Studying of SU(N) LGT in External Chromomagnetic Field with QCDGPU

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It is well-known that in extreme conditions the behavior of a system is different than in standard ones, see for example [1]. The presence of external (chromo)magnetic field is a particular case of these nonstandard conditions. One of the most powerful methods to investigate quantum field theory (also in extreme conditions) is the lattice Monte Carlo (MC) approach, which allows to investigate problems which cannot be solved analytically. Lattice methods became popular with the increase in performance of computers and the use of GPUs significantly increases the computing performance.

Major research collaborations develop software for MC simulations (see, for example, [2] for a review) that is adopted for their tasks and hardware. However, due to the severe competition between hardware manufacturers, cross-platform principles have to be considered while creating such type of programs.

The open-source package QCDGPU is a package that enables to perform MC lattice simulations of SU(N) gauge theories and O(N) models on AMD and nVidia GPUs. The main feature of the package is the possibility of investigating physical phenomena in presence of an external chromomagnetic field. All functions needed for MC simulations are performed on GPU. The CPU part of the program computes averages of measured quantities over the MC run, prepares the computational device, provides input/output functions, etc. The GPU kernels are implemented in OpenCL to enable execution on devices by various vendors. The host part of the code is written in C++. The package is optimized for execution on GPUs, but can also run on CPUs.

QCDGPU performs gauge configuration production as well as the measurements of the lattice observables. In particular, the package allows to measure the most common lattice observables: mean values of plaquette and action, the Wilson loop, the Polyakov loop, its square and 4-th power. Also some nonconventional measurements are implemented: measurement of components of the $F_{\mu\nu}$ tensor, spatial distribution of the Polyakov loop and action. The last quantities are useful for the investigation of phenomena in presence of external chromomagnetic fields.

The QCDGPU package allows to investigate SU(N) gluodynamics in n-dimensional space. By default the 4D hypercubic lattice is considered but the number n can be changed through the run parameters.

The standard Wilson action for lattice SU(N) theories is considered,

$$S_W = \beta \sum_{x,\nu>\mu} \left(1 - \frac{1}{N} \operatorname{Re} \operatorname{Tr} U_{\mu\nu}(x) \right),$$

summation is performed on all the lattice sites and on all the pairs of space-time directions μ and ν, $\beta = 2N/g^2$ is inverse coupling, $U_{\mu\nu}(x)$ is the plaquette in the point $x = (x, y, z, t)$,

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x), \tag{1}$$

$\hat{\mu}$ is the unit vector in direction μ. The field variables $U_\mu(x)$ are SU(N) matrices. For SU(2) and SU(3) groups the following symmetry properties are used to represent link variables:

$$\mathrm{SU}(2): \quad U = \begin{pmatrix} u1 & u2 \\ -u2^* & u1^* \end{pmatrix}$$

$$\mathrm{SU}(3): \quad U = \begin{pmatrix} u1 & u2 & u3 \\ v1 & v2 & v3 \\ w1 & w2 & w3 \end{pmatrix}, \ \vec{w} = \vec{u}^* \times \ \vec{v}^*.$$

So SU(2) matrices are defined through 4 real numbers and SU(3) matrices are defined through 12 reals instead of 18. Due to the GPU architecture, those matrices are represented as the appropriate number of 4-component vectors. For SU(2) this is one `double4` (`float4`) structure,

```
U.uv1 = (Re u1, Im u1, Re u2, Im u2),
```

while in SU(3) case three such structures are needed for one matrix:

```
U.uv1 = (Re u1, Re u2, Re u3, Re v3);
U.uv2 = (Im u1, Im u2, Im u3, Im v3);
U.uv3 = (Re v1, Re v2, Im v1, Im v2).
```

Other matrix elements are calculated during execution, when they are needed.

For link update the standard multi-hit heat-bath algorithm [3, 4] is implemented. To parallelize lattice update the checkerboard scheme is used.

One of the main purposes of QCDGPU is to enable the investigation of phenomena in external chromomagnetic field. The external Abelian chromomagnetic field is realized through twisted boundary conditions (t.b.c.) [5]. This approach is similar to one used in [6]. The field is introduced as additional flux through plaquettes. This allows to work with the external field as with a continuous quantity. The t.b.c. have the following form:

$$U'_\mu(N_x - 1, y, z, t) = U_\mu(N_x - 1, y, z, t)\Omega, \qquad \Omega = e^{i\varphi}\delta_{\mu 2}, \qquad \forall\, y, z, t,$$

$\varphi = a^2 N_x H$ is the field flux in z direction through the $N_x \times 1$ stripe of plaquettes, see Fig. 1.

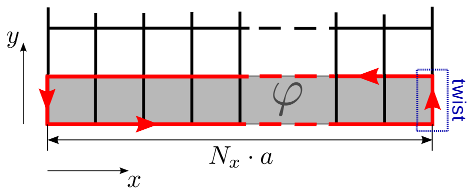

Figure 1: Chromomagnetic flux φ through a stripe of plaquettes

This method is valid for values of the external field flux small enough to neglect the commutators with the external field.

The possibility of varying the external field flux in the QCDGPU package allows to study the dependence of the deconfinement phase transition temperature, the gluon magnetic mass, etc., on the applied external field. One more problem that can be investigated with QCDGPU is the vacuum magnetization in SU(N) gluodynamics.

A general approach to investigate this phenomenon on the lattice is to find the (global) minimum of the action as function of the applied external field. A non-trivial minimum of the action means that a non-zero chromomagnetic field exists in the ground state.

The possibility of vacuum magnetization was investigated recently for SU(3) lattice gluodynamics [5]. The simulations were performed on 4×16^3 lattice at $\beta = 6$. There are two neutral chromomagnetic field components in this group. They correspond to the 3-rd and 8-th Gell-Mann matrices. So the action was considered as function of two field variables. Three sections of this 2D surface were investigated: $\varphi_8 = 0$, $\varphi_3 = 0$ and $\varphi_8 = 6.17\varphi_3$, to compare results with [7]. For the section $\varphi_8 = 0$ the non-trivial minimum of the action was obtained at 95% confidence level (Fig. 2).

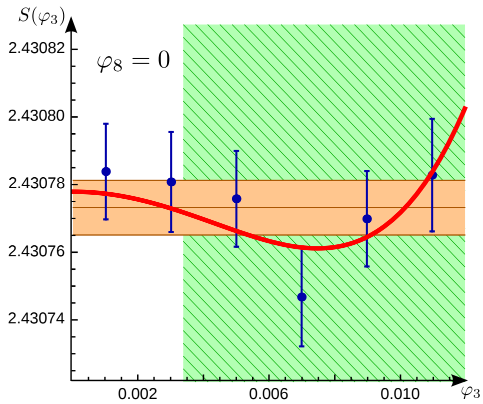

Figure 2: Dependence of action S on applied external field flux φ_3 at $\varphi_8 = 0$. The bars correspond to the 95% CI in the bins, the horizontal stripe is zero level (mean value and 95% CI), the thick curve corresponds to the χ^2-fit result. 95% CI for the position of the minimum is shown by the hatched background

From χ^2 analysis the following estimates and 95% confidence interval (CI) were obtained for generated field:

$$\varphi_3 = \left(7.48^{+11.3}_{-4.11}\right) \times 10^{-3} \qquad \text{or} \qquad H_3 = \left(488^{+734}_{-268}\right)\ \text{MeV}^2.$$

The estimated temperature is $T \simeq 370$ MeV.

Also the possibility of direct measurement of the Cartesian components of the SU(N) electromagnetic field tensor $F_{\mu\nu}$ is implemented in the QCDGPU package. The expansion of the plaquette (1) for small lattice spacing,

$$U_{\mu\nu}(n) = 1 + ia^2 F^b_{\mu\nu}(n)\lambda^b - \frac{1}{2}a^4 F^b_{\mu\nu}(n)F^{b'}_{\mu\nu}(n)\lambda^b\lambda^{b'} + \mathcal{O}(a^5), \tag{2}$$

where λ^b are the gauge group generators, is used to extract the field strength. This is done by multiplying (2) by the generators λ^b, and taking the trace [8]:

$$a^2 F^b_{\mu\nu} = -i\,\mathrm{Tr}\left[U_{\mu\nu}\lambda^b\right] + \mathcal{O}(a^4). \tag{3}$$

This function of the QCDGPU package provides an alternative approach to investigate the vacuum magnetization phenomenon [8]. The method is based on a study of the distribution function of the total chromomagnetic field strength H. This total field consists of the condensed $\vec{H}_c$ and "quantum" parts. The Cartesian components of $\vec{H}$ are obtained directly from MC run by Eq. (3). They are supposed to be Gaussian ones with mean values $\vec{H}_c$ and variance σ^2. The variance can be obtained as usual sample variance over the MC run. Thus, the absolute values of the field $\vec{H}$ have the following distribution (in dimensionless units):

$$p(\eta) = \frac{16\eta}{\zeta\pi^{3/2}}\, e^{-\zeta^2/4} e^{-4\eta^2/\pi} \sinh\frac{2\zeta\eta}{\sqrt{\pi}},$$

$\eta = H/H_0$, $\zeta = H_c\sqrt{2}/\sigma$, $H_0 = 4\sigma/\sqrt{2\pi}$ is the mean value of H at $H_c = 0$. Consequently, the mean value of η is

$$\bar{\eta} = \frac{16}{\zeta\pi^{3/2}}\, e^{-\zeta^2/4} \int_0^\infty \eta^2 e^{-4\eta^2/\pi}\, \sinh\frac{2\zeta\eta}{\sqrt{\pi}}\, d\eta = f(\zeta).$$

This can be associated with its estimator from lattice simulations and $f(\zeta)$ must be inverted numerically to calculate the condensate field.

The open-source QCDGPU package is developed as a tool for MC lattice simulations of SU(N) gluodynamics in external chromomagnetic field. The package performs gauge configurations production as well as measurements. All procedures for MC simulations are realized in OpenCL and are optimized for execution on GPUs. CPU performs only averaging of measured quantities over run and auxiliary work for the interaction with the compute device. Apart from common lattice quantities, QCDGPU provides measurement of several non-standard ones. This allows to investigate such problems as the dependence of different quantities on the applied chromomagnetic field, spontaneous vacuum magnetization, etc. The introduction of fermionic fields is among the plans for future developments.

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