The Rethermalizing Bose-Einstein Condensate of Dark Matter Axions

Nilanjan Banik, Adam Christopherson, Pierre Sikivie, Elisa Maria Todarello
University of Florida, Gainesville, FL 32611, USA

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The axions produced during the QCD phase transition by vacuum realignment, string decay and domain wall decay thermalize as a result of their gravitational self-interactions when the photon temperature is approximately 500 eV. They then form a Bose-Einstein condensate (BEC). Because the axion BEC rethermalizes on time scales shorter than the age of the universe, it has properties that distinguish it from other forms of cold dark matter. The observational evidence for caustic rings of dark matter in galactic halos is explained if the dark matter is axions, at least in part, but not if the dark matter is entirely WIMPs or sterile neutrinos.

1 Axion dark matter

The story we tell applies to any scalar or pseudo-scalar dark matter produced in the early universe by vacuum realignment and/or the related processes of string and domain wall decay. However, the best motivated particle with those properties is the QCD axion since it is not only a cold dark matter candidate but also solves the strong CP problem of the standard model of elementary particles [1, 2]. So, for the sake of definiteness, we discuss the specific case of the QCD axion.

The Lagrangian density for the axion field $\phi(x)$ may be written as

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + ... \tag{1}$$

where the dots represent interactions of the axion with the known particles. The properties of the axion are mainly determined by one parameter f with dimension of energy, called the 'axion decay constant'. In particular the axion mass is

$$m \simeq \frac{f_\pi m_\pi}{f}\frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 6\cdot 10^{-6}\text{eV}\ \frac{10^{12}\ \text{GeV}}{f} \tag{2}$$

in terms of the pion decay constant f_π, the pion mass m_π and the masses m_u and m_d of the up and down quarks, and the axion self-coupling is

$$\lambda \simeq \frac{m^2}{f^2}\frac{m_d^3 + m_u^3}{(m_u + m_d)^3} \simeq 0.35\frac{m^2}{f^2} \quad . \tag{3}$$

All couplings of the axion are inversely proportional to f. When the axion was first proposed, f was thought to be of order the electroweak scale, but its value is in fact arbitrary [3]. However the combined limits from unsuccessful searches for the axion in particle and nuclear physics experiments and from stellar evolution imply $f \gtrsim 3 \cdot 10^9$ GeV [4].

An upper limit $f \lesssim 10^{12}$ GeV is obtained from the requirement that axions are not overproduced in the early universe by the vacuum realignment mechanism [5], which may be briefly described as follows. The non-perturbative QCD effects that give the axion its mass turn on at a temperature of order 1 GeV. The critical time, defined by $m(t_1)t_1 = 1$, is $t_1 \simeq 2 \cdot 10^{-7}$ sec$(f/10^{12}\text{ GeV})^{\frac{1}{3}}$. Before t_1, the axion field ϕ has magnitude of order f. After t_1, ϕ oscillates with decreasing amplitude, consistent with axion number conservation. The number density of axions produced by vacuum realignment is

$$n(t) \sim \frac{f^2}{t_1}\left(\frac{a(t_1)}{a(t)}\right)^3 = \frac{4 \cdot 10^{47}}{\text{cm}^3}\left(\frac{f}{10^{12}\text{ GeV}}\right)^{\frac{5}{3}}\left(\frac{a(t_1)}{a(t)}\right)^3 \quad , \tag{4}$$

where $a(t)$ is the cosmological scale factor. Their contribution to the energy density today equals the observed density of cold dark matter when the axion mass is of order 10^{-5} eV, with large uncertainties. The axions produced by vacuum realignment are a form of cold dark matter because they are non-relativistic soon after their production at time t_1. Indeed their typical momenta at time t_1 are of order $1/t_1$, and vary as $1/a(t)$, so that their velocity dispersion is

$$\delta v(t) \sim \frac{1}{mt_1}\frac{a(t_1)}{a(t)} \quad . \tag{5}$$

The average quantum state occupation number of the cold axions is therefore

$$\mathcal{N} \sim \frac{(2\pi)^3\ n(t)}{\frac{4\pi}{3}(m\delta v(t))^3} \sim 10^{61}\left(\frac{f}{10^{12}\text{ GeV}}\right)^{\frac{8}{3}} \quad . \tag{6}$$

$\mathcal{N}$ is time-independent, in agreement with Liouville's theorem. Considering that the axions are highly degenerate, it is natural to ask whether they form a Bose-Einstein condensate [6, 7]. We discuss the process of Bose-Einstein condensation and its implications in the next section.

The thermalization and Bose-Einstein condensation of cold dark matter axions is also discussed in Refs. [8, 9, 10, 11] with conclusions that do not necessarily coincide with ours in all respects.

2 Bose-Einstein condensation

Bose-Einstein condensation occurs in a fluid made up of a huge number of particles if four conditions are satisfied: 1) the particles are identical bosons, 2) their number is conserved, 3) they are highly degenerate, i.e. $\mathcal{N}$ is much larger than one, and 4) they are in thermal equilibrium. Axion number is effectively conserved because all axion number changing processes, such as axion decay to two photons, occur on time scales vastly longer than the age of the universe. So the axions produced by vacuum realignment clearly satisfy the first three conditions. The fourth condition is not obviously satisfied since the axion is very weakly coupled. In contrast, for Bose-Einstein condensation in atoms, the fourth condition is readily satisfied whereas the third is hard to achieve. The fourth condition is a matter of time scales. Consider a fluid that satisfies the first three conditions and has a finite, albeit perhaps very long, thermal relaxation time scale τ. Then, on time scales short compared to τ and length scales large compared to a certain Jeans' length (see below) the fluid behaves like cold dark matter (CDM), but on time scales large compared to τ, the fluid behaves differently from CDM.

Indeed, on time scales short compared to τ, the fluid behaves as a classical scalar field since it is highly degenerate. In the non-relativistic limit, appropriate for axions, a classical scalar field is mapped onto a wavefunction ψ by

$$\phi(\vec{r},t) = \sqrt{2}Re[e^{-imt}\psi(\vec{r},t)] \quad . \tag{7}$$

The field equation for $\phi(x)$ implies the Schrödinger-Gross-Pitaevskii equation for ψ

$$i\partial_t\psi = -\frac{1}{2m}\nabla^2\psi + V(\vec{r},t)\psi \quad , \tag{8}$$

where the potential energy is determined by the fluid itself:

$$V(\vec{r},t) = m\Phi(\vec{r},t) - \frac{\lambda}{8m^2}|\psi(\vec{r},t)|^2 \quad . \tag{9}$$

The first term is due to the fluid's gravitational self-interactions. The gravitational potential $\Phi(\vec{r},t)$ solves the Poisson equation:

$$\nabla^2\Phi = 4\pi Gmn \quad , \tag{10}$$

where $n = |\psi|^2$. The fluid described by ψ has density n and velocity $\vec{v} = \frac{1}{m}\vec{\nabla}\arg(\psi)$. Eq. (8) implies that n and $\vec{v}$ satisfy the continuity equation and the Euler-like equation

$$\partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{m}\vec{\nabla}V - \vec{\nabla}q \quad , \tag{11}$$

where

$$q = -\frac{1}{2m^2}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} \quad . \tag{12}$$

q is commonly referred to as 'quantum pressure'. The $\vec{\nabla}q$ term in Eq. (11) is a consequence of the Heisenberg uncertainty principle and accounts, for example, for the intrinsic tendency of a wavepacket to spread. It implies a Jeans length [12]

$$\ell_J = (16\pi G\rho m^2)^{\frac{-1}{4}} = 1.01\cdot 10^{14}\ \mathrm{cm}\left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}}\left(\frac{10^{-29}\ \mathrm{gr/cm^3}}{\rho}\right)^{\frac{1}{4}} \quad . \tag{13}$$

where $\rho = nm$ is the energy density. On distance scales large compared to ℓ_J, quantum pressure is negligible. CDM satisfies the continuity equation, the Poisson equation, and Eq. (11) without the quantum pressure term. So, on distance scales large compared to ℓ_J and time scales short compared to τ, a degenerate non-relativistic fluid of bosons satisfies the same equations as CDM and hence behaves as CDM. The wavefunction describing density perturbations in the linear regime is given in Ref. [13].

On time scales large compared to τ, the fluid of degenerate bosons does not behave like CDM since it thermalizes and forms a BEC. Most of the particles go to the lowest energy state available to them through their thermalizing interactions. This behavior is not described by classical field theory and is different from that of CDM. When thermalizing, classical fields suffer from an ultraviolet catastrophe because the state of highest entropy is one in which each field mode has average energy $k_B T$, where T is temperature. In contrast, for the quantum field, the average energy of each mode is given by the Bose-Einstein distribution, and the ultraviolet catastrophe is removed. To see whether Bose-Einstein condensation is relevant to axions one must estimate the relaxation rate $\Gamma \equiv \frac{1}{\tau}$ of the axion fluid. We do this in the next section.

When the mass is of order 10^{-21} eV or smaller, the Jeans length is long enough to affect structure formation in an observable way [14]. Because we are focussed on the properties of QCD axions, we do not consider this interesting possibility here.

3 Thermalization rate

It is convenient to introduce a cubic box of size L with periodic boundary conditions. In the non-relativistic limit, the Hamiltonian for the axion fluid in such a box has the form

$$H = \sum_j \omega_j a_j^\dagger a_j + \sum_{j,k,l,m} \frac{1}{4} \Lambda_{jk}^{lm} a_j^\dagger a_k^\dagger a_l a_m \quad . \tag{14}$$

with the oscillator label j being the allowed particle momenta in the box $\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$, with n_x, n_y and n_z integers, and the Λ_{jk}^{lm} given by [7]

$$\Lambda_{\vec{p}_1,\vec{p}_2}^{\vec{p}_3,\vec{p}_4} = \Lambda_{s\ \vec{p}_1,\vec{p}_2}^{\ \vec{p}_3,\vec{p}_4} + \Lambda_{g\ \vec{p}_1,\vec{p}_2}^{\ \vec{p}_3,\vec{p}_4} \quad . \tag{15}$$

where the first term

$$\Lambda_{s\ \vec{p}_1,\vec{p}_2}^{\ \vec{p}_3,\vec{p}_4} = -\frac{\lambda}{4m^2 L^3}\ \delta_{\vec{p}_1+\vec{p}_2,\vec{p}_3+\vec{p}_4} \tag{16}$$

is due to the $\lambda\phi^4$ self-interactions, and the second term

$$\Lambda_{g\ \vec{p}_1,\vec{p}_2}^{\ \vec{p}_3,\vec{p}_4} = -\frac{4\pi G m^2}{L^3}\delta_{\vec{p}_1+\vec{p}_2,\vec{p}_3+\vec{p}_4}\ \left(\frac{1}{|\vec{p}_1-\vec{p}_3|^2} + \frac{1}{|\vec{p}_1-\vec{p}_4|^2}\right) \tag{17}$$

is due to the gravitational self-interactions.

In the particle kinetic regime, defined by the condition that the relaxation rate $\Gamma \equiv \frac{1}{\tau}$ is small compared to the energy dispersion $\delta\omega$ of the oscillators, the Hamiltonian of Eq. (14) implies the evolution equation

$$\dot{\mathcal{N}}_l = \sum_{k,i,j=1} \frac{1}{2}|\Lambda_{ij}^{kl}|^2 \left[\mathcal{N}_i\mathcal{N}_j(\mathcal{N}_l+1)(\mathcal{N}_k+1) - \mathcal{N}_l\mathcal{N}_k(\mathcal{N}_i+1)(\mathcal{N}_j+1)\right] 2\pi\delta(\omega_i+\omega_j-\omega_k-\omega_l) \tag{18}$$

for the quantum state occupation number operators $\mathcal{N}_l(t) \equiv a_l^\dagger(t)a_l(t)$. The thermalization rate in the particle kinetic regime is obtained by carrying out the sums in Eq. (18) and estimating the time scale τ over which the $\mathcal{N}_j$ change completely. This yields [6, 7]

$$\Gamma \sim n\ \sigma\ \delta v\ \mathcal{N} \quad . \tag{19}$$

where σ is the scattering cross-section associated with the interaction, and $\mathcal{N}$ is the average state occupation number of those states that are highly occupied. The cross-section for scattering by $\lambda\phi^4$ self-interactions is $\sigma_\lambda = \frac{\lambda^2}{64\pi m^2}$. For gravitational self-interactions, one must take the cross-section for large angle scattering only, $\sigma_g \sim \frac{4G^2m^2}{(\delta v)^4}$, since forward scattering does not change the momentum distribution.

However, the axion fluid does not thermalize in the particle kinetic regime. It thermalizes in the opposite "condensed regime" defined by $\Gamma >> \delta\omega$. In the condensed regime, the relaxation rate due to $\lambda\phi^4$ self-interactions is [6, 7]

$$\Gamma_\lambda \sim \frac{n\lambda}{4m^2} \tag{20}$$

and that due to gravitational self-interactions is

$$\Gamma_g \sim 4\pi G n m^2 \ell^2 \tag{21}$$

where $\ell = \frac{1}{m\delta v}$ is, as before, the correlation length of the particles. One can show that the expressions for the relaxation rates in the condensed regime agree with those in the particle kinetic regime at the boundary $\delta\omega = \Gamma$.

We apply Eqs. (20) and (21) to the fluid of cold dark matter axions described at the end of Section 1. One finds that $\Gamma_\lambda(t)$ becomes of order the Hubble rate, and therefore the axions briefly thermalize as a result of their $\lambda\phi^4$ interactions, immediately after they are produced during the QCD phase transition. This brief period of thermalization has no known implications for observation. However, the axion fluid thermalizes again due to its gravitational self-interactions when the photon temperature is approximately [6, 7]

$$T_{\rm BEC} \sim 500 \text{ eV} \left(\frac{f}{10^{12} \text{ GeV}}\right)^{\frac{1}{2}} \quad . \tag{22}$$

The axion fluid forms a BEC then. After BEC formation, the correlation length ℓ increases till it is of order the horizon and thermalization occurs on ever shorter time scales relative to the age of the universe.

4 Observational consequences

As was emphasized in Section 3, the axion fluid behaves differently from CDM when it thermalizes. Indeed when all four conditions for Bose-Einstein condensation are fulfilled, almost all the axions go to their lowest energy available state. CDM does not do that. One can readily show that, in first order of perturbation theory and within the horizon, the axion fluid does not rethermalize and hence behaves like CDM. This is important because the cosmic microwave background observations provide very strong constraints in this arena and they are consistent with CDM. In second order of perturbation theory and higher, axions generally behave differently from CDM. The rethermalization of the axion BEC is sufficiently fast that axions that are about to fall into a galactic gravitational potential well go to their lowest energy state consistent with the total angular momentum they acquired from nearby protogalaxies through tidal torquing [7]. That state is a state of net overall rotation. In contrast, CDM falls into galactic gravitational potential wells with an irrotational velocity field. The inner caustics are different in the two cases. In the case of net overall rotation, the inner caustics are rings [15] whose cross-section is a section of the elliptic umbilic D_{-4} catastrophe [16], called caustic rings for short. If the velocity field of the infalling particles is irrotational, the inner caustics have a 'tent-like' structure which is described in detail in Ref. [17] and which is quite distinct from caustic rings. There is observational evidence for caustic rings [18]. It was shown [19] that the assumption that the dark matter is axions explains not only the existence of caustic rings but also their detailed properties, in particular the pattern of caustic ring radii and their overall size. Furthermore, it was shown [20] that axion BEC solves the galactic angular momentum problem, the tendency of CDM to produce halos that are too concentrated at the center compared to observations.

In a recent paper [21], J. Dumas et al. compare the predictions of the caustic ring model with the rotation curve of the Milky Way and the observations of the Sagittarius sattelite's tidal disruption.

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