# Cosmological Search for Ultra-Light Axions

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Ultralight axions (ULAs) with masses in the range $10^{-33}$ eV $\leq m_a \leq 10^{-18}$ eV (motivated by string theory) might contribute to the dark-matter or dark-energy density of the Universe. ULAs would suppress the growth of structure on small scales and change the shape of the cosmic microwave background (CMB) anisotropy power spectra. In this work, we compute cosmological observables over the full ULA mass range and then use them to search for evidence of ULAs using CMB temperature data from the *Planck* satellite, large-scale CMB polarization data from Wilkinson Microwave Anisotropy Probe (WMAP), smaller-scale CMB experiments, as well as the WiggleZ galaxy-redshift survey. In the mass range $10^{-32}$ eV $\leq m_a \leq 10^{-25.5}$ eV, the ULA relic-density must obey the constraint $\Omega_a h^2 \leq 0.006$ at 95%-confidence. For $m_a \gtrsim 10^{-24}$ eV, ULAs are indistinguishable from standard cold dark matter on the length scales probed while for $m_a \lesssim 10^{-32}$ eV, ULAs are allowed to compose a significant fraction of the dark energy. If primordial gravitational waves are detected, limits to the primordial isocurvature fraction will put severe constraints on ULA dark matter. In the future, weak-lensing measurements of the CMB will yield even more powerful probes of the ULA hypothesis.

## 1 Motivation

Originally introduced to solve the strong *CP* problem [1, 2, 3], axions are a well-motivated dark-matter candidate [4, 5]. In the context of the axiverse scenario, in which there are many axions with masses spanning many orders-of-magnitude covering the range $10^{-33}$ eV $\lesssim m_a \lesssim 10^{-18}$ eV, ultra-light axions (ULAs) could compose significant fractions of both the dark matter and the dark energy [6, 7, 8]. More generally, axion-like particles (ALPs) arise in string theory [9, 10, 11, 12], often as the Kaluza-Klein zero modes of anti-symmetric tensors compactified on extra dimensions. As discussed in many of the other workshop contributions, a variety of creative laboratory techniques have emerged to probe a wide swathe of ULA/ALP parameter space. All these techniques depend, however, on the highly model-dependent two-photon couplings of ULAs/ALPs.

The gravitational imprint of ULAs [for example, on the cosmic microwave background (CMB) or the distribution of galaxies at redshifts $z \lesssim 1$], however, is nearly model-independent, once their mass and density is specified [8]. For masses $m_a \lesssim 10^{-20}$ eV, ULA dark matter exhibits suppressed structure formation on cosmological length scales. If the ULA is the Goldstone boson of a global symmetry broken during inflation (and not subsequently restored) the relative entropy fluctuation between ULAs and radiation yields a detectable *isocurvature* imprint on the CMB.

Here we apply these effects to search for evidence of ULA dark matter or dark energy using *Planck* CMB data and the WiggleZ survey. These proceedings are a summary of Ref. [13], whose results are reproduced with permission (Copyright 2012 by The American Physical Society). We built on past work (in which constraints are obtained without a Boltzmann code [14]) by extending the standard CMB Boltzmann code CAMB[1] to include the evolution of cosmological perturbations in the presence of ULAs with any $m_a$ value.

## 2 Ultra-light axion cosmology

As a first step in exploring the axiverse, we consider a single ULA, described by a real scalar field $\phi_0$ (subject to a harmonic potential) with equation of motion

$$\ddot{\phi}_0 + 2\mathcal{H}\dot{\phi}_0 + m_a^2 a^2 \phi_0 = 0. \tag{1}$$

Here $\mathcal{H} = aH$ is the conformal Hubble parameter, where $a$ is the usual cosmological scale factor and $H$ the usual Hubble parameter with respect to physical time. Early on, when $m_a \ll 3H$, the scalar field rolls slowly, has equation-of-state parameter $w \simeq -1$, and constant energy density. A transition when $m_a = 3H$, defining the transition scale factor $a \equiv a_{\rm osc}$. Thenceforth, on timescales longer than the oscillation period $\sim m^{-1}$, the ULA field is well described as a non-relativistic fluid with $\rho \propto a^{-3}$ and $w \simeq 0$. The ULA relic abundance is then readily obtained to be

$$\Omega_a = \begin{cases} \frac{1}{6}(9\Omega_r)^{3/4}\left(\frac{m_a}{H_0}\right)^{1/2}\left(\frac{\phi_{0,i}}{M_{pl}}\right)^2 \text{ if } a_{\rm osc} < a_{\rm eq}\,, \\ \frac{9}{6}\Omega_m\left(\frac{\phi_{0,i}}{M_{pl}}\right)^2 \text{ if } a_{\rm eq} < a_{\rm osc} \lesssim 1\,, \\ \frac{1}{6}\left(\frac{m_a}{H_0}\right)^2\left(\frac{\phi_{0,i}}{M_{pl}}\right)^2 \text{ if } a_{\rm osc} \gtrsim 1\,, \end{cases} \tag{2}$$

where $\phi_{0,i}$ is the initial scalar field displacement, $M_{pl}^2 = 1/(8\pi G)$ is the Planck mass. Here $\Omega_r$ and $\Omega_m$ are the radiation and matter energy-densities today relative to the critical density.

The observed dark matter or dark-energy relic densities can be obtained for the ULA mass-range $10^{-33}$ eV $\lesssim m_a \lesssim 10^{-18}$ eV for sub-Planckian axion global $U(1)$-symmetry breaking scales and initial field misalignments [16, 8]. One important moment is the epoch of matter-radiation equality, which occurs when $a = a_{\rm eq} = (1+z_{\rm eq})^{-1} \simeq 2.93 \times 10^{-4}$. If $a_{\rm osc} \lesssim a_{\rm eq}$, the homogeneous piece of the ULA homogeneous field behaves as a non-relativistic relic while most observed cosmological large-scale structure (LSS) forms, and we call such ULAs "dark-matter like." If $a_{\rm osc} \gtrsim a_{\rm eq}$, the homogeneous piece of the ULA field behaves as a cosmological constant while LSS forms [7], and we call such ULAs "dark-energy like." Both regimes are evident in Fig. 1, where energy densities from the full (scalar-field+matter+cosmological constant) evolution are shown. To actually obtain field histories for our exploration of parameter space (which also requires perturbation evolution), we solve the exact Klein-Gordon equation [Eq. (1)] including all components in $\mathcal{H}$ until $a = a_{\rm osc}$, and then use a simple $w \simeq 0$, $\rho_a \propto a^{-3}$ solution afterwards.

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[1] CAMB is distributed and described at http://camb.info/.

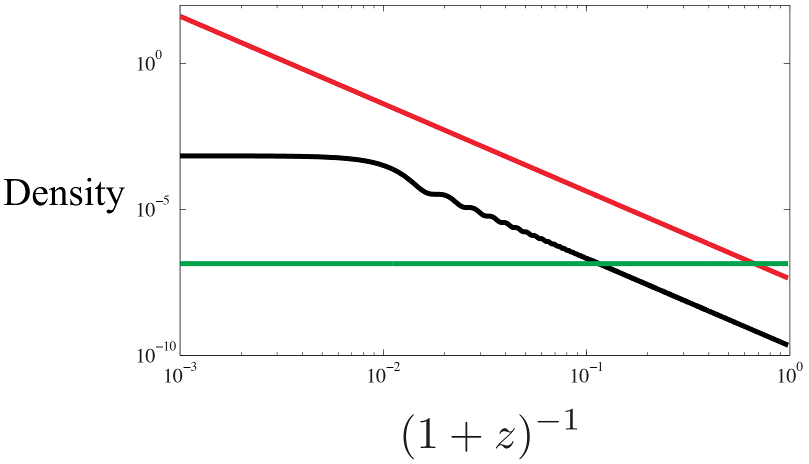

Figure 1: Time evolution of ULA density in arbitrary units (shown in black) for an $m_a = 10^{-30}$ eV axion comprising 1% of the dark matter, as a function of cosmological scale factor $a$. The red curve shows the matter density while the green curve shows the energy density associated with the cosmological constant. Modified and reproduced (with permission) from Ref. [15]. Copyright 2012 by The American Physical Society.

## 3 Perturbation evolution and observables

For the low values of $m_a$ we consider, the ULA de Broglie wavelength is macroscopic, and so is the ULA "Jeans" scale $k_J \sim \sqrt{m_a H}$, which today corresponds to a wavelength [6, 7, 17, 18, 20, 8]

$$\lambda_J \simeq 2.5 \text{ Mpc } h^{-1/2} \left(\frac{m_a}{10^{-25} \text{ eV}}\right)^{-1/2} . \tag{3}$$

For ULA in the "dark-matter like" regime, structure formation is suppressed at length scales $l < \lambda_J$ and in proportion to $\Omega_a/\Omega_d$, where $\Omega_d = (\Omega_a + \Omega_c)$ and $\Omega_c$ is the relic density of standard cold dark-matter (CDM). This fact is evident in the suppressed amplitude of the matter power-spectrum at small scales, as seen in Fig. 2. CMB anisotropies are altered in this regime for modes which enter the horizon when ULAs are still not redshifting as $\rho \propto a^{-3}$. In this case, gravitational potential wells decay more rapidly than they usually do, leading to enhanced higher-$l$ acoustic peak heights in the observed CMB power spectra, as seen in Fig. 2.

At lower masses still ($m_a \lesssim 10^{-25}$ eV), in the "dark-energy like" ULA regime, the expansion rate (and thus the growth rate of structure) is altered from its behavior in a $w = -1$ dark-energy cosmology. This leads to shifts of the CMB acoustic peak locations in $l$ at fixed values of other parameters, due to the altered angular-diameter distance to the surface of last-scattering, as seen in Fig. 2. Matter-radiation equality also occurs at a different time, changing the shape and amplitude of the matter power-spectrum, as seen in Fig. 2.

Our modified Boltzmann code, AXICAMB, handles perturbation evolution by solving the perturbed Klein-Gordon equation exactly when $a \leq a_{\rm osc}$. Later, when $a > a_{\rm osc}$, we treat ULAs as a fluid with an unusual sound speed, a result which can be rigorously justified using a WentzelKramersBrillouin (WKB) approximation. The resulting sound speed is [20]

$$c_a^2 = \frac{k^2/m_a^2}{4/\left(1+z\right)^2 + k^2/m_a^2}, \tag{4}$$

where $k$ is the perturbation wave number. This scale-dependent $c_a^2$ captures the suppression of small-scale structure by ULAs. This code is nearly ready for public release, and we will make it publicly available in coming months, so that observers can include new data in the search for ULAs.

# 4 Data and constraints

To determine the allowed parameter space, we use *Planck* 2013 temperature anisotropy data, WMAP large-scale CMB polarization data (to break the degeneracy between the perturbation amplitude $A_s$ and the optical depth to reionization $\tau$), as well as small-scale CMB data from the Atacama Cosmology Telescope (ACT) and South-Pole Telescope (SPT) [21, 22, 23, 24]. To complement this data, we also include measurements of the galaxy clustering power-spectrum from the WiggleZ galaxy survey [25]. We vary the standard 6$\Lambda$ CDM parameters, in addition to the ULA parameters $m_a$ and $\Omega_a h^2$. The degeneracy of ULA parameters with the standard 6 is strongly dependent on the mass, making the parameter space difficult to sample. To address this difficulty, we use a nested sampling technique, as described in Ref. [13]. We ultimately obtain the constraints to ULA parameter space shown in Fig. 3. Marginalizing over all other parameters, we find that in the constrained region of parameter space ($10^{-32}$ eV $\lesssim m_a \lesssim 10^{-25.5}$ eV), $\Omega_a h^2 \lesssim 6 \times 10^{-3}$ at 95%-confidence, while at higher and lower masses, ULAs can compose nearly all of the dark matter or dark energy, respectively.

# 5 Conclusions and future work

There are many exciting possibilities for future cosmological tests of ULAs and standard QCD axions. The most powerful is related to the phase structure of the CMB acoustic peaks. These reflect the initial conditions of the primordial plasma, which are now known to be predominantly adiabatic. This is consistent with simple inflationary models. If the Peccei-Quinn symmetry is broken during the inflationary era and not restored, the axion will carry *isocurvature* perturbations, altering the phase structure of the CMB acoustic peaks [26]. The 2013 *Planck* satellite data impose a limit (through the lack of isocurvature) of $\left(H_I/\phi_{i,0}\right)\left[\Omega_a/\left(\Omega_a+\Omega_c\right)\right] \lesssim 4 \times 10^{-5}$, where $H_I$ is the Hubble parameter during inflation [27, 28]. In the "dark-matter like" ULA mass range, or for QCD axions, the relic density may then be related to the initial field value using standard expressions.

This then yields the limits

$$\frac{\Omega_a}{\Omega_a+\Omega_c} \lesssim 10^{-3}\left(\frac{10^{14}\ \mathrm{GeV}}{H_I}\right) \tag{5}$$

for ULA dark matter and

$$\frac{\Omega_a}{\Omega_a+\Omega_c} \lesssim 10^{-12}\left(\frac{10^{14}\ \mathrm{GeV}}{H_I}\right)^{7/2} \tag{6}$$

for QCD axions. The fractional slope in the QCD case results from temperature-dependent corrections to the axion mass during the onset of coherent oscillation of the field. It is important to note that the corresponding limits are not known in the "dark-energy like" ULA mass regime, because the isocurvature transfer function of such ULAs, while known from our AXICAMB code using analytic initial conditions we have derived, has not yet been self-consistently included in a cosmological Monte-Carlo Markov chain analysis of CMB data. A robust detection of primordial gravitational waves at the level of the current limits [$H_I \sim 10^{14}$ GeV] would thus either severely constrain the cosmic relic density of axions/ULAs, or require a non-canonical scenario for their production. Alternatively, robust evidence for ULA or standard QCD axion dark matter could indicate a very dim forecast for experiments targeting primordial CMB $B$-mode polarization.

We note that our limits from the matter power-spectrum result from a simple treatment of the bias between the galaxy density field and the ULA density field which we will work to improve once a complete treatment of nonlinear structure formation from ULAs is developed. In the meantime, we will use our code with new data sets, such as the nearly $40\sigma$ observation [29] by *Planck* of weak lensing of the CMB by foreground structure. This data set tests both the kinematics of cosmic expansion when ULAs replace some of the dark matter or dark energy, and also the altered growth of structure in such a cosmology. We show an example case in Fig. 4, where the effect of low and high ULA mass-fractions is contrasted with observations of the lensing deflection-angle dimensionless power-spectrum by the ACT experiment. The era of precision cosmology promises ever more sensitive tests of axionic dark matter and dark energy.

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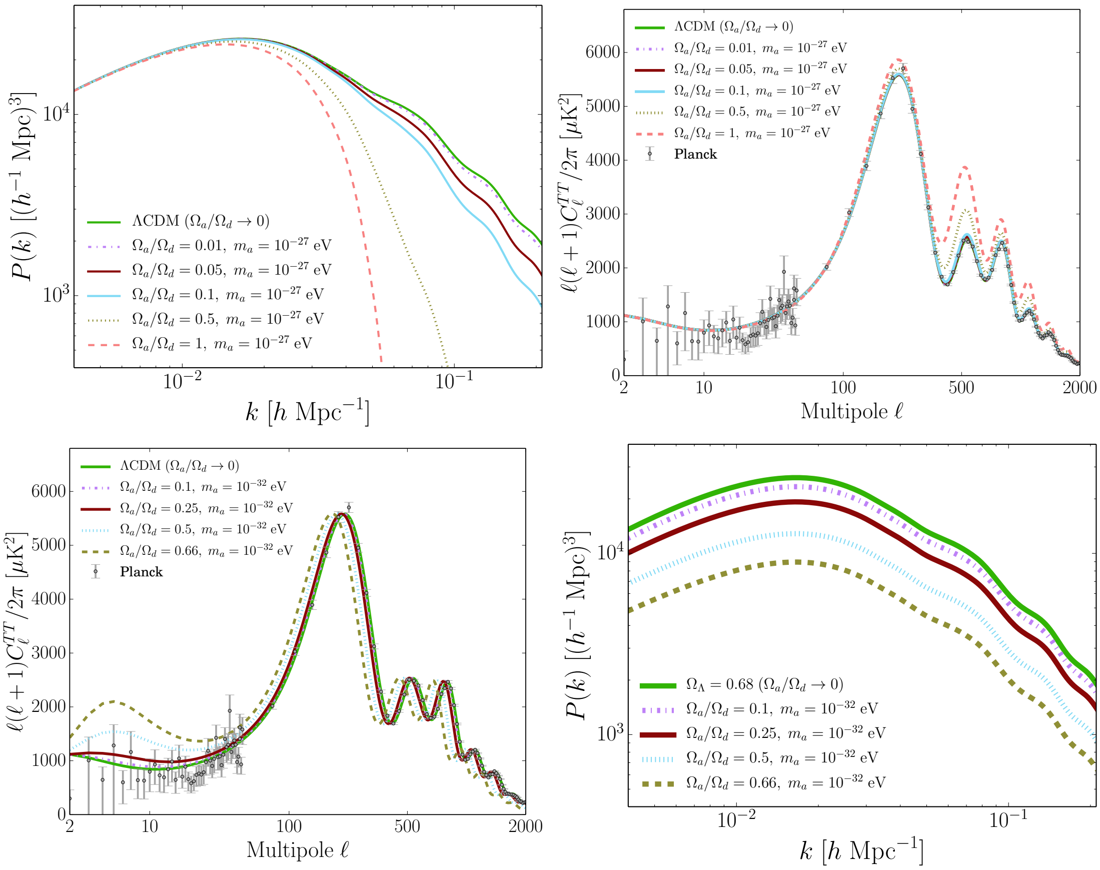

Figure 2: Top left panel shows total theoretical matter power-spectra when $m_a = 10^{-27}$ eV ULAs replace the indicated fraction of matter (CDM). The fraction is normalized as $\Omega_a/\Omega_d$, where $\Omega_d = \Omega_a + \Omega_c$, with $\Omega_c$ denoting the fractional density of ordinary CDM relative to the critical density. Top right panel shows theoretical CMB TT power spectra in the same ULA regime, along with *Planck* measurements of the TT power spectrum. Bottom left panel shows theoretical CMB TT power spectra when $m_a = 10^{-32}$ eV, deep into the "dark-energy like" ULA mass range. ULAs replace the indicated fraction of $\Omega_d$. Bottom right panel shows theoretical matter power-spectra over the same parameter range. Reproduced (with permission) from Ref. [13]. Copyright 2015 by The American Physical Society.

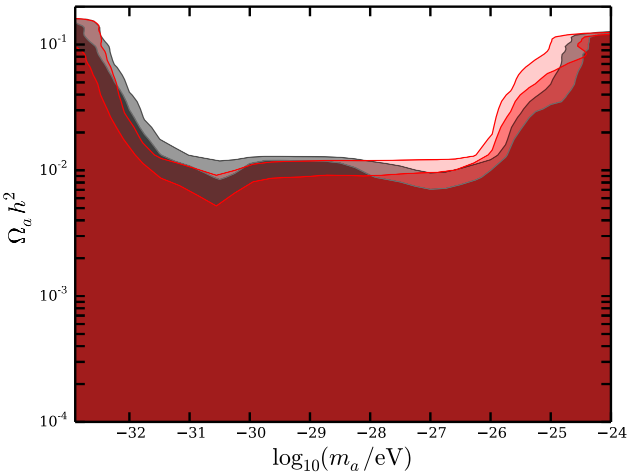

Figure 3: Marginalized 2 and $3\sigma$ contours in the $m_a - \Omega_a h^2$ plane for both the CMB-only and CMB+WiggleZ (large-scale structure survey) combinations of data sets. We obtain constraints of $\Omega_a h^2 \leq 0.006$ at 95% confidence level over some seven orders of magnitude in ULA mass $m_a$. Reproduced (with permission) from Ref. [13]. Copyright 2015 by The American Physical Society. Theoretical curves are compared here with TT power spectra from the *Planck* 2013 data release [21].

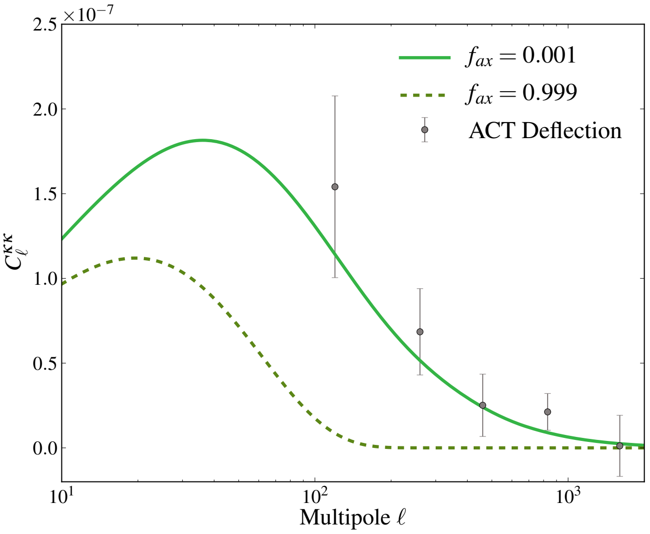

Figure 4: Dimensionless CMB-lensing deflection angle power-spectrum in ULA models [for $m_a = 10^{-28}$ eV and varying axion mass fraction $f_{ax} = \Omega_a/(\Omega_a + \Omega_c)$] compared with ACT data.