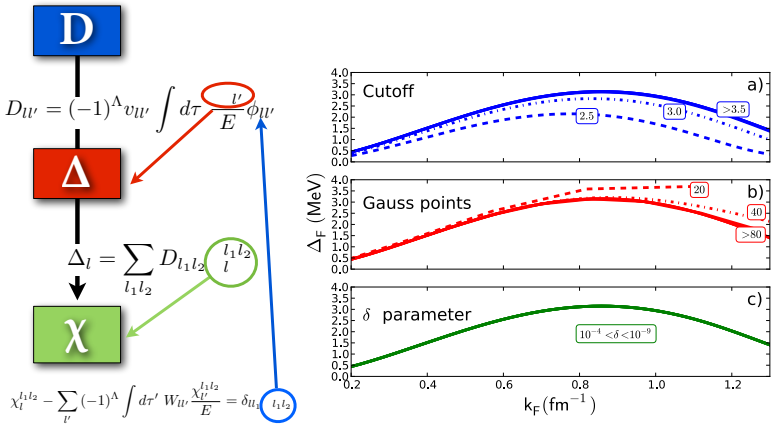

Figure 1: **Left**: Self-consistent procedure (Eqs. 7–9) for the solution of the gap equation according to Khodel's prescription [6]. **Right**: Numerical analysis of Khodel's procedure for the singlet channel in neutron matter: **a)** cutoff, **b)** Gaussian integration points and **c)** δ dependence. This method is a very stable procedure if satisfactory values of n_{gauss} and Λ_k are employed.

2 Results and Numerical analysis

In the neutron matter case, at the two-body level, there is good agreement with the gap computed from well known realistic potentials like the CD-Bonn or Nijmegen interactions [9], except for larger densities where the N3LO gap exhibits a higher value (phase shifts from the chiral N3LO potential exhibit more attraction than the CD-Bonn potential for high momenta [8]). We tested Khodel's method [6] against the variation of the following three parameters: n_{gauss} (number of Gauss integration points), Λ_k (cutoff for integrals in the momentum space, see Eq. (4)) and δ (the scale factor). In Fig. 1 (right side) we summarise our results. In the upper panel (**a**) we calculated Δ_F for different values of the momentum cutoff (using $n_{gauss} = 200$ and $\delta = 1 \times 10^{-10}$ MeV) where in the second panel (**b**) we varied n_{gauss} (keeping $\Lambda_k = 4.5$ fm^{-1} and $\delta = 1 \times 10^{-10}$ MeV) and in the lower panel (**c**) we changed δ (with $n_{gauss} = 200$ and $\Lambda_k = 4.5$ fm^{-1}) by orders of magnitude. Our conclusion is that the method proposed by Khodel [6] is a very stable procedure to study nuclear superfluidity if a reasonable number of Gaussian points (≥ 100) and a realistic momentum cutoff (≥ 4 fm^{-1}) are employed. In Fig. 2 we compare our full calculation for the gap, i.e., with the complete potential in Eq. (11) and the density-dependent effective mass, with recent results by Hebeler *et al.* [8], where the authors started from a chiral N3LO interaction and evolved to a sharp low-momentum interaction. Also presented for comparison are *ab-initio* results obtained in the last several years: Auxiliary Field Diffusion Monte Carlo (AFDMC) [10] with AV8' + UIX potentials, Quantum Monte Carlo (QMC) [11], where the authors have retained the S-wave part of the AV18 interaction, and Correlated Basis Functions (CBF) [12] still with AV8' plus UIX. We observe that at low densities the gap behaviors are very similar, with the exception of QMC, but beyond Fermi momenta

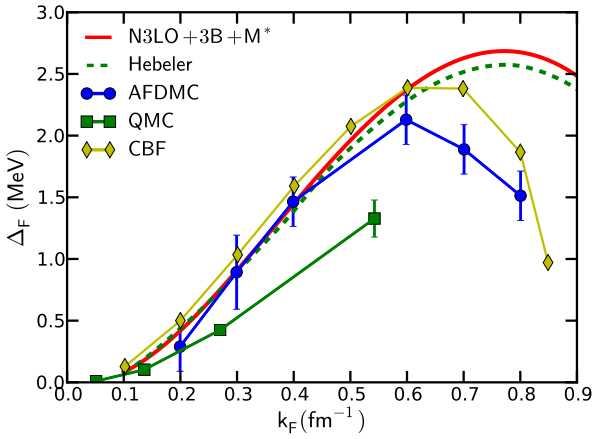

Figure 2: The $^1S_0$ gap for neutron matter computed with the realistic chiral potential of [2] at N3LO plus the three-body contribution of Eq. (11) and the inclusion of the effective mass in comparison with *ab-initio* simulations.

of $k_F \approx 0.6$ fm^{-1} the gaps computed with the Argonne potentials decrease rapidly in contrast to those from chiral interactions. At the present time, it is hard to assess if disagreement is due to different choices in the nuclear Hamiltonian or different many-body methods.

Acknowledgments

Work supported in part by US DOE Grant No. DE-FG02-97ER-41014.

References

[1] S. Maurizio, J. W. Holt and P. Finelli, Phys. Rev. C **90** (2014) 044003.
[2] R. Machleidt and D. R. Entem, Phys. Rept. **503** (2011) 1 and references therein.
[3] E. Epelbaum, H.-W.Hammer and Ulf-G. Meissner, Rev. Mod. Phys. **81** (2009) 1773 and references therein.
[4] J. W. Holt, N. Kaiser and W. Weise, Phys. Rev. C **79** (2009) 054331.
[5] J. W. Holt, N. Kaiser and W. Weise, Phys. Rev. C **81** (2010) 024002.
[6] V. V. Khodel, V. A. Khodel and J. W. Clark, Nucl. Phys. A **598** (1996) 390.
[7] J. W. Holt, N. Kaiser, and W. Weise, Eur. Phys. J. A **47** (2011) 128.
[8] K. Hebeler, A. Schwenk and B. Friman, Phys. Lett. B **648** (2007) 176.
[9] M. Hjorth-Jensen and D. J. Dean, Rev. Mod. Phys. **75** (2003) 607.
[10] S. Gandolfi, A. Y. .Illarionov, F. Pederiva, K. E. Schmidt and S. Fantoni, Phys. Rev. C **80** (2009) 045802.
[11] A. Gezerlis and J. Carlson, Phys. Rev. C **77** (2008) 032801(R).
[12] A. Fabrocini, S. Fantoni, A. Y. Illarionov and K. E. Schmidt, Phys. Rev. Lett. **95** (2005) 192501.