The QCD critical end point driven by an external magnetic field in asymmetric quark matter

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The effect of the isospin/charge asymmetry and an external magnetic field in the location of the critical end point (CEP) in the QCD phase diagram is investigated. By using the 2+1 flavor Nambu–Jona-Lasinio model with Polyakov loop (PNJL), it is shown that the isospin asymmetry shifts the CEP to larger baryonic chemical potentials and smaller temperatures, and in the presence of a large enough isospin asymmetry the CEP disappears. Nevertheless, a sufficiently high external magnetic field can drive the system into a first order phase transition again.

The QCD phase diagram under extreme conditions of density, temperature and magnetic field is the subject of intense studies [1]. Understanding the effect of an external magnetic field on the structure of the QCD phase diagram is very important: these extremely strong magnetic fields are expected to affect the measurements in heavy ion collisions (HIC) at very high energies, to influence the behavior of the first stages of the Universe and are also relevant to the physics of compact astrophysical objects like magnetars.

On the other hand, the effect of the isospin/charge asymmetry in the QCD phase diagram is also very interesting due to its role on the location of the critical end point (CEP): it was shown that for a sufficiently asymmetric system the CEP is not present [2, 3].

In the present work we describe quark matter subject to strong magnetic fields within the 2+1 PNJL model. The PNJL Lagrangian with explicit chiral symmetry breaking where the quarks couple to a (spatially constant) temporal background gauge field, represented in terms of the Polyakov loop and in the presence of an external magnetic field is given by [4]:

$$\mathcal{L} = \bar{q}\left[i\gamma_\mu D^\mu - \hat{m}_f\right] q \;+\; G\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]$$
$$-K\left\{\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right\} + \mathcal{U}\left(\Phi,\bar{\Phi};T\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where the quark sector is described by the SU(3) version of the Nambu–Jona-Lasinio model which includes the scalar-pseudoscalar (chiral invariant) and the t'Hooft six fermion interactions that breaks the axial $U_A(1)$ symmetry. The $q = (u, d, s)^T$ represents a quark field with three flavors, $\hat{m}_f = \mathrm{diag}_f(m_u^0, m_d^0, m_s^0)$ is the corresponding (current) mass matrix, $\lambda_0 = \sqrt{2/3}I$ where I is the unit matrix in the three flavor space, and $0 < \lambda_a \leq 8$ denote the Gell-Mann matrices. The coupling between the magnetic field B and quarks, and between the effective gluon field and quarks is implemented *via* the covariant derivative $D^\mu = \partial^\mu - iq_f A^\mu_{EM} - iA^\mu$ where q_f represents the quark electric charge ($q_d = q_s = -q_u/2 = -e/3$), A^{EM}_μ and $F_{\mu\nu} = \partial_\mu A^{EM}_\nu - \partial_\nu A^{EM}_\mu$ are used to account for the external magnetic field and $A^\mu(x) = g_{strong}\mathcal{A}^\mu_a(x)\frac{\lambda_a}{2}$ where $\mathcal{A}^\mu_a$ is the $\mathrm{SU}_c(3)$ gauge field. We consider a static and constant magnetic field in the z direction, $A^{EM}_\mu = \delta_{\mu 2} x_1 B$. In the Polyakov gauge and at finite temperature the spatial components of the gluon field are neglected: $A^\mu = \delta^\mu_0 A^0 = -i\delta^\mu_4 A^4$. The trace of the Polyakov line defined by $\Phi = \frac{1}{N_c}\langle\langle \mathcal{P}\exp i\int_0^\beta d\tau\, A_4(\vec{x},\tau)\,\rangle\rangle_\beta$ is the Polyakov loop which is the *exact* order parameter of the $\mathbb{Z}_3$ symmetric/broken phase transition in pure gauge.

To describe the pure gauge sector an effective potential $\mathcal{U}\left(\Phi,\bar{\Phi};T\right)$ is chosen in order to reproduce the results obtained in lattice calculations [5]:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{a\left(T\right)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\right], \qquad (2)$$

where $a\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$, $b(T) = b_3\left(\frac{T_0}{T}\right)^3$. The standard choice of the parameters for the effective potential $\mathcal{U}$ is $a_0 = 3.51$, $a_1 = -2.47$, $a_2 = 15.2$, and $b_3 = -1.75$. T_0 is the critical temperature for the deconfinement phase transition within a pure gauge approach: it was fixed to a constant $T_0 = 270$ MeV, according to lattice findings. The parameters of the model are $\Lambda = 602.3\,\mathrm{MeV}$, $m_u^0 = m_d^0 = 5.5\,\mathrm{MeV}$, $m_s^0 = 140.7\,\mathrm{MeV}$, $G\Lambda^2 = 1.385$ and $K\Lambda^5 = 12.36$.

The thermodynamical potential for the three flavor quark sector, Ω, in the mean field approximation is written as

$$\Omega(T,\,B,\,\mu_f) = 2G\sum_{f=u,\,d,\,s}\left\langle\bar{q}_f q_f\right\rangle^2 - 4K\left\langle\bar{q}_u q_u\right\rangle\left\langle\bar{q}_d q_d\right\rangle\left\langle\bar{q}_s q_s\right\rangle + \left(\Omega_f^{vac} + \Omega_f^{mag} + \Omega_f^{med}\right), \qquad (3)$$

where the vacuum Ω_f^{vac}, the magnetic Ω_f^{mag}, the medium contributions Ω_f^{med} and the quark condensates $\langle\bar{q}_f q_f\rangle$ have been evaluated with great detail in [6, 7]. The mean field equations are obtained by minimizing the thermodynamical potential (3) with respect to the order parameters $\langle\bar{q}_f q_f\rangle$, Φ and $\bar{\Phi}$.

We start the discussion of our results by the location of the CEP when no external magnetic field is present.

It has been shown that the location of the CEP depends on the isospin [8]: as an example, in $\beta-$equilibrium matter the CEP occurs at larger baryonic chemical potentials and smaller temperatures [8]. Indeed, we are interested in d-quark rich matter as it occurs in neutron stars and in HIC: isospin asymmetry in neutron matter has $\mu_d \sim 1.2\mu_u$, and presently the attained isospin asymmetry in HIC corresponds to $\mu_u < \mu_d < 1.1\mu_u$. In the present work the effect of isospin on the CEP is studied: we increase systematically μ_d with respect to μ_u taking the s-quark chemical potential equal to zero ($\mu_s = 0$ leads to all CEP's occur at $\rho_s = 0$).

The results for the CEP in the previous conditions are presented in Fig. 1. For reference we also show the red full point that corresponds to the CEP with $\mu_u = \mu_d = \mu_s$. When the isospin asymmetry is increased the CEP moves to smaller temperatures and larger baryonic chemical potentials.

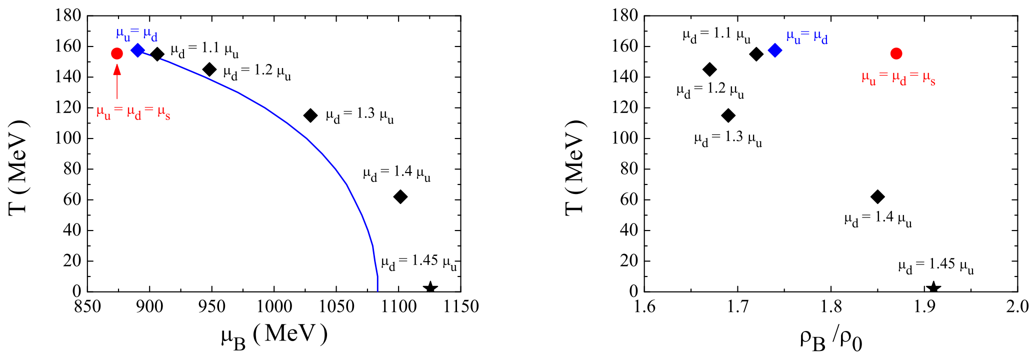

Figure 1: The influence of the isospin in the location of the CEP within the PNJL model: the full line is the first order phase transition line for zero isospin matter ($\mu_u = \mu_d$, $\mu_s = 0$). The chemical potential for the strange quark is always taken equal to zero, except the for the red point ($\mu_u = \mu_d = \mu_s$) which is given for reference. When $\mu_d > 1.45\mu_u$ the CEP doesn't exist anymore.

When the asymmetry is large enough, $\mu_d = 1.45\mu_u$, the CEP disappears (this CEP is represented in Fig. 1 by a star at $T = 0$). This scenario leads to $|\mu_u - \mu_d| = |\mu_I| = |\mu_Q| = 130$ MeV, below the pion mass and, accordingly, no pion condensation occurs under these conditions.

The CEP for (T, ρ_B) plane is shown in the right panel of Fig. 1. When $\mu_u < \mu_d < 1.2\mu_u$ the baryonic density of the CEP decreases with asymmetry but for $\mu_d \gtrsim 1.2\mu_u$ the opposite occurs and at the threshold ($\mu_d = 1.45\mu_u$) $\rho_B \sim 1.91\rho_0$.

Now, we investigate how a static external magnetic field will influence the localization of the CEPs previously calculated. The results are plotted in Fig. 2. In the left panel of Fig. 2 the red dots correspond to symmetric matter ($\mu_u = \mu_d = \mu_s$) and reproduce qualitatively the results previously obtained within the NJL model [9] being the trend qualitatively similar: the increasing of the intensity of the magnetic field leads to an increase of the CEP's temperature and to a decrease of the CEP's baryonic chemical potential until the critical value $eB \sim 0.4$ GeV^2; for stronger magnetic fields, both T and μ_B increase. In the right panel of Fig. 2 the CEP is given in a T vs. ρ_B plane. The results show that when eB increases from 0 to 1 GeV^2 the baryonic density at the CEP increases from $2\rho_0$ to $14\rho_0$.

Taking the isospin symmetric matter scenario $\mu_u = \mu_d$ and $\mu_s = 0$, the effect of the magnetic field on the CEP is very similar to the previous one (see blue diamonds in Fig. 2): the CEP's temperature is only slightly larger and the CEP's baryonic density is slightly smaller.

Also interesting is the case that occurs for the very asymmetric matter scenario: a first order phase transition driven by the magnetic field takes place if $\mu_d \gtrsim 1.45\mu_u$. Taking the threshold value $\mu_d = 1.45\mu_u$ it is seen that for eB <0.1 GeV^2 two CEPs may appear. Indeed, for sufficiently small values of eB the T^{CEP} is small and the Landau level effects are visible.

A magnetic field affects in a different way u and d quarks due to their different electric charge. A consequence is the possible appearance of two or more CEPs for a given magnetic field intensity. Two critical end points occur at different values of T and μ_B for the same magnetic field intensity for fields $0.03 \lesssim eB \lesssim 0.07$ GeV^2. Above 0.07 GeV^2 only one CEP remains. For stronger fields we get $T^{CEP} > 100$ MeV: Landau level effects are completely

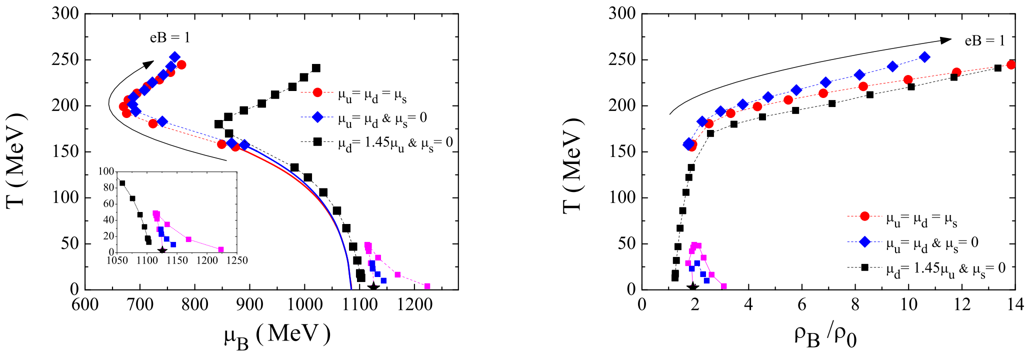

Figure 2: Effect of an external magnetic field on the location of the CEP: T^{CEP} vs μ_B^{CEP} (left panel) and T^{CEP} vs ρ_B^{CEP} (right panel). The full lines correspond to the first order transitions at $eB = 0$. Three scenarios are shown: $\mu_u = \mu_d = \mu_s$ (red dots), $\mu_u = \mu_d$; $\mu_s = 0$ (blue diamonds) and $\mu_d = 1.45\mu_u$, $\mu_s = 0$ (black squares) corresponding to the threshold isospin asymmetry above which no CEP occurs. In the last scenario for strong enough magnetic fields and low temperatures two or more CEP exist at different temperatures for a given magnetic field intensity (pink and blue squares).

washed out at these temperatures.

Acknowledgments

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